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Analysis and synthesis of the walking linkage of Theo Jansen with a flywheel

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Abstract. This paper presents the results of cinematic and dynamic calculations of Theo Jansen's walking linkage on the worksheet of Mathcad. To validate the cinematic calculations, a video clip with simulation of the motion of Theo Jansen's mechanism is composed. The synthesis of a flywheel for Theo Jansen's linkage input link to decrease the fluctuation in its rotation is considered in detail.

Key words: theory of mechanisms and machines, Theo Jansen linkage, flywheel, Mathcad.

INTRODUCTION

Humans are mainly using wheeled vehicles for on the ground transportation. But if we look around in the nature, there is no biological creature moving on wheels. To move on the ground, living creatures use legs or crawl. Compared to wheel locomotion, walking has many advantages: lower energy consumption, no need for roads, better to cross over obstacles, the contact with ground is in a determined point, the ground is damaged less (Moldovan et al., 2011). Hence, the scientists are trying to design vehicles which are using legs or other locomotion ways that are inspired from nature.

A Dutch physicist and artist Theo Jansen started to use a twelve-bar mechanism for legs in his kinetic sculptures (Jansen, 2007). This mechanism is now called the 'Theo Jansen linkage' and it can be used for walking vehicles.

There are several studies for the Theo Jansen linkage. Komoda and Wagatsum (2011) proposed an extension mechanism for climbing over bumps. They demonstrated that lifting up of the linkage centre (pivot O_1, Fig. 1) will raise the leg's orbit upward and the combination of a cycle and up-down motion provides a new elliptic orbit for climbing over bumps with about 10 times the height of the original. In their later work, they propose to move the pivot O_1 (look Fig. 1) periodically, this will also ensure movement over bumps (Komoda & Wagatsum, 2012). Moldovan et al. (2011) showed a walking robot as a part of a mechatronic system and the design of the leg structure based upon the CAD design and forward kinematics. Giesbrecht et al. (2012) optimize the design of the Jansen linkage. The optimization is set up to minimize the energy input and maximize the stride length; they based the optimization on the dynamic of the force analysis.

In this paper, the Jansen linkage movement in a resisting environment is analysed. The fluctuating forces in the resisting environment and the reduced moment of inertia on the driving link are causing speed fluctuation. To reduce speed fluctuation, a method for calculating a flywheel to the linkages on a Mathcad worksheet is demonstrated. A virtual model of the Jansen mechanism is composed on a worksheet of Mathcad for validation of kinematic calculations. A video clip is composed from the virtual model.

MATERIAL AND METHODS

Description of the walking mechanism of Theo Jansen

The twelve bars of the linkage with measurements are OA = 250 mm, OO_1 = 690 mm, AB = 910 mm, BO_1 = 650 mm, CO_1 = 970 mm, BC = 790 mm, AE = 1,030 mm, EO_1 = 660 mm, ED = 790 mm, DF = 1,150 mm, EF = 670 mm and CD = 920 mm. There are the same measurements for rear links (Fig. 1). The measurements originate from the thesis of Ingram (2006).

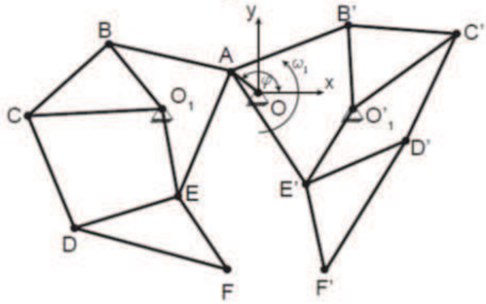

Figure 1. Theo Jansen mechanism.

The link OO_1 is fixed and the constructive angle between the x-axis and the link is $\gamma = 11.16$ degrees. In the calculations, the link OA has constant counter-clockwise angular velocity $\omega_1 = 30$ rad s^{-1}. The movement of the link OA will finally cause movement of the point F and F', these points are like the leg's feet. The loops of the points F and F' are triangular with these links alignments.

In the calculations, the link masses are found by $\mathrm{m} = 0.5 \cdot l$, where l is the length of the link. To determine the moment of inertia of the links, the following formula is used: $\mathrm{I} = \mathrm{m} \cdot l^2/10$. For the triangles BCO_1, DEF and $B'C'O'_1$, the D'E'F' moment of inertia is found by the Huygens-Steiner theorem (Lepik & Roots, 1971).

Determination of coordinates, velocities and accelerations analogues

The kinematic calculations were made on a worksheet of Mathcad; this method has been introduced in previous papers (Aan et al., 2012a; Aan & Heinloo, 2012).

On the basis of kinematic calculations, the simulation of linkage movement with pivot F and F' velocity and acceleration vectors, a video clip was composed on a Mathcad worksheet (Aan, 2014). A frame from the video clip is on Fig. 2.

Analysis of the loops of the pivots F and F'

If we look at Fig. 2, the loops of the pivots F and F' look triangular. On Fig. 3, the lower part of the pivot F loop is zoomed. From there one can see that the lower side of the triangle is not a straight line. By analysis of Fig. 3, the starting point, where the body weight of the pivot F is taken is set to $x_1 = -1.1498$ m, $y_1 = -1.3708$ m and the end point $x_2 = -0.2055$ m, $y_2 = -1.3708$ m. For the pivot F', the corresponding values are $x'_1 = 0.2055$ m, $y'_1 = -1.3708$ m and $x'_2 = 1.1498$ m, $y'_2 = -1.3708$ m. According to the value of the rotating angle φ, the pivot F is connected with the ground when $\varphi = \varphi_1 = 252$ degrees and leaves the ground when $\varphi = \varphi_2 = 132$ degrees. Pivot F' is connected with the ground when $\varphi = \varphi'_1 = 48$ degrees and leaves the ground when $\varphi = \varphi'_2 = 288$ degrees.

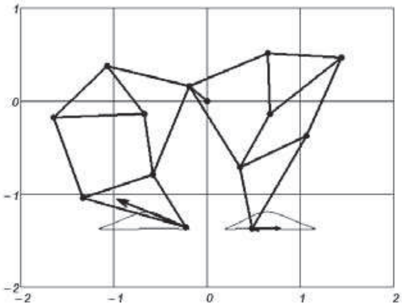

Figure 2. A frame from video clip (Aan, 2014).

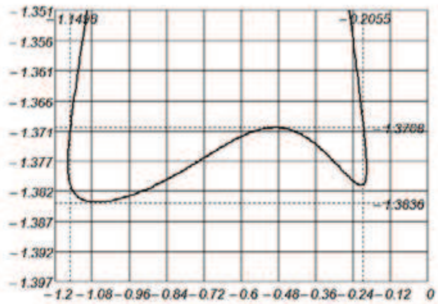

Figure 3. Zoomed lower part of the pivot F loop.

From Fig. 3, one can see that the leg (points F and F') is moving from right to left, it will leave the ground at the point (x_2, y_2) and touch it again at the point (x_1, y_1). In this study, the foot is moving in the environment (snow, water, mud, etc.) between these points, where resisting forces are applied to it. The resisting force projection on the x axis is $F_{tx} = 250$ N and on the y axis $F_{ty} = 150$ N, same for the rear leg. According to this analysis, the dependency of the resisting force on the rotation angle φ and from points (x_1, y_1) and (x_2, y_2) is defined by the following program on a worksheet of Mathcad (Fig. 4).

$$F_t(\varphi) = \left| \begin{array}{l} F_t \leftarrow F_t \quad if\ \varphi_2 \le \varphi \le \varphi_1 \\ 0 \quad otherwise \end{array} \right. \qquad F'_t(\alpha) = \left| \begin{array}{l} F_t \leftarrow F_t \quad if\ \varphi \le \varphi'_1 \vee \varphi \ge \varphi'_2 \\ 0 \quad otherwise \end{array} \right.$$

Figure 4. Programs for determination of resisting forces (left point F, right point F'), where F_t – magnitude resisting force.

Reduced resisting torque and moment of inertia

The resisting forces and moment of inertia will be reduced on the link OA. For the resisting forces, the following equation is used:

$$M_r \cdot \omega_1 = \sum_k (F_k \cdot v_k \cdot \cos \alpha_k) + \sum_u (M_u \cdot \omega_u), \tag{1}$$

where: M_r – magnitude of resisting torque on link one (OA in this case); F_k – magnitude of resisting force; v_k – magnitude of velocity; α_k – angle between force and velocity; M_u – pivot's resisting torque and ω_u – pivot angular velocity. In this case, the equation (3) is in the form of

$$M_r(\varphi) = F_t(\varphi) \cdot v_F(\varphi) \cdot \cos 180° + F'_t(\varphi) \cdot v'_F(\varphi) \cdot \cos 180°. \tag{2}$$

The kinetic energy of the linkage was determined by the well-known König theorem (Lepik & Roots, 1971). In order to use the König theorem, the following mechanism links parameters must be determined: moments of inertia, angular velocities, masses and centre of mass velocities; these values are found by the methods which are described above.

The reduced moment of inertia (Artobolevski, 1961) on the link OA can be presented in the form:

$$I_r(\varphi) = I_{OA} + m_{AB} \cdot v_{AB}(\varphi)^2 + I_{AB} \cdot \omega_{AB}(\varphi)^2 + I_{BCO1} \cdot \omega_{BCO1}(\varphi)^2 + m_{CD} \cdot v_{CD}(\varphi)^2 + I_{CD} \cdot \omega_{CD}(\varphi)^2 + I_{EO1} \cdot \omega_{EO1}(\varphi)^2 + m_{AE} \cdot v_{AE}(\varphi)^2 + I_{AE} \cdot \omega_{AE}(\varphi)^2 + m_{EDF} \cdot v_{EDF}(\varphi)^2 + I_{DEF} \cdot \omega_{DEF}(\varphi)^2 + m_{AB} \cdot v'_{AB}(\varphi)^2 + I_{AB} \cdot \omega'_{AB}(\varphi)^2 + I_{BCO1} \cdot \omega'_{BCO1}(\varphi)^2 + m_{CD} \cdot v'_{CD}(\varphi)^2 + I_{CD} \cdot \omega'_{CD}(\varphi)^2 + I_{EO1} \cdot \omega'_{EO1}(\varphi)^2 + m_{AE} \cdot v'_{AE}(\varphi)^2 + I_{AE} \cdot \omega'_{AE}(\varphi)^2 + m_{EDF} \cdot v'_{EDF}(\varphi)^2 + I_{DEF} \cdot \omega'_{DEF}(\varphi)^2, \quad (3)$$

where: I – moment of inertia; m – link mass; ω – link angular velocity; and v – link centre of mass velocity.

Reduction of the fluctuation of the input link's coefficient of speed with a flywheel

Input link OA angular velocity can be determined, in this case with reduced resisting torque (2) and moment of inertia (3), with the following equation (Artobolevski, 1961)

$$\omega(\varphi) = \sqrt{\frac{I_r(0) \cdot \omega_0^2}{I_r(\varphi)} + \frac{2}{I_r(\varphi)} \cdot \Delta T(\varphi)}, \quad (4)$$

where: $I_r(0)$ – value of reduced moment of inertia at $\varphi = 0$; ω_0 – unknown value of $\omega(\varphi)$ at $\varphi = 0$; and $\Delta T(\varphi)$ – change of the linkage kinetic energy. The exact formula for the determination of ω_0 cannot be derived. To find ω_0, one can use an approximate formula (Lepikson, 1998)

$$\omega_0 = \frac{n \cdot \omega_1}{\sum_{i=0}^{n} \sqrt{\frac{I_r(0)}{I_{ri}} + \frac{4 \cdot \Delta T_i}{I_{ri} \cdot \omega_1^2}}}, \quad (5)$$

where in this case n = 360, because the calculations are made with a step 1 degree, started at $\varphi = 0$ until $\varphi = 360$ degrees.

In equation (4), change of the linkage kinetic energy is determined with the following equation

$$\Delta T(\varphi) = M_m \cdot \varphi + \int_0^{\varphi} M_r(\varphi) d\varphi, \quad (6)$$

where M_m – constant driving torque of the link OA.

The results of equation (4) are shown on Fig. 5. Because of the reduced moment of inertia in equation (3) and the fluctuating resisting forces, angular velocity is also not constant.

The coefficient of speed fluctuation is defined as

$$\delta = \frac{\max(\omega(\varphi)) - \min(\omega(\varphi))}{\frac{\max(\omega(\varphi)) + \min(\omega(\varphi))}{2}} \quad (7)$$

Accordingly to equations (4) and (7), with these linkage parameters, the speed fluctuation is $\delta = 0.47$. When the fluctuation is ≥ 0.2, large variation must be allowed in the mechanism (Shigley et al., 2004). To decrease fluctuation, a flywheel must be added to the mechanism and equation (4) takes the form

$$\omega(\varphi) = \sqrt{\frac{\left(I_r(0)+I_f\right)\cdot\omega_0^2}{I_r(\varphi)+I_f} + \frac{2}{I_r(\varphi)+I_f}\cdot\Delta T(\varphi)}, \tag{8}$$

where I_f is the moment of inertia of the flywheel.

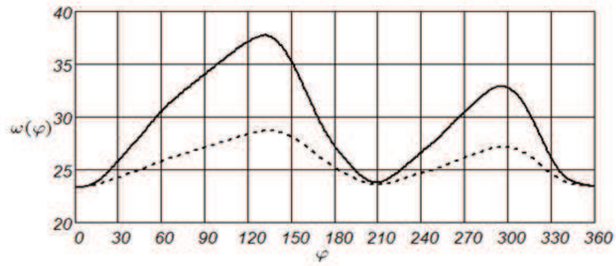

Figure 5. Link OA angular velocity according to formulas (6) and (10) $I_f = 1kg \cdot m^2$ (dotted line).

In consideration of equations (7) and (8), a program can be used on a worksheet of Mathcad to find fluctuation as a function of the flywheel's moment of inertia (Fig. 6).

$$U(I_f) = \left| \begin{array}{l} \text{for } n \in 0..360 \\ \quad \omega(\varphi)_n \leftarrow \sqrt{\dfrac{\left(I_r(0)+I_f\right)\cdot\omega_0{}^2}{I_{r_n}+I_h} + \dfrac{2}{I_{r_n}+I_f}\cdot\Delta T_n} \\ \delta \leftarrow \dfrac{max(\omega(\varphi)) - min(\omega(\varphi))}{\dfrac{max(\omega(\varphi))+min(\omega(\varphi))}{2}} \end{array} \right.$$

Figure 6. Program to find fluctuation as a function of the flywheel's moment of inertia.

The results according to the program on Fig. 6 are shown on Fig. 7. For example, if fluctuation must be $\delta \leq 0.1$, then according to Fig. 7, the flywheel's moment of inertia must be $I_f \geq 3 \ kg \cdot m^2$.

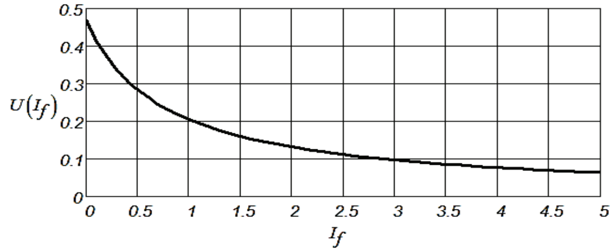

Figure 7. Fluctuation according to the flywheel's moment of inertia.

CONCLUSIONS

1. In this paper, the results of the calculations of Theo Jansen's mechanism are presented on a worksheet of the Computer Package Mathcad. According to the kinematic calculations, a video clip was composed on a Mathcad worksheet, where linkage motion is simulated together with the pivots' F and F' velocity and acceleration vectors.
2. The loops of the pivots F and F' are analysed and their movement under resisting forces determined.
3. The results of synthesis of a flywheel for the Theo Jansen's linkage are presented.

The method presented in this paper can be used in the teaching process of the engineering subject 'Mechanics of Machinery' and also by engineers in their everyday work.

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