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Theory of the oscillations of a toothed disc opener during its movement across irregularities of the soil surface

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Abstract. The paper presents the main provisions of the new theory of oscillations of the versatile combined opener assembly of the breeding seed drill with a spring-suspended furrow opening toothed disc in the vertical longitudinal plane during its movement across irregularities of the soil surface. Basing on the improved design of the opener assembly, an equivalent schematic model has been developed, which takes into account the forces applied to the structural components of the opener, forces in the springs as well as the reaction of the soil acting on the toothed disc, the hoe-type seed conductor and the packing wheel. The system of differential equations has been set up, which describes the movement of the opener across irregularities of the soil surface depending on the opener's design parameters and the kinematic modes of performing the drilling work process. The derived mathematical model makes it possible to determine the amplitudes and frequencies of the translational oscillations of the device in order to assess their impact on the drilling work process. The developed theory provides also tools for the assessment and lowering of the energy characteristics of the versatile breeding seed drill related to the oscillating movements of its openers in soil.

Key words: seed drill, combined opener, toothed disc, oscillations, frequency.

INTRODUCTION

The present-day intensive energy-efficient technologies of production of cereals and other agricultural crops make it necessary to carry out agronomical research studies with the use of breeding seed drills. And these studies have to be done as under the standard breeding experiment conditions so dropping the breeding material with minimal tillage or even without any preparation of the soil before drilling. This makes the application of openers with various design features, capable of performing the drilling work process with proper quality under various conditions, one of the key issues (Chaudhuri, 2001; Hasimu & Chen, 2014; Lin et al., 2014).

The SS-16 (SN-16) tractor-mounted breeding grain drill with a mechanical seed-feeding unit is widely used today, but it has some disadvantages, the main of which is the underdevelopment of its opener assemblies, which is especially notable, when the use of the drill entails the considerable consumption of power.

Thus, the development of versatile improved openers (opener assemblies) for breeding seed drills (along with general purpose seed drills), the justification of their design and kinematic parameters with the aim of reducing the power consumption and improving the quality of operation are of great theoretical and practical importance (Jingling et al., 2011; Liu & Ma, 2013).

Double-disc, runner and hoe-type openers supplied as standard equipment with the most popular breeding grain drills are widely used for various cereal drilling technologies employed in breeding experiments (Vamerali et al., 2006; Karayel & Özmerzi, 2007; Altikat et al., 2013). While more and more publications have recently been appearing about the research into the direct drilling (including the pedigree seed drilling), specifically into soil mulched with plant residues (Karayel, 2009; Bai et al., 2014; Šarauskis & Vaitauskiene, 2014).

One of the ways to extend the area of application of seed drills, including breeding ones, is the utilisation of the capabilities of different types of opener groups delivering the high quality sowing of cereals under various tillage systems. The wide application of diverse opener design features, such as solid disc coulters, turbo coulters, coulters with wavy and serrated surfaces is stipulated by the quality of tillage (Chen et al., 2004; Karayel & Özmerzi, 2007; Bianchini & Magalhães, 2008; Lian et al., 2012).

Quite a number of literary sources, starting from the first publications (Turbin et al., 1967), have been concerned with the oscillations of opener assemblies during the movement of their discs in the soil. It is obvious that the vibration processes of interaction between the implements and the soil provide, first of all, reduction of the coefficient of internal friction between the soil particles, the draught resistance of the soil to the vibrating implement is considerably reduced (Endrerud, 1999). For example, the use of oscillating digging out implements on beet harvesters reduces the draught resistance during their movement by 26–53% on the average (Prisyazhnyuk et al., 2013).

Moreover, theoretical research has been made into the operation of individual mechanisms of opener assemblies, in particular, studies have been carried out to find out the variation of the penetration force exerted by the spring suspension mechanism, depending on the position of the operating area of the opener suspension arm (Belov & Belov, 2007). Within these studies, the pattern of movements, the scheme for determining the tooth setting angle and blade shape have been developed, mathematical models have been put forth for the estimation of the optimal parameters of opener groups (Lisoviy, 2013). Nevertheless, the mentioned studies leave out of account the energy component rising from the vibration processes during the work of openers.

Reduction of the energy consumption in the work process of operation of the breeding grain drill through the utilisation of the vibration effect caused by the interaction of implements with the soil, by the theoretical justification of the rational design and kinematic parameters of the toothed disc opener assemblies.

MATERIALS AND METHODS

The research has been carried out with the use of the analytical method of the generation of mathematical models of machines and process operations, which is based on the laws of theoretical mechanics and higher mathematics. The derived analytical dependencies can be solved on PCs with the use of the prepared computer programmes. We have developed the improved opener assembly with arm-and-spring suspension-mounted implements, the configuration of which is composed in accordance with the pedigree seed planting technology selected for the breeding grain drill.

The developed new opener assembly (Fig. 1) has furrow opening toothed disc 1 mounted freely on the axle at the end of rod fastened at the lower end of the bracket attached to spar 2, which is pivotally connected to draw-bar 3. Meanwhile, draw-bar 3 is also attached at its front part through a cylindrical hinge to the seed drill's square beam 4. Draw-bar 3 is also linked to the seed drill's frame via pressure spring 5. Behind furrow opening toothed disc 1 (in one plane with the disc) hoe-type seed conductor 6 is installed on spar 2. At the rear end of spar 2, packing wheel 7 is installed, while its forward end is equipped with spring assembly 8 producing vibration during the motion in soil. The depth of running in soil of the opener assembly's furrow opening toothed disc 1 is adjusted by changing the position of packing wheel 7 with the use of adjustment device 9. The teeth of disc 1 in its front part are pointing upwards, i.e. they are tilted against the direction, in which disc 1 rotates due to its engagement with the soil. This tilt ensures the release of the soil and plant residues stuck to the disc, when they arrive to the rear part of disc 1, and this release is also facilitated by cleaning device 10 in the plane of disc 1.

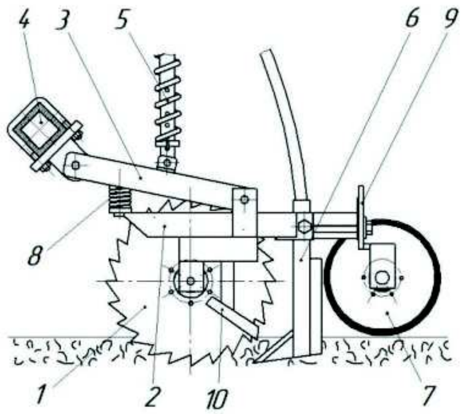

Figure 1. A scheme of combined opener assembly: 1 – tooted disc, 2 – spar, 3 – draw-bar, 4 – square beam, 5 – pressure spring, 6 – hoe-type seed conductor, 7 – packing wheel, – spring assembly, 9 – adjustment device, 10 – cleaning device.

Furrow opening toothed disc 1 has a distinctive feature, which is the special V-notches equally spaced along its circumference with one side of each notch aligned radially and the other side aligned at an angle to the radius and accordingly to the radial cutting edge of the notch (Magalhães et al., 2007).

For the analytical treatment of the operation of the improved design opener assembly we have to turn from its design and process schematic model to its equivalent schematic model. In the equivalent schematic model (Fig. 2) furrow opening toothed disc 10 is situated at the end of rod 9 and mounted freely rotating on the axle. Rod 9 is fastened to spar 8, mounted on draw-bar 5 with the use of a cylindrical hinge, while this draw-bar in its turn is hinge-mounted on draw-bar beam 1 of the breeding seed drill. The draw-bar is additionally connected with the breeding seed drill's frame through spring 3 and pressure rod pivot journal 2. Also, one end of spar 8 is connected with draw-bar 5 by spring 7, while hoe-type seed conductor 11 and packing wheel 12 are attached to its other end. The teeth of furrow opening toothed disc 10 are tilted contrary to the disc's sense of rotation (caused by its engagement with the soil), which speeds up the release of plant residues from the disc.

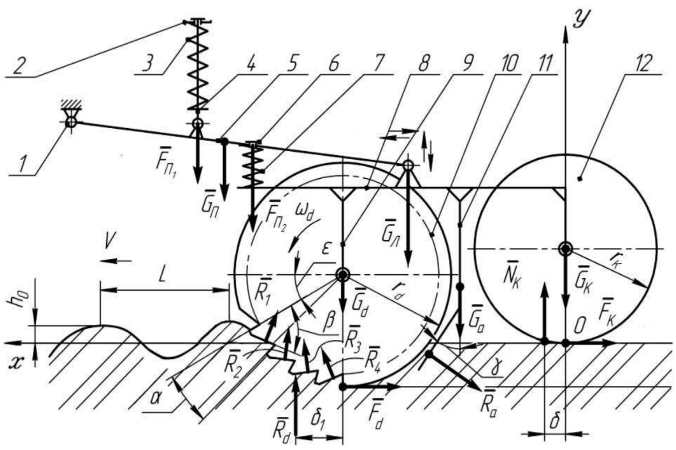

Figure 2. Theoretical schematic model of the improved design toothed disc opener assembly: 1 – breeding seed drill's draw-bar beam, 2 – pressure rod pivot journal, 3 – pressure spring, 4 – pressure rod, 5 – draw-bar, 6 – vibrator rod, 7 – vibrator spring, 8 – spar, 9 – rod, 10 – toothed disc, 11 – hoe-type seed conductor, 12 – packing wheel.

To generate the differential equations describing the translational oscillating movements of the toothed disc opener assembly during its movement across irregularities of the soil surface it is necessary first to analyse and assume the pattern of forces acting on the opener assembly in the process of its movement. For this purpose we will use the approach described in (Vasilenko, 1996). To generate an equivalent schematic model, it is necessary first to decide on the necessary assumptions (Prisyazhnyuk et al., 2013). Thus, the examined movement of the toothed disc opener assembly takes place only in the vertical longitudinal plane. The seed drill, where the opener assembly is mounted, moves uniformly along a straight line. The soil surface irregularities crossed by the toothed disc in its movement can be approximated by a harmonic function. The forces applied to the components of the examined system can be represented by point forces.

Basing on the design features of the improved toothed disc opener assembly as well as the above-said assumptions, the equivalent schematic model (combined with the process layout) has the representation shown in Fig. 2.

RESULTS

First of all, we have to designate in the equivalent schematic model the forces of weights of the main structural components in the improved design of the toothed disc opener assembly. They are:

$\bar{G}_{П}$ – force of weight of the draw-bar;

$\bar{G}_d$ – force of weight of the toothed disc;

$\bar{G}_л$ – force of weight of the spar;

$\bar{G}_a$ – force of weight of the hoe-type seed conductor;

$\bar{G}_к$ – force of weight of the packing wheel.

Accordingly, the masses of the listed structural components will be designated as $m_П$, m_d, $m_л$, m_a and $m_к$. Now, the tension forces in the first and second springs are designated in the equivalent schematic model: $\bar{F}_{П1}$ and $\bar{F}_{П2}$, respectively.

Apparently, these tension forces will have the following values:

$$\begin{aligned} F_{П1} &= C_{П1}\, l_{П1}, \\ F_{П2} &= C_{П2}\, y, \end{aligned} \tag{1}$$

where: $C_{П1}$, $C_{П2}$ – deflection rates of the first and second springs, respectively, (N m^{-1}); $l_{П1}$, y – deflections of these springs (m). Force $\bar{F}_{П1}$ can be considered at a first approximation as having a constant value.

It is obvious that the action of the forces of weights of the opener assembly structural components and the forces exerted by the springs results in the generation of support reactions by the soil, which act on the toothed disc, hoe-type seed conductor and packing wheel.

We assume that the profile line of the travel (irregularities of the soil surface) changes by the following sinusoidal law (Macmillan, 2002):

$$h(t) = h_0 \sin\left(\frac{2\pi V t}{L}\right), \tag{2}$$

where: V – constant translational velocity of the opener assembly (m s^{-1}); h_0 – maximum height of a soil surface irregularity (m); L – length of a soil surface irregularity (distance between two adjacent ridges) (m); t – current time (s).

We assume at a first approximation that the support reactions exerted by the soil on the teeth of the toothed disc during the movement of the opener assembly across irregularities of the soil surface change by the same sinusoidal law:

$$R_i(t) = R_0 + H \sin\left(\frac{2\pi V t}{L}\right), \quad i = 1, 2, 3, 4, \tag{3}$$

where: R_0 – reaction of soil during the movement of the toothed disc on the perfectly even surface of the soil (N); $H \sin\left(\frac{2\pi V t}{L}\right)$ – excitation component of the soil reaction caused by the irregularities of the soil surface; H – amplitude of this excitation (N).

Such an assumption for a first approximation can be made following the fact that the motion of the seed drill's carrying wheels across the irregularities of the soil surface varying under law (2) causes the self-induced oscillation of the seed drill's draw-bar beam 1, draw-bar 5 itself and opener assembly spar 8 together with toothed disc 10. This results in the generation of dynamic loads imposed by the oscillating masses of the above-listed members of the opener assembly structure, which follow a law similar to (2). They are active forces giving rise to the respective reaction of the soil, therefore it is reasonable, to a first approximation, to assume that the reaction of the soil also varies under a law similar to (2), but generally with its own amplitude H.

Further, it is to be stressed that the deeper the penetration of toothed disc 10 into the soil is, the greater the total soil covered area of its teeth, to which the soil's reaction is applied, will be. As we can see from the schematic model in Fig. 2, when toothed disc 10 penetrates the soil down to the optimal depth, simultaneously four teeth in the disc's front part (which is in immediate contact with the unopened part of soil) move in soil. Moreover, the tooth preceding the first of those four teeth shown in the figure is not yet in contact with the soil surface irregularity, while the tooth succeeding the fourth of the shown teeth is already out of contact with the soil, moving in the furrow that has been cut in the soil. So, the number of the teeth simultaneously making contact with the soil (effectively moving in soil) depends also on the size of those teeth. Hence, in every particular case both the size of the teeth and the depth of the disc's penetration into the soil (disc running depth) have to be taken into account. In our case we assume, according to the schematic model (Fig. 2), that only four front teeth of disc 10 are in contact with the soil.

The soil also exerts reaction $\bar{R}_a$ on the hoe-type seed conductor, which also affects, although only moderately, the movement of the opener assembly.

Lastly, when the packing wheel rolls on the opened soil, the soil's normal reaction $\bar{N}_k$ is applied to the packing wheel as well as rolling friction force $\bar{F}_k$, the value of which is:

$$F_k = \delta \frac{N_k}{r_k}, \tag{4}$$

e: δ – coefficient of rolling friction (m); $r_к$ – packing wheel radius (m).

The sense of the toothed disc's rotation ω_d due to its engagement with the soil is shown with an arrow. The equivalent schematic model in Fig. 2 shows the necessary linear and angular dimensions. Now we establish the system of Cartesian rectangular coordinates Oxy with the origin at point O. Axis Ox runs along the line of translational movement of the opener assembly (co-directional with translational movement velocity vector V), axis Oy runs upward (Fig. 2).

Now we can write down the equation of the opener assembly movement in the vector form:

$$\begin{aligned} M\,\bar{a} = {} & \bar{F}_{П1} + \bar{F}_{П2} + \bar{G}_П + \bar{G}_d + \bar{G}_л + \bar{G}_a + \bar{G}_k + \bar{R}_1 + \bar{R}_2 + \bar{R}_3 + \bar{R}_4 + \\ & + \bar{R}_a + \bar{N}_k + \bar{F}_k + \bar{F}_d, \end{aligned} \tag{5}$$

where: M – mass of the opener assembly (kg); $\bar{a}$ – acceleration of the opener assembly (m s^{-2}).

The value of the mass of the opener assembly is found as follows:

$$M = m_П + m_d + m_л + m_a + m_к. \tag{6}$$

Next, we will write vector equation (5) using the projections on axes Ox and Oy. Initially we assume that the two springs (Fig. 2) are aligned parallel to axis Oy. Also we assume at a first approximation that the reactions of the soil acting on the teeth of the disc are perpendicular to the tooth surface, as shown in Fig. 2. Apparently, the adjacent teeth of the disc are spaced at an angular pitch of $\alpha = \frac{2\pi}{z}$, where z – number of teeth on the disc.

Further, we designate ε – angle between axis Ox and the upper side surface of the first tooth coming in contact with the soil surface, and β – angle between side surfaces of the teeth (Fig. 1). In this case, force projections $\bar{R}_i$ on axis Oy will be as follows:

$$
\begin{aligned}
&R_{1y} = R_1 \cos(\beta - \varepsilon);\\
&R_{2y} = R_1 \cos(\beta - \varepsilon + \alpha) = R_1 \cos(\beta - \varepsilon + \tfrac{2\pi}{z});\\
&R_{3y} = R_1 \cos(\beta - \varepsilon + 2\alpha) = R_1 \cos(\beta - \varepsilon + \tfrac{4\pi}{z});\\
&R_{4y} = R_1 \cos(\beta - \varepsilon + 3\alpha) = R_1 \cos(\beta - \varepsilon + \tfrac{6\pi}{z}).
\end{aligned} \tag{7}
$$

Similarly, the projections on axis Ox for the same four teeth will be:

$$
\begin{aligned}
&R_{1x} = R_1 \sin(\beta - \varepsilon);\\
&R_{2x} = R_1 \sin(\beta - \varepsilon + \tfrac{2\pi}{z});\\
&R_{3x} = R_1 \sin(\beta - \varepsilon + \tfrac{4\pi}{z});\\
&R_{4x} = R_1 \sin(\beta - \varepsilon + \tfrac{6\pi}{z}).
\end{aligned} \tag{8}
$$

The force of rolling friction of the toothed disc at a first approximation can be calculated as follows:

$F_d = \frac{4R_1}{r_d}\,\delta_1$,

or, using formula (3),

$$
F_d = \frac{4\left[R_0 + H \sin\left(\frac{2\pi V t}{L}\right)\right]\delta_1}{r_d}, \tag{9}
$$

where δ_1 – coefficient of rolling friction (m); r_d – disc radius (m).

The projection of force $\bar{R}_{\boldsymbol{a}}$ (the soil's reaction acting on the hoe-type seed conductor) on coordinate axes x and y will be equal to:

$$
\begin{aligned}
&R_{ax} = -R_a \cos\gamma ,\\
&R_{ay} = -R_a \sin\gamma .
\end{aligned} \tag{10}
$$

Angle γ is shown in Fig. 2.

Hence, taking into account formulae (5), (7), (8) and (10), we obtain the system of differential equations of the movement of the opener assembly in the projections to axes Ox and Oy:

$$
\begin{aligned}
M\ddot{x} = &-R_1\left[\sin(\beta-\varepsilon) + \sin\left(\beta-\varepsilon+\frac{2\pi}{z}\right) + \sin\left(\beta-\varepsilon+\frac{4\pi}{z}\right)\right.\\
&\left.- \sin\left(\beta-\varepsilon++\frac{6\pi}{z}\right)\right] - R_{\mathrm{a}}\cos\gamma - F_d - F_k;\\
M\ddot{y} = &-F_{\Pi 1} - F_{\Pi 2} - G_{\Pi} - G_d - G_{\text{л}} - G_{\mathrm{a}} - G_k\\
&+ R_1\left[\cos(\beta-\varepsilon) + \cos\left(\beta-\varepsilon+\frac{2\pi}{z}\right) + \cos\left(\beta-\varepsilon+\frac{4\pi}{z}\right)\right.\\
&\left.+ \cos\left(\beta-\varepsilon+\frac{6\pi}{z}\right)\right] - R_{\mathrm{a}}\, sin\gamma + N_k.
\end{aligned} \tag{11}
$$

Substituting formulae (1), (3), (4), (6) and (9) into differential equation system (11), we arrive at the following system of differential equations:

$$\left.\begin{aligned}
&M\ddot{x} = -\left[R_0 + H\sin\left(\frac{2\pi Vt}{L}\right)\right]\left[\sin(\beta-\varepsilon) + \sin\left(\beta-\varepsilon+\frac{2\pi}{z}\right) + \right.\\
&\left.+\sin\left(\beta-\varepsilon+\frac{4\pi}{z}\right) + +\sin\left(\beta-\varepsilon+\frac{6\pi}{z}\right)\right] - R_a\cos\gamma - \frac{4\left[R_0 + H\sin\left(\frac{2\pi Vt}{L}\right)\right]\delta_1}{R_d} - \delta\frac{N_к}{r_к};\\
&M\ddot{y} = -C_{П1}l_{П1} - C_{П2}y - (m_П + m_d + m_л + m_a + m_к)g + \left[R_0 + H\sin\left(\frac{2\pi Vt}{L}\right)\right]\left[\cos(\beta-\varepsilon) + \cos\left(\beta-\varepsilon+\frac{2\pi}{z}\right) + \cos\left(\beta-\varepsilon+\frac{4\pi}{z}\right) + \right.\\
&\left.\cos\left(\beta-\varepsilon+\frac{6\pi}{z}\right)\right] - R_a\sin\gamma + N_к.
\end{aligned}\right\} \quad (12)$$

Differential equation system (12) characterises the process of the horizontal and vertical translational oscillations of the opener assembly (its spar) during the movement of the opener assembly across the irregularities of the soil surface. In these equations, the component $H\sin\left(\frac{2\pi Vt}{L}\right)$ plays the role of the exciting force and the component C_{n2} y acts as the restoring force.

To reduce the formulae of differential equation system (12), the following designations are introduced:

$$\left[\sin(\beta-\varepsilon) + \sin\left(\beta-\varepsilon+\frac{2\pi}{z}\right) + \sin\left(\beta-\varepsilon+\frac{4\pi}{z}\right) + \sin\left(\beta-\varepsilon+\frac{6\pi}{z}\right)\right] = A;$$
$$\left[\cos(\beta-\varepsilon) + \cos\left(\beta-\varepsilon+\frac{2\pi}{z}\right) + \cos\left(\beta-\varepsilon+\frac{4\pi}{z}\right) + \cos\left(\beta-\varepsilon+\frac{6\pi}{z}\right)\right] = B.$$

Then differential equation system (12) will have the following representation:

$$\left.\begin{aligned}
&\ddot{x} = -A\frac{R_0}{M} - \frac{AH}{M}\sin\left(\frac{2\pi Vt}{L}\right) - \frac{R_a\cos\gamma}{M} - \frac{4R_0\delta_1}{MR_d} - 4H\frac{\sin\left(\frac{2\pi Vt}{L}\right)\delta_1}{MR_d} - \frac{\delta N_k}{Mr_k};\\
&\ddot{y} + \frac{C_{п2}y}{M} = -\frac{C_{п1}l_{п1}}{M} - g + \frac{R_0B}{M} + \frac{BH}{M}\sin\left(\frac{2\pi Vt}{L}\right) - \frac{R_a\sin\gamma}{M} + \frac{Nк}{M}.
\end{aligned}\right\} \quad (13)$$

Since the differential equations in system (13) are independent and they can be integrated separately, we start with integrating the first equation of the system. The first integral of the first equation is equal to:

$$\dot{x} = -\left(\frac{R_0A}{M} + \frac{R_a\cos\gamma}{M} + \frac{4R_0\delta_1}{MR_d} + \frac{\delta N_k}{MR_k}\right)t + \left(\frac{LAH}{2\pi VM} + \frac{4LH\delta_1}{2\pi VMR_d}\right)\cos\left(\frac{2\pi Vt}{L}\right) + C_1. \quad (14)$$

The second integral of the first equation is equal to:

$$\begin{aligned}
x = &-\left(\frac{R_0A}{M} + R_a\frac{\cos\gamma}{M} + 4R_0\frac{\delta_1}{M}R_d + \delta\frac{N_k}{M}R_k\right)\frac{t^2}{2} + \left(\frac{L^2AH}{4\pi^2V^2M} + \right.\\
&\left.+\frac{L^2H\delta_1}{\pi^2V^2MR_d}\right)\sin\left(\frac{2\pi Vt}{L}\right) + C_1t + C_2.
\end{aligned} \quad (15)$$

Arbitrary constants C_1 and C_2 can be derived from the following initial conditions:

at $t = 0$: $x = 0, \dot{x} = 0, y = 0, \dot{y} = 0.$ (16)

This gives the following values of the arbitrary constants:

$$C_1 = -\left(\frac{L\,AH}{2\pi\,V\,M} + \frac{2L\,H\delta_1}{\pi\,V\,M\,R_d}\right),\ C_2 = 0\,. \tag{17}$$

Thus, the first integral of the first differential equation of system (13), complying with initial conditions (16), is expressed as follows:

$$\dot{x} = -\left(\frac{R_0 A}{M} + \frac{R_\text{a}\cos\gamma}{M} + \frac{4R_0\delta_1}{M\,R_d} + \frac{\delta N_k}{M\,R_k}\right)t + \left(\frac{L\,AH}{2\pi\,V\,M} + \right.$$
$$\left. + \frac{2L\,H\delta_1}{\pi\,V\,M\,R_d}\right)\cos\left(\frac{2\pi Vt}{L}\right) - \left(\frac{L\,AH}{2\pi\,V\,M} + \frac{2L\,H\delta_1}{\pi\,V\,M\,R_d}\right). \tag{18}$$

The second integral, i.e. the solution of the equation, complying with initial conditions (16), is expressed as follows:

$$x = -\left(\frac{R_0 A}{M} + \frac{R_\text{a}\cos\gamma}{M} + \frac{4R_0\delta_1}{MR_d} + \frac{\delta N_k}{MR_k}\right)\frac{t^2}{2} + \left(\frac{L^2AH}{4\pi^2V^2M} + \right.$$
$$\left. \frac{L^2H\delta_1}{\pi^2V^2MR_d}\right)\sin\left(\frac{2\pi Vt}{L}\right) - \left(\frac{LAH}{2\pi VM} + \frac{2LH\delta_1}{\pi VMR_d}\right)t. \tag{19}$$

Formula (19) characterises the process of the translational oscillations of the opener assembly along axis Ox. The amplitude of these oscillations, as may be inferred from formula (19), is found as the multiplier at function $\left(\frac{2\pi Vt}{L}\right)$. Further, we give consideration to the second differential equation of system (13). Doing this, we introduce the designation $\sqrt{\frac{C_{\Pi 2}}{M}} = k.$

Then the representation of this differential equation transforms as follows:

$$\ddot{y} + k^2 y = -\frac{C_{\Pi 1} l_{\Pi 1}}{M} - g + \frac{BR_0}{M} + \frac{BH}{M}\sin\left(\frac{2\pi Vt}{L}\right) - \frac{R_\text{a}\sin\gamma}{M} + \frac{N_k}{M}. \tag{20}$$

For convenience we introduce the following designation:

$$-\frac{C_{\Pi 1} l_{\Pi 1}}{M} - g + \frac{BR_0}{M} - \frac{R_\text{a}\sin\gamma}{M} + \frac{N_\text{к}}{M} = D \tag{21}$$

Then differential equation (20) obtains the following representation:

$$\ddot{y} + k^2 y = \frac{BH}{M}\sin\left(\frac{2\pi Vt}{L}\right) + D. \tag{22}$$

Equation (22) is a linear differential equation of second order with constant coefficients and a right-hand side. Its solution is the sum of the solution of the homogeneous differential equation:

$$\ddot{y} + k^2 y = 0 \tag{23}$$

and the partial solution, which depends on the right-hand side of the equation. It is known that differential equation (23) has the following solution:

$$y_{hom} = L_1 \sin(kt) + L_2 \cos(kt)\ . \tag{24}$$

The partial solution of a non-homogeneous equation with a right-hand side is derived via the following expression:

$$y_{\text{part}} = R \sin\left(\frac{2\pi Vt}{L}\right) + S \cos\left(\frac{2\pi Vt}{L}\right) + T\ , \tag{25}$$

where R, S, T – unknown coefficients.

These coefficients can be found using the method of undetermined coefficients. Thus:

$$\dot{y}_{part} = 2R\pi v \cos\left(\frac{2\pi Vt}{L}\right)/\,L - 2S\pi v \sin\left(\frac{2\pi Vt}{L}\right)/\,L, \tag{26}$$

$$\ddot{y}_{part} = -\frac{4R\pi^2V^2}{L^2}\sin\left(\frac{2\pi Vt}{L}\right) - \frac{4S\pi^2V^2}{L^2}\cos\left(\frac{2\pi Vt}{L}\right). \tag{27}$$

Substituting formulae (25) and (27) into equation (22), we obtain:

$$-\frac{4R\pi^2V^2}{L^2}\sin\left(\frac{2\pi Vt}{L}\right) - \frac{4S\pi^2V^2}{L^2}\cos\left(\frac{2\pi Vt}{L}\right) + k^2R\sin\left(\frac{2\pi Vt}{L}\right) + k^2S\cos\left(\frac{2\pi Vt}{L}\right) + k^2T = \frac{BH}{M}\sin\left(\frac{2\pi Vt}{L}\right) + D\ . \tag{28}$$

Equating the coefficients at identical trigonometric functions in (28), we arrive at the following system of equations:

$$\left(-\frac{4\pi^2V^2}{L^2} + k^2\right)R = \frac{BH}{M}\ , \quad \left(-\frac{4\pi^2V^2}{L^2} + k^2\right)S = 0\ , \ K^2\,T = D. \tag{29}$$

From system of equations (29) we find:

$$R = \frac{BH}{M\left(k^2 - \frac{4\pi^2V^2}{L^2}\right)}, \qquad S = 0, \qquad T = \frac{D}{k^2}. \tag{30}$$

Substituting formulae (30) into formula (25), we find the needed partial solution:

$$y_{part} = \frac{BH}{M\left(k^2 - \frac{4\pi^2V^2}{L^2}\right)}\sin\left(\frac{2\pi Vt}{L}\right) + \frac{D}{k^2},$$

or, in a more convenient representation:

$$y_{part} = \frac{L^2BH}{M(L^2k^2 - 4\pi^2V^2)}\sin\left(\frac{2\pi Vt}{L}\right) + \frac{D}{k^2} \tag{31}$$

Thus, the general solution of differential equation (20) will be equal to:

$y = y_{hom} + y_{part}$,

or, taking into account (24) and (31), we obtain the following expression:

$$y = L_1 \sin(kt) + L_2 \cos(kt) + \frac{L^2 BH}{M(L^2k^2 - 4\pi^2 V^2)} \sin\left(\frac{2\pi Vt}{L}\right) + \frac{D}{k^2} \; . \qquad (32)$$

Arbitrary constants L_1 and L_2 can be found from initial conditions (16). From formula (32) for $t = 0$ we obtain:

$L_2 = -\frac{D}{k^2}$.

To find arbitrary constant L_1, we differentiate expression (32) with respect to t:

$$y = C_1 k \cos(kt) - C_2 k \sin(kt) + \frac{2\pi VLBH}{M(L^2k^2 - 4\pi^2 V^2)} cos\left(\frac{2\pi Vt}{L}\right). \qquad (33)$$

From formula (33) for $t = 0$ we find the value of arbitrary constant C_1:

$C_1 = -\frac{2\pi VLBH}{kM(L^2k^2 - 4\pi^2 V^2)}$.

Hence, we obtain the solution of differential equation (22), complying with initial conditions (16):

$$y = -\frac{2\pi VLBH}{kM(L^2k^2 - 4\pi^2 V^2)} \sin(kt) - \frac{D}{k^2} \cos(kt) + \frac{L^2 BH}{M(L^2k^2 - 4\pi^2 V^2)} \sin\left(\frac{2\pi Vt}{L}\right) + \frac{D}{k^2}. \qquad (34)$$

Formula (34) characterises the translational oscillations of the opener assembly along axis Oy in the presence of exciting force $R_0 + H\sin\left(\frac{2\pi Vt}{L}\right)$ and restoring force C_{n2} y of the vibrator spring.

In formula (34), the first two terms of sum characterise the natural vertical oscillations of the opener assembly, the third term – the forced vertical oscillations of the opener. Meanwhile, the amplitude of the natural oscillations, as may be inferred from formula (34), has a value of:

$$A_1 = \sqrt{\frac{4\pi^2 V^2 L^2 B^2 H^2}{k^2 M^2 (L^2k^2 - 4\pi^2 V^2)^2} + \frac{D^2}{k^4}}, \qquad (35)$$

The amplitude of forced oscillations of the opener assembly is found from the following formula:

$$B_1 = \frac{L^2 BH}{M(L^2k^2 - 4\pi^2 V^2)}. \qquad (36)$$

The frequency of the natural oscillations is determined as follows:

$$k = \sqrt{\frac{C_{п2}}{M}}, \qquad (37)$$

while the frequency of the forced oscillations, as is known, equals the frequency of the exciting force:

$$k_1 = \frac{2\pi V}{L}. \qquad (38)$$

Thus, the formulae have been obtained for determining the amplitude (35) and frequency of the natural oscillations (37) and the amplitude of the forced oscillations

(36) of the opener assembly depending on its main design parameters and modes of operation during its uniform movement across irregularities of the soil surface. These formulae take into account the following values: the number of teeth on the toothed disc of the opener, the deflection rates of its suspension springs and the velocity of the translational movement.

The obtained final expressions for the amplitude and frequency of vibration of the opener assembly spar provide the basis for numerical calculations with the use of a PC. The computer programme developed in the MathCAD environment has been used to carry out the numerical calculation of the amplitude of the vibration of the furrow opening toothed disc generated by the system of spring devices (two-spring suspension) comprising the pressure spring and the additional self-induced oscillation spring during the interaction between the disc and soil surface irregularities.

The results of the calculations have been used to construct the graphs of opener assembly vibration amplitudes $A(V)$ (m) for different values of spring constants $C_{\Pi 1}$ and $C_{\Pi 2}$ (N m^{-1}), depending on the translational motion velocity V (m s^{-1}). Those graphs are presented in Fig. 3.

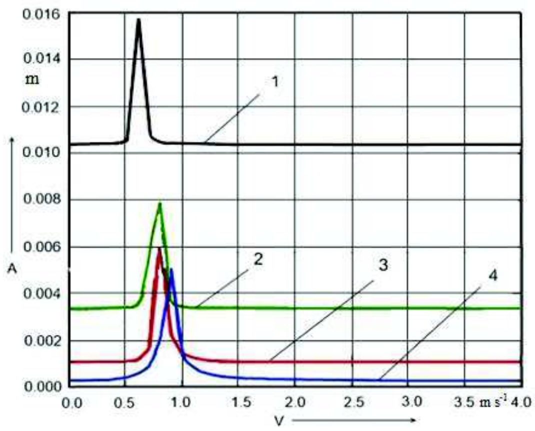

Figure 3. Curves of denoting the relation between the amplitude of vibration of opener assembly with a two-spring suspended toothed disc and the velocity of its motion at following deflection rates (N m^{-1}):
1 – $C_{\Pi 1}$ = 16815, $C_{\Pi 2}$ = 17150 (25% of rated deflection rate);
2 – $C_{\Pi 1}$ = 33635, $C_{\Pi 2}$ = 34300 (50% of rated deflection rate);
3 – $C_{\Pi 1}$ = 50450, $C_{\Pi 2}$ = 51450 (75% of rated deflection rate);
4 – $C_{\Pi 1}$ = 67270, $C_{\Pi 2}$ = 68600 (100% of rated deflection rate).

It appears from the presented graphs that resonance amplitudes of vibration are observed, when the opener assembly perturbation frequency is equal to its natural vibration frequency at velocities of 0.5 m s^{-1} to 1.0 m s^{-1}. During the following increase of the translational motion velocity from 1.2 m s^{-1} to 4 m s^{-1} the amplitude values remain stable.

CONCLUSIONS

7. The system of differential equations has been set up for the translational oscillations of the improved design opener assembly initiated by the action of the exciting force generated by the soil surface irregularities during the uniform movement of the opener assembly down the field.

8. For the mentioned differential equation system, a solution has been found that characterises the law of the oscillatory movement of the opener assembly along the axes of the Cartesian coordinate system.

9. The finite analytical expressions have been found for determining the amplitude and frequency of the mentioned oscillations depending on the design parameters and kinematic modes of operation of the opener assembly.

10. The obtained mathematical model allows to assess the conditions of the system and subsequently optimise the energy characteristics of the breeding grain drill equipped with improved design opener assemblies with toothed discs.

REFERENCES

Altikat, S., Celik, A. & Gozubuyuk, Z. 2013. Effects of various no-till seeders and stubble conditions on sowing performance and seed emergence of common vetch. *Soil & Tillage Research* **126**, 72–77.

Bai, X., Lin, J., Lü, C. & Hu, Y. 2014. Analysis and experiment on working performance of disc coulter for no-tillage seeder. *Nongye Gongcheng Xuebao/Transactions of the Chenese Society of Agricultural Engineering* **30**(15), 1–9.

Belov, V.V. & Belov, S.V. 2007. On the operating area of the opener suspension mechanism. *Machinery in agriculture* **5**, 10–12. (in Russian).

Bianchini, A. & Magalhães, P.S.G. 2008. Evalution of coulters for cutting sugar cane residue in a soil bin. *Biosystems Engineering* **100**, 370–375.

Chaudhuri, D. 2001. Performance evaluation of various types of furrow oponers on seed drills – A review. *Journal of Agricultural Engineering Research* **79**(2), 125–137.

Chen, Y., Tessier, S. & Irvine, V. 2004. Drill and crop performances for no-till seeding. *Soil & Tillage Research* **77**, 145–155.

Hasimu, A. & Chen, Y. 2014. Soil disturbance and draft force of selected seed openers. *Soil & Tillage Research* **140**, 48–54.

Jingling, S., Zidong, Y., Shandong, Y., Guohai, Z. & Hongweng, L. 2011. Development of a new type seeder. *Intenational Agricultural Engineering Journal* **20**(2), 57–60.

Endrerud, H.C. 1999. Dynamic performance of drill coulters in a soil bin. *J. agric. Engng Res.* **74**, 391–401.

Karayel, D. 2009. Performance of a modified precision vacuum seeder for no-till sowing of maize and soybean. *Soil & Tillage Research* **104**, 121–125.

Karayel, D. & Özmerzi, A. 2007. Comparison of vertical and lateral seed distribution of furrow openers using a new criterion. *Soil & Tillage Research* **95**, 69–75.

Lin, J., Song, Y. & Li, B. 2014. Mechanical no-tillage sowing technology in ridge area of Nordheast China. *Nongye Gongcheng Xuebao/Transactions of the Chenese Society of Agricultural Engineering* **30**(9), 50–57.

Lisoviy, I.A. 2013. Justification of direct drilling opener parameters. *Kirovograd: A Thesis for applying for the degree of Doctor of Philosophy in Agricultural Engineering*.

Lian, Z., Wang, J., Yang, Z. & Shang, S. 2012. Development of plot-sowing mechanization in China. Nongye Gongcheng *Xuebao/Transactions of the Chenese Society of Agricultural Engineering* **2**8(2), 140–145.

Liu, S. & Ma, Y. 2012. Development history, status, and trends of plot seeder. *Applied Mechanics and Materials* **268**(1), 1966–1969.

Macmillan, R.H. 2002. The Mechanics of Tractor – Implement Performance. Theory and Worked Examples. *University of Melbourne*.

Magalhães, P.S.G., Bianchini, A. & Braunbeck, O.A. 2007. Simulated and experimental analyses of a toothed rolling coulter for cutting crop residues. *Biosystems Engineering* **96**(2), 193–200.

Prisyazhnyuk, N.V., Adamchuk, V.V. & Bulgakov, V.M. 2013. Theory of vibration machines in agricultural industry. *Kiev: Agricultural Science*.

Šarauskis, E. & Vaitauskiene, K. 2014. Reseach of mechanicaltraction characteristics of direct sowing equipment. *Mechanica* **20**(5), 506–511.

Turbin, B.G., Lurie, A.B. & Grigoryev, S.M. 1967. Agricultural machinery. Theory and process design. *Leningrad: Mechanical Engineering, 2nd ed. revision and updated*.

Vamerali, T., Bertocca, M. & Sartori, L. 2006. Effects of a new wide-sweep for no-till planter on seed zone properties and root establishment in maize (Zea mays, L.): A comparison with double-disk opener. *Soil & Tillage Research* **89**, 196–209.

Vasilenko, P.M. 1996. Introduction to agricultural mechanics. *Kiev: Agricultural Education*.