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Analysis of the rheological model of the process of chip formation with metal machining

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Abstract. The present paper discusses mathematical modelling with regard to deformation processes resulting from chip forming and removal when cutting heavilyprocessed materials in a condition of non-homogeneous stress state, which can evoke undesirable self-excited oscillation. By describing the chip forming and removal process a synthesized rheological model was used.

Key words: cutting materials, elastic and plastic deformation, relaxation, rheological model

INTRODUCTION

The principal factor in determining the deformation and fracture of metals is defective structure, caused by formation of local stresses and the probability of their partial relaxation. The non-homogeneous stress state relieves the rotatory instability and the formation of a revolving zone related to the base material (Veitz & Maksarov, 2000)**.** For the adequate mapping of the process of plastic deformation with chip formation, taking into account elastic-plastic properties in the dynamics of contact interaction of tool with the billet and the rheological special features in the zone of active plastic deformation, it is necessary to construct a generalized mathematical model of the process of chip formation, which will make it possible to reveal the conditions for passage from the unsteady state to the stable operation of metal machining.

MATERIALS AND METHODS

In accordance with the works published (Savin & Rushitskiy, 1976; Veitz et al., 2000; Veitz & Maksarov, 2002), the defined relation between stress σ and deformation ε can be presented in the form of the following rheological equation

$$a_4\ddot{\varepsilon} + a_5\dot{\varepsilon} + a_6\varepsilon = a_1\ddot{\sigma} + a_2\dot{\sigma} + a_3\left(\sigma + \sigma_{pl}\right), \qquad (1)$$

where a_1, a_2, a_3, a_4, a_5, a_6 – partially constant functions, including rigidity criteria (c_1, c_2, c_3) and energy scattering parameters (β_2, β_3);
σ_{pl} – plasticity limit.

Equation (1) is the basis for constructing an adequate dynamic model of chip forming and removal. It presents an intricate rheological model for connecting sequentially the elements of Ishlinski elastic-ductile-plastic relaxing medium (describing the dynamic plastification process of the cut metal layer) and the elastic-dissipative elements of Voigt medium (describing the off-peeling chip deformation and friction processes).

To motivate the adequacy of the derived complicated rheological model (1) describing the chip forming and removal process, the computer-based mathematical modeling was performed. By modeling the rheological equation the deformation character in-time was presupposed to be known. The stress dynamic was studied by solving the stress equation. Substituting σ by a new variable $x \equiv \sigma - a_3 \cdot \sigma_{pl}$, the equation (1) takes the following shape:

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = f(\varepsilon, \dot{\varepsilon}, \ddot{\varepsilon}), \text{ if } a_1 \neq 0, \tag{2}$$

where

$$\omega_0 = \frac{1}{\sqrt{a_1}};$$

$$\gamma = \frac{a_2}{2 \cdot a_1};$$

$$f(\varepsilon, \dot{\varepsilon}, \ddot{\varepsilon}) = \frac{(a_4 \cdot \ddot{\varepsilon} + a_5 \cdot \dot{\varepsilon} + a_6 \cdot \varepsilon)}{a_1}.$$

Specifying the variation law $\varepsilon = \varepsilon(t)$ and presenting in the form $f(\varepsilon, \dot{\varepsilon}, \ddot{\varepsilon}) = h(t)$, variation of stress σ (or new variable x) was expressed:

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = h(t), \tag{3}$$

where parameters γ and ω_0 in their physical content according to equation (2) are positive real quantity. Depending on the relation between frequency ω_0 and relaxation index γ, the vibration mode with vibration frequency $\omega = \sqrt{\omega_0^2 - \gamma^2}$; or relaxation mode ($\gamma < \omega_0$) are realized.

After transition from variable x to variable σ and making the corresponding substitutions, by the following initial terms $\sigma(0) = \sigma_0$ and $\dot{\sigma}(0) = const$, an equation describing the stress variations ($\omega = \sqrt{\omega_0^2 - \gamma^2}$; $\gamma < \omega_0$) is realized:

$$\sigma(t) = a_3 \cdot \sigma_{pl} + \left[-\frac{a_4 \cdot \varepsilon_0}{a_1 \cdot \omega^2} + \frac{b-a}{2} \cdot \exp(-\omega \cdot t) + \frac{b+a}{2} \cdot \exp(\omega \cdot t) \right] \cdot \exp(-\gamma \cdot t), \tag{4}$$

where

$$a = \frac{\gamma}{\omega}\left(\sigma_0 - a_3 \cdot \sigma_{pl}\right) + \frac{\dot{\sigma}(0)}{\omega};$$

$$b = \left(\sigma_0 - a_3 \cdot \sigma_{pl}\right) + \frac{a_4 \varepsilon_0}{a_1 \omega^2}.$$

From equation (4) we get that stress is relaxed to its equilibrium value $\sigma \to a_3 \cdot \sigma_{pl}$ not in accordance with a simple exponential law, but with a significantly more complicated law with two relaxation speeds, ($\omega + \gamma$) and ($\omega - \gamma$). Consequently, we get two relaxation times:

$$\tau_{rel}^{(1)} = \frac{1}{\gamma + \omega}; \quad \tau_{rel}^{(2)} = \frac{1}{\gamma - \omega}. \tag{5}$$

THEORETICAL RESULTS

Analysis of stress variation in the rheological equation under discussion (1), actively reflected plastic deformation of metal in the chip forming and removal process rendering possible the condition

$$\omega \leq \gamma \tag{6}$$

upon which realization the microlevel of the primary plastic deformation zone, the processes, evoking chip forming and removing unstability, occur.

Variations changes of deformation as the time factor in the synthesized rheological model by constantly maintained stresses

$$\sigma - a_3\sigma_{pl} = \sigma_0 = const$$

and by initial conditions

$$\varepsilon(0) = \varepsilon_0; \dot{\varepsilon}(0) = const$$

are specified by equation

$$\varepsilon(t) = \left| \left[-\frac{\sigma_0}{a_6 \cdot \widetilde{\omega}^2} + \frac{\widetilde{b} - \widetilde{a}}{2} \cdot \exp(-\widetilde{\omega} \cdot t) + \frac{\widetilde{b} + \widetilde{a}}{2} \cdot \exp(\widetilde{\omega} \cdot t) \right] \cdot \exp(-\widetilde{\gamma} \cdot t) \right|, \tag{7}$$

where

$$\widetilde{a} = \frac{\widetilde{\gamma}}{\widetilde{\omega}} \cdot \varepsilon_0 + \frac{\dot{\varepsilon}(0)}{\widetilde{\omega}};$$

$$\widetilde{b} = \varepsilon_0 + \frac{\sigma_0}{a_6 \cdot \widetilde{\omega}^2}.$$

Solution (7) shows that the reverting process to the systems' non-deformed state of aggregation can be characterized by two time lags, which are represented as the main characteristics within the following rheological model:

$$\tau_{ret}^{(1)} = \frac{1}{\widetilde{\gamma} + \widetilde{\omega}} = \frac{c_3}{2\beta_3} + \frac{c_2}{2\beta_2} + \left| \frac{c_3}{2\beta_3} - \frac{c_2}{2\beta_2} \right|; \tag{8}$$

$$\tau_{ret}^{(2)} = \frac{1}{\widetilde{\gamma} - \widetilde{\omega}} = \frac{c_3}{2\beta_3} + \frac{c_2}{2\beta_2} - \left| \frac{c_3}{2\beta_3} - \frac{c_2}{2\beta_2} \right|; \tag{9}$$

As realised in the case above, consecutive connection of the elements of Ishlinski elastic-ductile-plastic relaxing medium representing the dynamic plasticity process of the cut metal layer as well as the elastic-dissipative elements of Voigt medium representing the off-peeling chip deformation and friction processes of the model, may be characterized by two time lags, (8) and (9).

EXPERIMENT RESULTS

Experimental studies were carried out in the broader band of back rake angle γ = -5°–25° and values of cutting speed v = 10–75 m min^{-1}. In the paper the extreme values are selected, which confirm our studies on the origin of the primary relaxation phenomena, which, at their early stage depending on the state of technological system (hardness, grade of machined material, tool, the dynamic properties of lathe etc.), induce the origination of the self-oscillating process by the turning of materials.

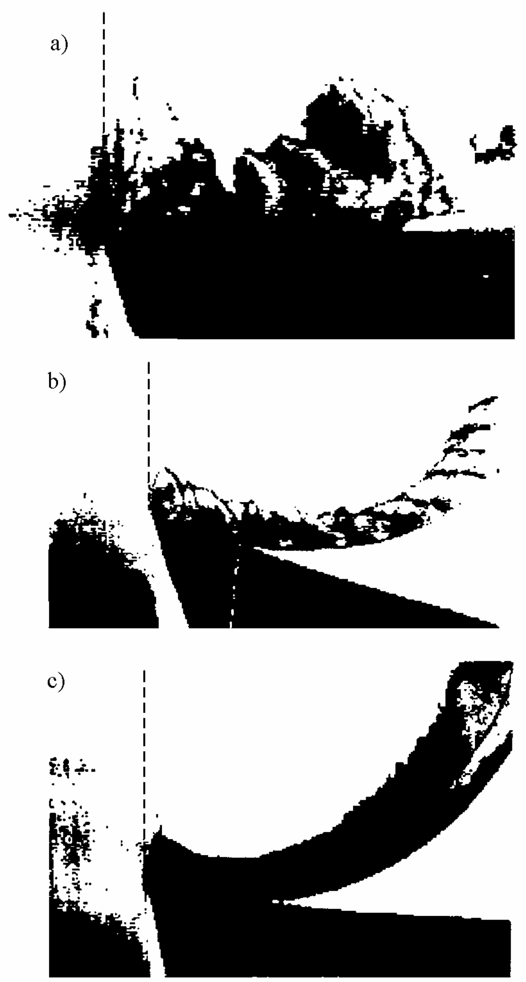

Fig. 1. Form of chip, formed by processing steel 45 with cutting tool, where:

a) γ = - 5°; v = 10 m min^{-1} – discontinuous chip,
b) γ = 25°; v = 25 m min^{-1} – built-up edge chip,
c) γ = 10°; v = 75 m min^{-1} – continuous chip.

CONCLUSIONS

1. The analysis of differential equation (1) describing a synthesized rheological model proves that the prescribed model enables an adequate description of the deformation process by chip forming and determination of the criteria by its transition to a non-stable mode (Fig. 1), provoking self-excited oscillation.
2. The present paper deals with the origin of the relaxation phenomena on the micro level and also with their influence on the development of self-oscillating process in the technological system of lathe. Theoretical studies have been confirmed by experiment.

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