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Theory of motion of a material point along a plane curve with a constant pressure and velocity

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Abstract. The theory of motion of a material point has been developed, as a result of which plane curves as orthogonal sections of cylindrical surfaces with horizontal generators that provide a constant force of pressure during motion of a particle along a curve at a constant velocity have been found. New differential equations of motion of a material point along a plane trajectory on the surface of the cylinder have been made. Visualisation of the obtained results has been performed. Individual cases of motion where the force of pressure on the surface was bigger, smaller or equal to the weight of the particles, and where reaction of the surface equalled zero have been found. The given theory can be successfully used for design of mouldboard surfaces of cultivator machines.
Keywords: engineering design, plane curve, arc length, natural parameter, pressure, velocity.

INTRODUCTION

The cultivator tools used today have a significant shortcoming – uneven wear during operation. This leads to additional untimely technological expenses on repair or even replacement of the tool. Uneven wear is caused by the variable force of pressure of the soil as a combination of material particles along the surface of the tool. The given problem can be solved by finding such surfaces during velocity on which the constant force of pressure of the soil on the surface of the tool will be ensured.

At the same time, cylindrical surfaces will be analysed, as they are widely used in agricultural machines as tools that interact with different technological materials.

Analysis of latest publications. Research of motion of material particles along a surface, including along a cylindrical one, has been performed by the academicians V.P. Goryachkin (1968), P.M. Vasilenko (1960) and P.M. Zaika (1992). Motion of a particle along an inner surface of an inclined stationary cylinder under the influence of the force of own weight was analysed in the paper (Linnik, 2006). Motion of particles along gravitational surfaces was studied in the papers (Pilipaka, 2003a; 2003b; 2003c; 2002). The paper (Pilipaka, 2009) focused on analysis of motion of a particle along an inner surface of a horizontal cylinder revolving round its axis, and in the paper (Pilipaka, 2010) – along an inner surface of an inclined cylinder. Similar cases of a refined theory of motion of a material point (particle) were presented in the papers (Adamchuk et al., 2008; Pilipaka et al.; 2009; Bulgakov, 2009).

Purpose of the paper. Finding surfaces of even wear by using study of plane curves of orthogonal sections of horizontal cylinders, along which the particle moves at a constant velocity and has a constant force of pressure on the surface.

MATERIALS AND METHODS

Results of the research

Motion of a particle along a plane curve – the orthogonal section of a cylindrical surface with horizontal generators, will now be analysed.

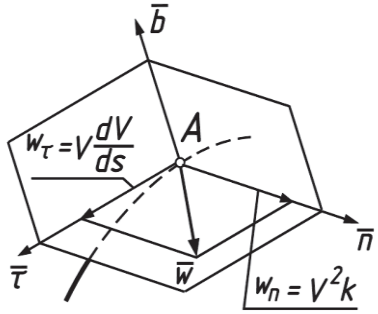


Figure 1. Tangential w_τ and centripetal w_n acceleration of the vertex A of a natural trihedral in projections on its unit vectors.

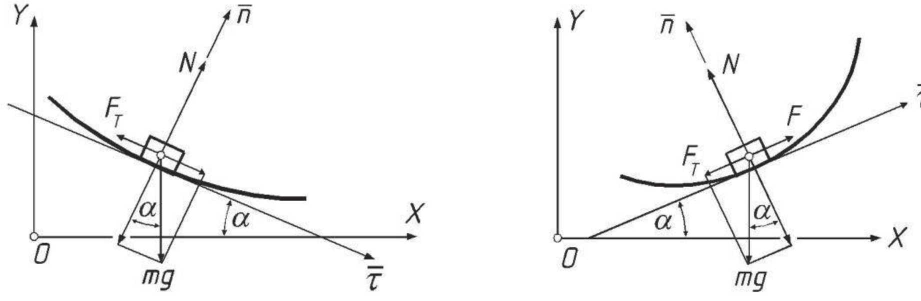
Motion of a point along a curve will always lead to emergence of acceleration w, even if the velocity V is constant. It consists of two components (Fig. 1): one component characterises speed of change of the value of the vector of velocity, projected on the unit vector of the tangent $\bar{\tau}$ and is called tangential acceleration. Its value is defined by differentiating the velocity V with respect to time t: $w_\tau = dV/dt$. The other component – normal or centripetal acceleration, characterises speed of change of the direction of the vector of velocity and it is projected on the unit vector of the main normal $\bar{n}$. Its value is designated by the product of the curvature by the velocity squared V: $w_n = V^2k$. Respectively, the following can be written in a vector form: $\bar{w} = \bar{\tau}\, dV/dt + \bar{n}V^2k$. In case the velocity V is the function of the length of arc s (of the path), i.e. $V = V(s)$, then the tangential acceleration will be: $w_\tau = dV/dt = dV/ds \cdot ds/dt = V \cdot dV/ds$. Respectively, the vector of acceleration in projection on unit vectors of a natural trihedral will be written as follows:

$$\bar{w} \quad \left\{ V\frac{dV}{ds};\; V^2k;\quad 0 \right\}. \tag{1}$$

According to resolution of the vector of acceleration into unit vectors of the natural trihedral (1) the differential equations of motion of the particle projected on unit vectors will be written as follows:

$$mV\frac{dV}{ds} = F_\tau; \qquad mV^2k = F_n, \tag{2}$$

where F_τ and F_n – are the projections of forces applied to the particle.



a b

Figure 2. Resolution of the forces having influence on the particle into unit vectors of the natural trihedral: a) the particle travels under the influence of the force of own weight; b) the particle travels under the influence of the applied force *F*.

It will now be assumed that the particle is moving along the curve under the influence of the force of its own weight *mg* (Fig. 2 a). In this case it will be resolved into unit vectors of the trihedral according to the angle α, which is a variable angle and which is the angle between two systems: the system of a moving trihedral and the fixed Cartesian system *OXY*. The particle is still under the influence of the force of reaction *N*, directed along the normal, and the force of friction $F_т$, directed along the unit vector of the tangent in the direction opposite to the direction of motion (Fig. 2 a). Given these forces the equations (2) will look as follows:

$$mV\frac{dV}{ds} = mg\sin\alpha - F_\tau; \qquad mV^2k = N - mg\cos\alpha. \tag{3}$$

As it is known, the force of friction $F_т$ is numerically equal to the product of the force of normal reaction *N* by the coefficient of friction *f*: $F_т = fN$. It follows from the second equation (3) that: $N = mV^2k + mg\cos\alpha$. Given these expressions the first equation (3) will look as follows:

$$mV\frac{dV}{ds} = mg\sin\alpha - f\left(mV^2k + mg\cos\alpha\right). \tag{4}$$

The differential equation (4) can be reduced by the mass *m* of the particle. In order to solve the equation, it is necessary to specify the curve using the natural equation $k = k(s)$ and to search for the law of motion looking as $V = V(s)$, or to set the law of motion $V = V(s)$ and to search for the respective curve.

For example, the speed of motion of the particle will be constant (V = const). The respective line that will ensure such speed will now be found. According to (4) the following will be obtained:

$$g\sin\alpha - f\left(V^2k + g\cos\alpha\right) = 0\,. \tag{5}$$

The equation (5) has two solutions. The first one is the straight line $k = 0$. By solving (5) with $k = 0$, the following will be obtained: $f = \mathrm{tg}\alpha$, which means that the straight line must be inclined at an angle of friction to the horizon. According to the second solution $k \neq 0$, which means that the line will be a curved one. This solution will be analysed later.

It will now be assumed that the particle moves along the curve under the influence of the applied force F (Fig. 2 b). The equations (2) will look as follows:

$$mV\frac{dV}{ds} = -mg\sin\alpha - F_{\text{т}} + F; \qquad mV^2k = N - mg\cos\alpha\,. \tag{6}$$

The condition will now be set for the particle to travel upwards at a constant velocity $V = const$, while the force of reaction N (force of pressure) also remains constant. In practice the surface with the target section will wear equally, and in case of cultivator machines it will possibly be less prone to sticking. The second equation will now be transformed (6):

$$\frac{V^2}{g}k + \cos\alpha = \frac{N}{mg}\,. \tag{7}$$

The relation $N\,mg^{-1}$ has a constant value and it shows what part of the weight of the particle is represented by the force of pressure on the surface. It will now be designated using a_N and the equation (7) will be solved with respect to $k=d\alpha/ds$:

$$\frac{d\alpha}{ds} = \frac{g}{V^2}(a_{\mathrm{N}} - \cos\alpha)\,. \tag{8}$$

After division of the variables integration of the equation (8) is possible for two cases: $a_N > 1$ (meaning that the force of pressure on the surface is bigger than the weight of the particle) and $a_N < 1$ (the force of pressure is smaller than the weight of the particle). These integrals will now be written down (the constant of integration will be left out):

$$\begin{aligned} s &= \frac{V^2}{g}\int\frac{d\alpha}{a_{\mathrm{N}} - \cos\alpha} = \frac{2V^2}{g\sqrt{a_{\mathrm{N}}^2 - 1}}\,\mathrm{arctg}\sqrt{\frac{a_{\mathrm{N}}+1}{a_{\mathrm{N}}-1}}\,\mathrm{tg}\frac{\alpha}{2}, && (a_{\mathrm{N}} > 1)\\ s &= \frac{V^2}{g}\int\frac{d\alpha}{a_{\mathrm{N}} - \cos\alpha} = \frac{V^2}{g\sqrt{1 - a_{\mathrm{N}}^2}}\ln\frac{(1+a_{\mathrm{N}})\mathrm{tg}\frac{\alpha}{2} - \sqrt{1-a_{\mathrm{N}}^2}}{(1+a_{\mathrm{N}})\mathrm{tg}\frac{\alpha}{2} + \sqrt{1-a_{\mathrm{N}}^2}}. && (a_{\mathrm{N}} < 1)\end{aligned} \tag{9}$$

In the equations (9) next step is transition from the relation $s = s(\alpha)$ to the natural equation $k = k(s)$. This can be done using two methods: either by changing in the equations (9) to the relations $\alpha = \alpha(s)$ and differentiating with respect to s, or by solving (9) jointly with (8), excluding the common parameter α. The natural equations for the first case look as follows:

$$k = \frac{g(a_N^2 - 1)}{V^2\left[a_N + \cos\left(\frac{gs}{V^2}\sqrt{a_N^2 - 1}\right)\right]}. \tag{10}$$

For the second case ($a_N < 1$):

$$k = \frac{g(1 - a_N^2)}{V^2\left(-a_N + \cosh\frac{g\sqrt{1 - a_N^2}}{V^2}s\right)}. \tag{11}$$

The natural relations (10) and (11) set curves irrespective of their position and turn on the plane. In order to build them it is necessary to change to the coordinate form of writing, for example to parametric equations. The required position of the curves on the plane according to the acting forces is chosen when the change is performed, by setting initial conditions (by assigning the required values to the constants of integration). In order to perform the change, the known relations (Pilipaka, 2002b) will be used, and change to the independent variable α will be performed:

$$\frac{dx}{d\alpha}\frac{d\alpha}{ds} = \cos\alpha, \qquad \text{and} \qquad \frac{dx}{d\alpha} = \cos\alpha \div \frac{d\alpha}{ds}. \tag{12}$$

Similarly:

$$\frac{dy}{d\alpha} = \sin\alpha \div \frac{d\alpha}{ds}. \tag{13}$$

Having inserted into (12) and (13) the expression $d\alpha/ds$ from (8), the relations for finding x and y of the curve will be obtained:

$$\begin{aligned} x &= \frac{V^2}{g}\int\frac{\cos\alpha d\alpha}{a_N - \cos\alpha} = \frac{a_N V^2}{g}\int\frac{d\alpha}{a_N - \cos\alpha} - \frac{V^2}{g}\alpha; \\ y &= \frac{V^2}{g}\int\frac{\sin\alpha d\alpha}{a_N - \cos\alpha} = \frac{V^2}{g}\ln(a_N - \cos\alpha). \end{aligned} \tag{14}$$

It is evident from (14) that after integration the relation $y = y(\alpha)$ has a simple form, and the expression for the coordinate $x = x(\alpha)$ reduces to the integrals (9), and that's why it is split into two relations for $a_N > 1$ and $a_N < 1$:

$$x = \frac{2a_N V^2}{g\sqrt{a_N^2 - 1}}\operatorname{arctg}\sqrt{\frac{a_N + 1}{a_N - 1}}\operatorname{tg}\frac{\alpha}{2} - \frac{V^2}{g}\alpha; \qquad (a_N > 1) \tag{15}$$

$$x = \frac{a_N V^2}{g\sqrt{1-a_N^2}} \ln \frac{(1+a_N)\mathrm{tg}\frac{\alpha}{2} - \sqrt{1-a_N^2}}{(1+a_N)\mathrm{tg}\frac{\alpha}{2} + \sqrt{1-a_N^2}} - \frac{V^2}{g}\alpha. \qquad (a_N < 1) \qquad (16)$$

Respectively, it is possible to write the parametric equations of the curve, along which the particle travels at a constant velocity and has a constant force of pressure on the surface, for two cases. In the first case the constant $a_N > 1$, meaning that the force of pressure is bigger than the force of own weight of the particle:

$$\begin{aligned} x &= \frac{2a_N V^2}{g\sqrt{a_N^2-1}} \mathrm{arctg}\sqrt{\frac{a_N+1}{a_N-1}}\mathrm{tg}\frac{\alpha}{2} - \frac{V^2}{g}\alpha; \\ y &= \frac{V^2}{g}\ln(a_N - \cos\alpha). \qquad (a_N > 1) \end{aligned} \qquad (17)$$

In the second case the constant $a_N < 1$, meaning that the force of pressure is smaller than the force of own weight of the particle:

$$\begin{aligned} x &= \frac{a_N V^2}{g\sqrt{1-a_N^2}} \ln\left[\frac{(1+a_N)\mathrm{tg}\frac{\alpha}{2} - \sqrt{1-a_N^2}}{(1+a_N)\mathrm{tg}\frac{\alpha}{2} + \sqrt{1-a_N^2}}\right] - \frac{V^2}{g}\alpha; \\ y &= \frac{V^2}{g}\ln(a_N - \cos\alpha). \qquad (a_N < 1) \end{aligned} \qquad (18)$$

By using the expression of the length of the arc (9), it is possible to change to the parametric equations in the function of the natural parameter s, at the same time excluding the angle α. For example, by jointly solving (17) and the first equation (9), after exclusion of α the following will be obtained:

$$\begin{aligned} x &= a_N s - \frac{2V^2}{g}\mathrm{arctg}\left[\sqrt{\frac{a_N-1}{a_N+1}}\mathrm{tg}\left(\frac{gs}{2V^2}\sqrt{a_N^2-1}\right)\right]; \\ y &= \frac{V^2}{g}\ln\left[\frac{a_N^2-1}{a_N+\cos\left(\frac{gs}{V^2}\sqrt{a_N^2-1}\right)}\right]. \qquad (a_N > 1) \end{aligned} \qquad (19)$$

Using the same approach the parametric equations for the curve in case of $a_N < 1$, will be obtained, by jointly solving (18) and the second equation (9):

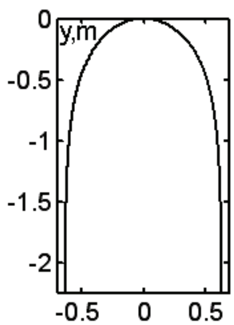
$$x = -a_{\mathrm{N}} s + \frac{2V^2}{g} \operatorname{arctg}\left[\sqrt{\frac{1 + a_{\mathrm{N}}}{1 - a_{\mathrm{N}}}} \tanh\left(\frac{gs}{2V^2}\sqrt{1 - a_{\mathrm{N}}^2}\right)\right];$$

$$y = \frac{V^2}{g} \ln\left[\frac{1 - a_{\mathrm{N}}^2}{-a_{\mathrm{N}} + \cosh\left(\frac{gs}{V^2}\sqrt{1 - a_{\mathrm{N}}^2}\right)}\right]. \qquad (a_{\mathrm{N}} < 1) \tag{20}$$

In order to understand the essence of separation of curves by cases of $a_N > 1$ and $a_N < 1$, the analysis will start with the second case where $a_N = 0$ (this value corresponds to the second case and in a certain way divides the respective set of curves into two subsets). In case of $a_N = 0$ the natural equation (11) is significantly simplified and looks as follows:

$$k = \frac{g}{V^2} \operatorname{sec\,h}\left(\frac{g}{V^2} s\right). \tag{21}$$

In a scientific literature the curve described with the natural equation (21), is known under the name of centenary line of equal resistance. The relation $g\,V^{-2}$ serves as the constant value in the equation (21) of this line. In order to construct it the parametric equations (18) will be used, that are also significantly simplified in case of $a_N = 0$. The curves are presented in Fig. 3.



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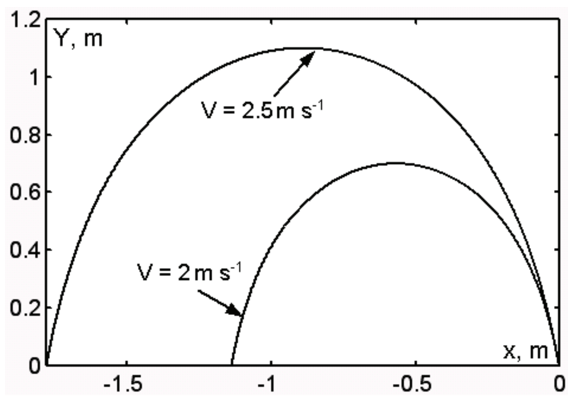


b

Figure 3. The curve having the natural equation (21) and described by the parametric equations (18) and (20) in case of $a_N = 0$: a) $V = 2$ m s^{-1}, $\alpha = -90^0 \ldots 90°$; б) $\alpha = -80° \ldots 80°$.

The equation $a_N = 0$ means that the reaction of the surface equals zero, which means that such curve is a trajectory of flight of the particle (point) at a constant velocity of motion without taking the air resistance into account. For example, the lower areas of the trajectory of motion (Fig. 3 a) almost constitute vertical lines, that's why the particle will not exert any pressure on the respective surface. On the other part of the curve the component of the force of weight is balanced by the component of the centrifugal force.

From the course of theoretical mechanics it is known that a body thrown at an angle to the horizon travels along a parabolic curve, however in such case the velocity of motion is a variable. Fig. 3 b demonstrates the built trajectories of the particle with different constant velocity of motion with the initial angle of ascent of 80° (that's why in order to build a curve in this case it is more convenient to use the equations (18), and not (20), as the limits of change of the variable α are known at once). The curves look like a parabola, and their insignificant differences from it are caused by the fact that the particle travels at a constant velocity V. In order to ensure such flight it is necessary to maintain a constant velocity, which is ensured by the force F, acting tangentially (Fig. 2 b). If necessary, it can be found from the first equation (6). Due to the fact that the velocity of motion is constant and the force of friction is absent, the equation will look as follows:

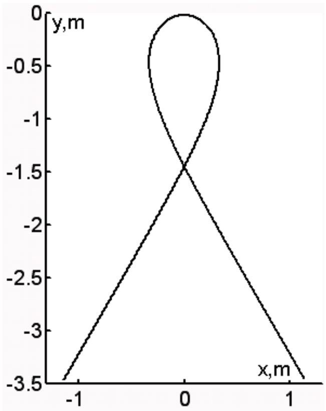
$$F = mg \sin \alpha . \tag{22}$$

Respectively, in the lower part of the trajectory, when the angle α is almost equal to 90° (Fig. 3a), the force F is equal to the weight of the particle, which means that it almost overcomes the force of gravity during ascent. As the particle ascends, the force is reduced and becomes equal to zero at the top point of the trajectory (with $\alpha = 0$), and then, according to the same law it is increased, having changed the sign, i.e. slowing down the fall. That's why the centenary line of equal resistance can be considered as a prototype of a parabola with respect to the trajectory of free flight of a body in the field of gravity of the Earth, with the only difference being that the body travels along a parabolic curve only under an influence of the force of own weight, and along a centenary line of equal resistance it travels with an additional force that ensures constant speed of velocity.

If a goal is set to make the particle having a constant velocity of motion of 2.5 m s^{-1} travelling not along the upper trajectory (Fig. 3 b), but along the lower one, it is necessary to make a limiting cover in a shape of a cylindrical surface with the section corresponding to the lower curve. In this case the force of reaction N of the surface emerges, which in case of a respective selection of the curve can also be constant. At the same time it can be bigger than the weight of the particle (body), or smaller. The equations of the respective curves have been found. The following question arises: what line corresponds to the coefficient $a_N = 1$, i.e. to the case where the force of reaction is equal to the force of weight of the particle? Obviously this will be a horizontal straight line. The same result can be obtained from the natural equation (11) where $a_N = 1$, respectively obtaining: $k = 0$.

The group of curves for which the force of reaction is smaller than the weight of the particle will now be analysed. Obviously, in such case $0 < a_N < 1$. The curves for various values of a_N from this interval are built in Fig. 4.

All three curves have a loop. During motion of the particle along the curve within the range of the loop it is located inside of it. Such motion will be called the motion along the inner side of the surface. As the force of reaction of the surface increases from zero to the value close to one, the curve is transformed, but the loop does not disappear. The branches of the curve going into infinity approach the straight line. In particular, in case of $a_N = 0.5$ (Fig. 4 b) the branch is inclined to horizon at an angle equal approximately to 60^0, which corresponds to position of the particle on the plane inclined at the same angle. In case of a_N close to one (Fig. 4 c), in infinity the branches approach the horizontal line.



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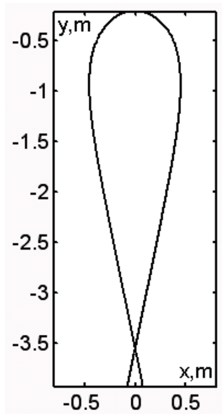

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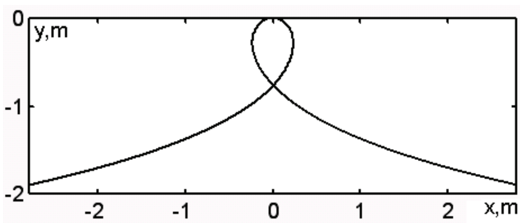
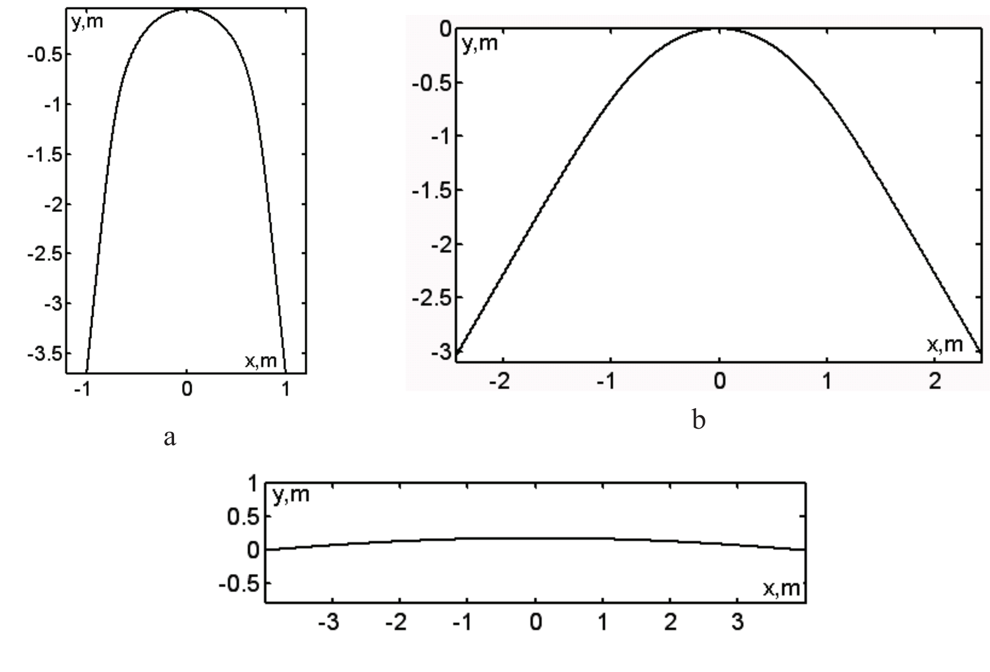


c

Figure 4. Curves built according to the parametric equations (20) in case of $V = 2\ \mathrm{m\ s^{-1}}$ and various values of a_N: a) $a_N = 0.2$; b) $a_N = 0.5$; c) $a_N = 0.995$.

The analysed curves constitute the first subset, as there is also another subgroup of curves built with the same values of the coefficient a_N, but with the 'minus' sign. At the same time the condition $a_N < 1$ is not breached, and that's why these two subsets are united into a group of curves, in case of motion of the particle along which at a constant velocity the emerging force of reaction is smaller than the force of weight of the particle. These curves are presented in Fig. 5.

The given curves don't have loops. During motion along the curve the particle is located above it at all time. Such motion will be called the motion along the outer side of the plane. It must be noted that if the values of a_N are equal in absolute magnitude (i.e. in case of equal force of pressure) the curves of the first and the second subsets have branches going into infinity and having the same inclination (Figs 4 b and 5 b). In case of motion of the particle along the inner side of the surface in the first case, or along the outer side of the surface in the second case, the particle on the branch going into infinity has the same position – above the curve. As the force of reaction of the surface increases from zero to the value equal to one, the curve of this subset is transformed, smoothly approaching the horizontal plane.



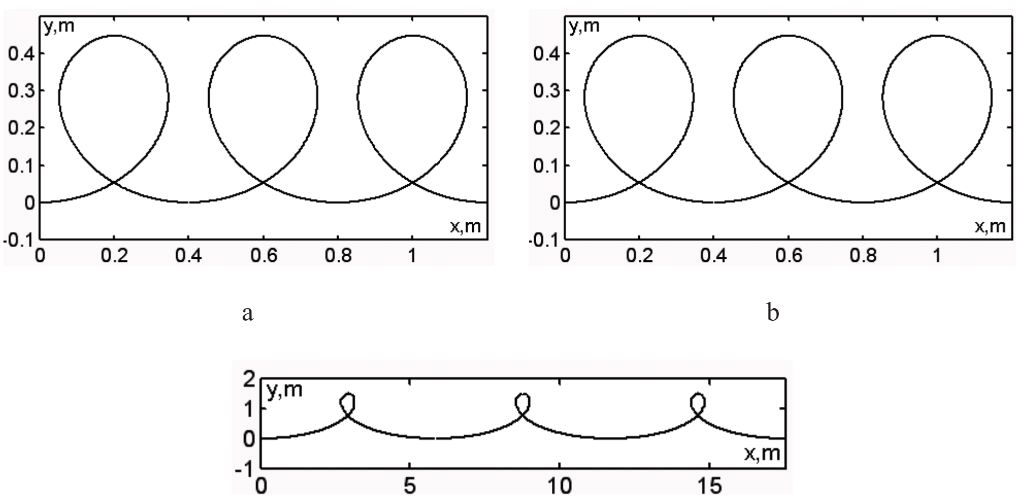
a

b

c

Figure 5. Curves built according to the parametric equations (20) in case of $V = 2$ m s^{-1} and various values of a_N: a) $a_N = -0.1$; b) $a_N = -0.5$; c) $a_N = -0.99$.

Now, the second group of the curves corresponding to the value $a_N > 1$ will be analysed. These curves, built according to the equations (14) or (19), are presented in Fig. 6.



a

b

c

Figure 6. Curves built according to the parametric equations (17) in case of $V = 2$ m s^{-1} and various values of a_N: a) $a_N = 2$; b) $a_N = 1.5$; c) $a_N = 1.05$.

For the presented curves only one-way motion of the particle, which, according to the accepted definition, corresponds to the motion along the inner side, is typical. The curves are periodical ones and have loops. In Fig. they are presented in different scales. Given this circumstance, it is quite apparent that as the coefficient a_N approaches one, the sizes of the curve increase and its shape changes: the period is significantly increased compared to the size of the loop. Smooth transition to the straight line when a_N approaches one is absent. Respectively, when the coefficient a_N approaches one, for two groups of curves only in one of the three cases smooth transition to the straight line is possible.

Influence of the velocity V of the motion of the particle on the shape of the curve is the same in all cases. Analysis of the parametric equations of the curves (17), (19) leads to a conclusion that V^2 plays a role of a scale coefficient. In case of the given value of the coefficient a_N, increase of the velocity of motion by two times leads to increase of the sizes of the curve by four times.

The following example will now be analysed. It will be assumed that a stunt motorcyclist has to make a loop in shape of a curve, presented in Fig. 6. Taking the mass of the motorcyclist with the motorcycle as the material particle, the task is to calculate the difference in height between the highest and lowest points of the curve at the velocity V = 100 km h^{-1} = 27.8 m s^{-1} and overload of 20% (a_N = 1.2).

The lowest point will be at α=0^0, and the highest point – at α = 180^0. According to the equation $y = y(\alpha)$ (14) the following will be obtained:

$$\Delta y = y_{\alpha=180} - y_{\alpha=0} = \frac{v^2}{g}\ln\frac{\alpha_N + 1}{\alpha_N - 1} = \frac{27.8^2}{9.81}\ln\frac{1.2+1}{1.2-1} = 189\,\text{m}. \tag{23}$$

CONCLUSIONS

The new theory of motion of a material point has been developed, and flat curves as orthogonal sections of cylindrical surfaces with horizontal generators that can provide a constant force of pressure (force of reaction) during motion of a particle along a curve at a constant velocity have been found. Such surface will wear evenly and in case of cultivator machines it will be not only less prone to sticking, but also to wear.

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