Calculations of heated floor panel for resting places of piglets

V. Zagorska[1] and U. Iljins[2]

[1]Institute of Mechanical Engineering, Latvia University of Agriculture, Liela iela 2, LV-3000, Jelgava, Latvia; e-mail: vzagorska@gmail.com
[2]Latvia University of Agriculture, Faculty of Information Technologies; e-mail: uldis.iljins@llu.lv

Abstract. The purpose of this paper is to show the use of mathematical physics in solving engineering problems with the help of the separation of variables method, using optimizing heating element – placement of the electrical wire or hot water tube in concrete panel housing. During the research heat flows from all 4 surfaces (right, left, upper, and bottom) were calculated, using obtained solutions for heat flow calculation. These calculations are required for constructing a more comfortable and energy efficient heating panel for newborn piglets.

Key words: Method of separation of variables, heating distribution.

INTRODUCTION

The first days are the most critical in a piglet's life (Fahmy & Bernhard, 1971; Furniss, 1986; Edwards & Malkin, 1986). Up to 80% of piglet losses occur in the first 3 days of their life and the majority of deaths are attributed to crushing (English & Morrison, 1984; Weary et al., 1998). To reduce piglet losses by crushing by the sow the nest area must be warm. However, because of economic and hygienic disadvantages, straw is becoming less common (Hoy & Ziron, 1998) and the modern practice is to use either electric or gas infrared heaters, or underground heaters with solid plates (Hoy & Ziron, 1998).

In this article we will pay attention to the solid plates. For ensuring the comfort of piglets, concrete floor panels heated by electric current or hot water are used. If an electro-heated cable is placed in the panel's body, the amount of heat conducted from the cable is the same along the whole cable length (Iljins & Ziemelis, 1996). If hot water circulating through tube is used, the amount of heat energy taken off the heater decreases along its length. Distribution of temperature over the working surface of the panel at different position x and y coordinates of heating elements, different heat transfer coefficients and variable heating element intensity have been calculated previously (Zagorska et al, 2010), using MS Excel software and obtained temperature distribution solution over the cross-section of the panel. The aim of the research is to find the solution for the heat flow's determination from all 4 sides of the panel, evaluating variable: heating element intensity and heat transfer coefficients of the surfaces.

MATERIALS AND METHODS

Mathematical physics problem always contains a certain equation. In our case we have to solve the special steady state case of general heat-conduction equation (Riekstins, 1969):

$$\Delta T = 0, \tag{1}$$

where: $\Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ - Laplace operator;

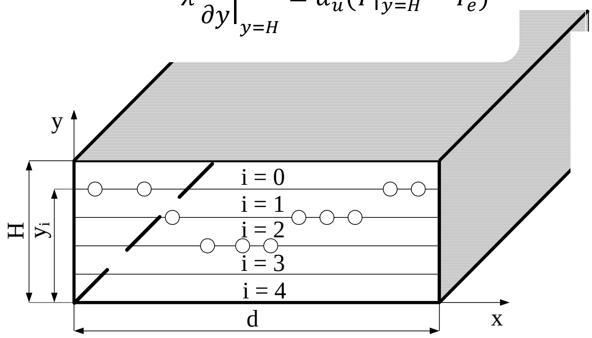
T – temperature, °C .

The problem is being solved at the following limit conditions (Fig. 1):

$$-\lambda \frac{\partial T}{\partial y}\bigg|_{y=H} = \alpha_u (T|_{y=H} - T_e)$$

$$-\lambda \frac{\partial T}{\partial x}\bigg|_{x=0} = \alpha_{s_1} (T|_{x=0} - T_e)$$

$$-\lambda \frac{\partial T}{\partial x}\bigg|_{x=d} = \alpha_{s_2} (T|_{x=d} - T_e)$$



$$\lambda \frac{\partial T}{\partial x}\bigg|_{y=0} = \frac{q_i}{\Delta x_i} \frac{\frac{H - y_i}{\lambda} + \frac{1}{\alpha_u}}{\frac{H}{\lambda} + \frac{1}{\alpha_u} + \frac{d_{iz}}{\lambda_{iz}}}$$

Figure 1. Calculation scheme: H – panel thickness, m; h – heating element (tube) placement depth, m; x_{01}, x_{02}, ..., x_{0i} – heating element twine coordinates, m; d – width of the panel, m; i – number of layer, where heating element is placed; T – temperature of the panel, °C; T_e – ambient air temperature, °C; λ – thermal conductivity coefficient, W $(m\ K)^{-1}$; $\alpha_{s_1}, \alpha_{s_2}, \alpha_u$ – surface heat transfer coefficients from upper and side surfaces, W $m^{-2} \cdot K^{-1}$; q_i specific heat flow from the i-th heating element, W m^{-1}; Δx_i – width of the operation zone, m; q_{iz} – thickness of insulation from the bottom, m.

As the number of zones i_{max} = n can change, it is necessary to look for an expression which would allow to calculate the temperature in such a way that the temperature would not change when going from zone number i to zone number i+1. It will be the following expression (Riekstins, 1969):

$$T_i = \\ = \quad T_e + \sum_{k=1}^{\infty} \sin(\xi_k x + \varphi_k) \begin{Bmatrix} A_k \left[\mathrm{a}_{\mathrm{k_i}} \mathrm{sh}\, \xi_k (y_i - y) + b_{k_i} \mathrm{ch}\, \xi_k (y_i - y) \right] + \\ \mathrm{c}_{\mathrm{k_i}} \mathrm{sh}\, \xi_k (y_i - y) + d_{k_i} \mathrm{ch}\, \xi_k (y_i - y) \end{Bmatrix} \tag{2}$$

where: $A_k, \mathrm{a}_{\mathrm{k_i}}, b_{k_i}, \mathrm{c}_{\mathrm{k_i}}, d_{k_i}$– free chosen constants which are unchangeable when going to zone i+1;
ξ_k – particular value, m^{-1};
φ_k – free chosen constant.

Constants $A_k, \mathrm{a}_{\mathrm{k_i}}, b_{k_i}, \mathrm{c}_{\mathrm{k_i}}, d_{k_i}$were obtained previously (V. Zagorska et al., 2010), which enabled calculation of the temperature distribution along the working surface by changing coordinates of the discrete heater coils. Using the formula (2), we have got the best coordinates for the temperature evenness over the working surface (V. Zagorska et al., 2010). Heat flow from different sides is a very important point as well, therefore we will further find out a solution for the heat flows.

It is necessary to find derivatives at different boundary values of the expressions (3, 4, 5, and 6) and to integrate the found derivatives at different boundary values to obtain a solution for the calculation the heat flows. It is being shown below in equations (7, 8, 9, and 10).

$$Q_{s_1} = -\lambda \int_0^H \left.\frac{\partial T}{\partial x}\right|_{x=0} dy \tag{3}$$

$$Q_U = -\lambda \int_0^d \left.\frac{\partial T}{\partial y}\right|_{y=H} dx \tag{4}$$

$$Q_{s_2} = \lambda \int_H^0 \left.\frac{\partial T}{\partial x}\right|_{x=d} dy \tag{5}$$

$$Q_B = \lambda \int_d^0 \left.\frac{\partial T}{\partial y}\right|_{y=0} dx \tag{6}$$

where: Q_{s_1} - heat flow from the right side, W m^{-1};
Q_U - heat flow from the upper surface, W m^{-1};
Q_{s_2} - heat flow from the left side, W m^{-1};
Q_B - heat flow from the bottom surface, W m^{-1}.

After carrying out the above mentioned mathematical actions for the case when i_{max} =2, the following is obtained:

$$Q_{s_1} = \lambda \int_0^H \frac{\partial T}{\partial x}\bigg|_{x=0} dy =$$
$$= -\lambda \sum_{k=1}^{\infty} \cos(\varphi_k) \begin{Bmatrix} A_k\left[\mathrm{a}_{\mathrm{k}_2} - \mathrm{a}_{\mathrm{k}_2} \mathrm{ch}\, \xi_k(y_2) - b_{k_2} \mathrm{sh}\, \xi_k(y_2)\right] + \\ + \mathrm{c}_{\mathrm{k}_2} - \mathrm{c}_{\mathrm{k}_2} \mathrm{ch}\, \xi_k(y_2) + d_{k_2} \mathrm{sh}\, \xi_k(y_2) \end{Bmatrix} +$$
$$- \lambda \sum_{k=1}^{\infty} \cos(\varphi_k) \begin{Bmatrix} A_k\left[a_{k_1} - \xi_k\left(\mathrm{a}_{\mathrm{k}_1} \mathrm{ch}\, \xi_k(y_1 - y_2) - b_{k_1} \mathrm{sh}\, \xi_k(y_1 - y_2)\right)\right] + \\ + \xi_k \mathrm{c}_{\mathrm{k}_1} \mathrm{ch}\, \xi_k(y_1 - y_2) + \xi_k d_{k_1} \mathrm{sh}\, \xi_k(y_1 - y_2) \end{Bmatrix} \qquad (7)$$
$$-\lambda \sum_{k=1}^{\infty} \cos(\varphi_k) \begin{Bmatrix} A_k\left[\mathrm{a}_{\mathrm{k}_0} - \mathrm{a}_{\mathrm{k}_0} \mathrm{ch}\, \xi_k(y_0 - y_1) - b_{k_0} \mathrm{sh}\, \xi_k(y_0 - y_1)\right] + \\ - \mathrm{c}_{\mathrm{k}_0} \mathrm{ch}\, \xi_k(y_0 - y_1) - d_{k_0} \mathrm{sh}\, \xi_k(y_0 - y_1) \end{Bmatrix}$$

$$Q_u = -\lambda \int_0^d \frac{\partial T}{\partial y}\bigg|_{y=H} d = -\lambda \sum_{k=1}^{\infty} \left(\cos(\xi_k d + \varphi_k) - \cos \varphi_k\right) \cdot \left[A_k \mathrm{a}_{\mathrm{k}_0} + \mathrm{c}_{\mathrm{k}_0}\right] \qquad (8)$$

$$Q_{s2} = -\lambda \int_H^0 \frac{\partial T}{\partial x}\bigg|_{x=d} dy =$$
$$= \lambda \sum_{k=1}^{\infty} \cos(\xi_k d \varphi_k) \begin{Bmatrix} A_k\left[\mathrm{a}_{\mathrm{k}_0} - \mathrm{a}_{\mathrm{k}_0} \mathrm{ch}\, \xi_k(H - y_1) - b_{k_0} \mathrm{sh}\, \xi_k(H - y_1)\right] + \\ - \mathrm{c}_{\mathrm{k}_0} \mathrm{ch}\, \xi_k(H - y_1) - d_{k_0} \mathrm{sh}\, \xi_k(H - y_1) \end{Bmatrix}$$
$$+ \lambda \sum_{k=1}^{\infty} \cos(\xi_k d \varphi_k) \begin{Bmatrix} A_k\left[a_{k_1} - \mathrm{a}_{\mathrm{k}_1} \mathrm{ch}\, \xi_k(y_1 - y_2) - b_{k_1} \mathrm{sh}\, \xi_k(y_1 - y_2)\right] + \\ + \mathrm{c}_{\mathrm{k}_1} - \mathrm{c}_{\mathrm{k}_1} \mathrm{ch}\, \xi_k(y_1 - y_2) - d_{k_1} \mathrm{sh}\, \xi_k(y_1 - y_2) \end{Bmatrix} \qquad (9)$$
$$+ \lambda \sum_{k=1}^{\infty} \cos(\xi_k d \varphi_k) \begin{Bmatrix} A_k\left[a_{k_2} - \mathrm{a}_{\mathrm{k}_2} \mathrm{sh}\, \xi_k(y_2 - y_3) - b_{k_2} \mathrm{ch}\, \xi_k(y_2 - y_3)\right] + \\ + c_{k_2} - \mathrm{c}_{\mathrm{k}_2} \mathrm{sh}\, \xi_k(y_2 - y_3) - d_{k_2} \mathrm{ch}\, \xi_k(y_2 - y_3) \end{Bmatrix}$$

$$Q_B = \lambda \int_d^0 \frac{\partial T}{\partial y}\bigg|_{y=0} dx =$$
$$= \lambda \sum_{k=1}^{\infty} \left(\cos(\xi_k d + \varphi_k) - \cos \varphi_k\right) \cdot \begin{Bmatrix} A_k\left[\mathrm{a}_{\mathrm{k}_2} \mathrm{ch}\, \xi_k(y_2) + b_{k_2} \mathrm{sh}\, \xi_k(y_2)\right] + \\ \mathrm{c}_{\mathrm{k}_2} \mathrm{ch}\, \xi_k(y_2) + d_{k_2} \mathrm{sh}\, \xi_k(y_2) \end{Bmatrix} \qquad (10)$$

RESULTS AND DISCUSSION

Assuming that concrete heat transfer coefficient $\lambda = 1\ W\ (m \cdot K)^{-1}$, panel width $d = 1$ m, heat transfer coefficient through the working surface $\alpha_{s_u} = 10\ W\ m^{-2} \cdot K^{-1}$, pigsty air temperature 20 °C, panel thickness $H = 0.1\ m$, bottom side insulation thickness 0.1m , insulation heat transfer coefficient $\lambda = 0.04\ W\ (m \cdot K)^{-1}$ the Ms Excel software temperature on the working surface was used and heat flows from the concrete panel were calculated.

As it can be seen in Table 1, where four cases of simulation are represented, the necessary temperature on the upper surface of a panel can be reached by using smaller steps among outer twines to compensate the minimal decrease of the heating element intensity. If we do not take into account the area of 5cm from the side surfaces, then in the 1st case (variable step is applied) amplitude of temperature oscillation accounts 0.374°C. For example, in the case when panel from the left side is enclosed to the 0.1m wide wooden lath, then in the second case (variable step, and variable surface heat transfer coefficients) the amplitude of temperature oscillation only of 0.653°C is achieved. To land it is necessary to use smaller step among outer twine at the right side surface, and little variations of steps over the panel are required as well (to compensate the variety of heating element intensity). It is important to add that heat source flows from the right side where heat transfer coefficient is higher. Speaking about the 3rd case with equal steps used, the amplitude of temperature oscillation of 1,814°C appears. The best results are placing heat elements in two levels using variable step (4th case), in which case the amplitude of temperature oscillation achieves only 0.345°C.

Table 1. Variation of parameters of the panel for achieving temperature distribution evenness over the panel and higher heat flow value from the upper surface.

Nr. of case	1	2	3	4
Heat flow	Variable step $\alpha_{s_1} = \alpha_{s_2} = 10\ W\ m^{-2}K^{-1}$, $y = 0.06\ m$	Variable step $\alpha_{s_1} = 1.26\ W\ m^{-2}K^{-1}$, $\alpha_{s_2} = 10\ W\ m^{-2}K^{-1}$, $y = 0.06\ m$	Equal step $\alpha_{s_1} = \alpha_{s_2} = 10\ W\ m^{-2}K^{-1}$ $y = 0.06\ m$	Variable step $\alpha_{s_1} = \alpha_{s_2} = 10\ W\ m^{-2}K^{-1}$, $y_1 = y_{10} = 0.065\ m$ $y_2 \cdots y_9 = 0.055\ m$
Q_{s1}	7.78%	1.38%	6.77%	7.81%
Q_u	79.31%	85.64%	81.93%	79.15%
Q_{s2}	7.72%	9.59%	6.29%	7.74%
Q_L	5.19%	5.19%	5.01%	5.30%
Amplitude of oscillation, °C	0.3735	0.6527	1.8137	0.3448
Average upper surface temperature, °C	29.891	30.788	30.135	29.884

Taking into consideration the 2nd case with insulation only from the one side and heat flow from the upper side increasing till 85.64%, it can be seen that insulation from the right and left side is recommended as well to increase the heat flow from the upper surface. After making calculations for the case with the panel being insulated from both sides, it was obtained that heat flow from the upper surface was 91.84% and the calculated amplitude of temperature oscillation was only 0.2970°C (5th case in Fig. 1).

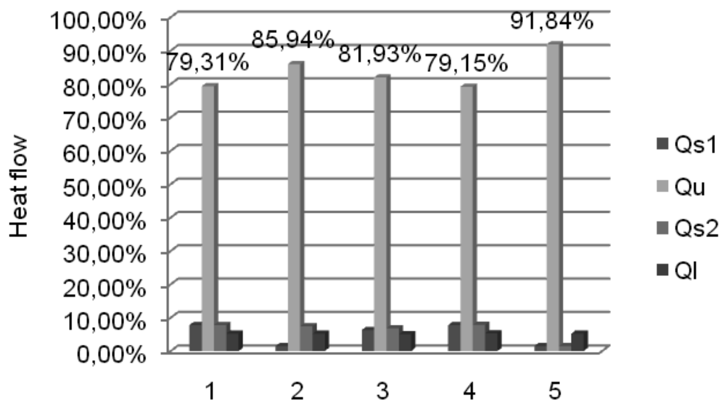


Table 1. Heat flows from all 4 sides of the panel at specific heat capacity 126.5W m^{-1}.

In the 5th case it was assumed that from the right and the left side the panel was insulated with a 10cm wooden lath, $\alpha_{s_1} = \alpha_{s_2} = 1.26W\ m^{-2}K^{-1}$. It is important to add that placing heating elements in two layers leads to increase in heat losses to the ground from 5.19% till 5.30%. In the third case the heat flow through the upper surface is quite high, accounting for 81.93%, but the amplitude of temperature oscillation is too high, which is why it is not advisable to use the equal step.

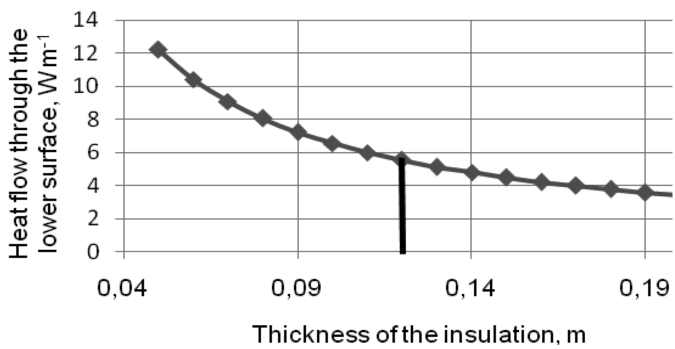


Figure 2. Heat losses through the bottom surface depending on the thickness of insulation.

From Fig. 2 it can be seen that heat flow through the lower surface dramatically increases, starting from the 12cm of insulation till 5cm of insulation. (in our case wool insulation). After 12cm insulation from the bottom heat losses are decreasing very moderately.

ACKNOWLEDGEMENT: The paper has been written with financial support from European Structural Fund – Attraction of human resources for investigation of renewable energy resources –executed by Project Department of Latvia University of Agriculture (contract No. 2009/0225/1DP/1.1.1.2.0/09/APIA/VIAA/129).

CONCLUSIONS

1. It is necessary to insulate the panel from the bottom no less than 12cm because of the sharp increase in heat losses decreasing insulation thickness below 12cm (in our case). Obtained formula (10) enables calculation of the optimal insulation thickness from the bottom, making well-founded decision from the point of view of heat loss reduction.
2. It is advisable to insulate the panel from all the sides (left, right, and bottom) which helps to make temperature distribution over the panel surface more even and to decrease heat losses gradually. Using solid insulation material, insulation can perform a casing function as well.
3. The displacement of the heating elements only slightly improves temperature distribution over the panel's upper surface. There is no need to complicate panel construction.
4. It is necessary to make an economical assessment, comparing the physical and economical parameters of different materials for the panel construction, calculating heat losses and their expenses.

REFERENCES

Iljins, U., Ziemelis, I. Providing the equal temperature on the working surface of a heated floor panel. *Proceedings of the International Scientific Conference Dedicated to the 50-th Anniversary of Water and land Management Faculty: Hydraulic Engineering and Land Management*. Lithuania, Kaunas, 1996, p. 60-67.

Riekstins, E. 1969. Methods of mathematical physics. Zvaigzne, Riga, 623 pp. (in Latvian)

Zagorska, V., Iljins, U., Ziemelis, I. Calculation of a Heated Floor Panel for Piglets Resting Places. *Proceedings of the 4th International Conference Trends in Agricultural Engineering 2010*. Prague, Czech Republic, 2010, 648-653.

Fahmy, M. H., Bernhard, C., 1971. Causes of mortality in Yorkshire pigs from birth to 20 weeks of age. *Canad. J. Anim. Sci*. **51**, 351.

Furniss, S. R., 1986. An evaluation of split-litter suckling as a method of reducing pre-weanling mortality in pigs. *Anim. Prod.* **42**, 470.

Edwards, S. A., Malkin, S. J., 1986. An analysis of piglet mortality with behavioural observation. *Anim. Prod*. **42**, 470.

English, P. R., Morrison, V., 1984. Causes and prevention of piglets mortality. *Pig News Inf.* **5**, 369–376.

Weary, M. D., Phillips, A. P., Pajor, E. A., Fraser, D., Thomson, B. K., 1998. Crushing of piglets by sows: effects of litter features and sow behaviour. *Appl. Anim. Behav. Sci*. **61**, 103–111.

Hoy, St., Ziron, M., 1998. Water bed qualities appeal to newborns. *Pig Prog*. **14**, 35–37.