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Use of Euler equations in research into three-dimensional oscillations of sugar beet root during its vibration-assisted lifting

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Abstract. Following the results of the research into the physical process of the vibratory interaction between the digging tool and the beet root, it has been found that the latter, while standing in soil, i.e. amid an elastic medium, has strong attachment to the soil in its lower (the densest and driest) part, which virtually implies one conventional fixed point. This finding provides the basis for examination of the three-dimensional motion of the beet root's body during its lifting from the ground in case of its asymmetric interaction with one of the shares of the vibrating digging tool. We have studied the gyration of the beet root's body about a point initiated by its interaction with the inclined face of the vibrating digging tool share that makes oscillatory movements in the longitudinal vertical plane. The aim of the study is to establish the values of the angular displacements of the root's body at the moment of its getting in asymmetric contact with the vibrating digging tool followed by the breaking of its bonds with the surrounding elastic medium, i.e. to develop a new mathematical model of the vibration-assisted digging of a beet root out of the soil. Basing on the use of the original equations of Euler, a new differential equation system has been obtained, which facilitates the analytical treatment of the mentioned work process. That system of differential equations for the three-dimensional oscillations of the root caused by the action of a perturbing force comprises three dynamic and three kinematic equations. It is a determined system, which makes possible its solution, i.e. the numerical modelling of the process of root lifting from the ground under different digging conditions, because it includes all necessary parameters of the vibrating digging tool, the sugar beet root and the soil surrounding it.

Key words: sugar beet root, vibrating digging tool, three-dimensional motion, lifting, modelling.

INTRODUCTION

The theoretical and field research into the work processes and the use of its results for the development of improved tools for digging sugar beet roots out of the ground are critically important tasks for the sugar beet growing industry, because just this final operation is the most complex and energy-intensive work process in the cultivation of sugar beet (Bulgakov & Ivanovs, 2010; Lammers, 2011; Lammers & Schmittmann, 2013; Gu et al., 2014). The extensive use of vibrating digging tools for the digging out of beet roots observed recently is stipulated by the lowest energy input for the break-up of the soil surrounding roots, the reduced loss and damage of roots in the harvesting process in this case. However, the described advantages are achieved under relatively favourable harvesting conditions, especially when the soil has low hardness indices and a sufficient moisture content. Under all other conditions vibrating digging tools do not display the said advantages.

Hence, that is exactly the work process that needs in-depth theoretical and field research followed by the development and implementation of improved vibrating digging tools.

Study (Dubrovski, 1968) should be regarded as the first analytical treatment of the vibration-assisted process of sugar beet root digging out of the soil. Still, the paper did not offer a mathematical model of the vibration-assisted lifting of the root from the ground. The oscillatory process of the interaction between the vibrating digging tool and the root's body was assumed to take place in the transverse vertical plane, which would not be implemented later in any digging tool employed in any commercially produced root crop harvester.

The further analytical study of the oscillations of the root's body fixed in soil was presented in publication (Vasilenko et al., 1970). As the paper itself states, the process of lifting sugar beet roots from the ground per se is analysed here with the use of the generated kinetostatic equations, which, in the authors' opinion, allow to find the conditions for the complete lifting of a sugar beet root from the soil. However, the detailed analysis of the mentioned equations has proved that they effectively do not characterise the work process of digging roots from soil.

Publication Pogorely et al. (2004) features the setting up of the Ostrogradsky-Hamilton functional that characterises the natural transverse oscillations of a sugar beet root's body modelled as a fixed low end bar. Meanwhile, it considers the case when the perturbing force is applied to the root crop in the transverse horizontal plane, which has also never been implemented in any real vibrating digging tool.

Paper (Pogorely et al., 1983), which is an essentially first published study on sugar beet harvesting machinery, adopts the main provisions and assumptions stated in the previous studies (Vasilenko et al., 1970). It should be pointed out that this work also does not furnish a model of the vibration-assisted lifting of a sugar beet root from the soil.

The further elaborations on the theory of the vibration-assisted sugar beet root digging out of the soil with application of the perturbing forces exactly in the longitudinal vertical plane were highlighted in studies (Bulgakov, 2005; Sarec et al., 2009; Bulgakov et al., 2014). Nevertheless, to advance further on it is necessary to consider separately and in more depth the dynamic system 'beet root – vibrating digging tool', in order to analyse both the process of oscillations of the beet root itself in the soil and the very process of lifting of the root as a solid body from the soil, the latter taking place under the effect of the vibrating digging tool that oscillates in the longitudinal vertical plane and also due to its translational motion.

The analysis of many literary sources (Vermeulen & Koolen, 2002; Lammers, 2011; Wang & Zhang, 2013; Wu et al., 2013; Gu et al., 2014) makes it evident that the research into new sugar beet harvesting technologies attracts substantial attention worldwide.

MATERIALS AND METHODS

In order to explore the process of sugar beet root digging, let's start with one of the first stages in this work process, when the beet root is still sufficiently strongly attached to the soil. The vibrating digging tool, advancing linearly at a preset running depth in the soil and at the same time oscillating in the longitudinal vertical plane, approaches the beet root from one side and breaks the surrounding soil. The bonds between the beet root and the soil in the immediate zone of movement of the work faces of the vibrating digging tool (on that side, from which the tool linearly advances and on which it starts making contact with the root), can be considered almost broken, while the soil surrounding the root in its upper part (within the tool running depth in the soil) is already sufficiently loosened (longitudinal fissures are possible in the soil layers across the whole thickness of the upper part). Meanwhile, the actual lifting of the beet root from the ground has not yet begun, even if the vibrating digging tool work faces have already started making contact with the root's body. It is quite apparent that at this stage of the work process of root digging out of the soil the root has already started carrying out its motions in the ground as a solid body with one fixed attachment point or a solid body pivoting about a fixed axis. It is to be stressed in the first instance that the soil surrounding the beet root provides resistance to the latter's lifting positively all over the whole conic surface, because the root gets sufficiently strongly embedded in the soil in the process of its growth.

Thus, during the straining of top soil by the work faces of the vibrating digging tool that move at a certain running depth and the subsequent breaking of bonds between the beet root and the soil at the upper part, the lower part of the root continues to reside in the unstrained and very dense layer of soil. The said circumstances substantiate the assumption that this is exactly the part of the soil, where the point on the root's symmetry axis, which can be conventionally considered the fixing point of the root in the soil, is located. But, this stage of digging the root out of soil continues only for a short while and is succeeded by the second stage, where the vibrating digging tool actually starts imparting its forces to the root body. Therefore, the motions of the beet root at the second stage can be characterised with the use of kinematic and dynamic Euler equations considering it as a symmetrical solid body with one fixed attachment point or a solid body pivoting about a fixed axis in the longitudinal vertical plane. At the same time, the beet root begins oscillating in the soil as an elastic medium, because just at this stage of the lifting the soil surrounding the root (especially on the side opposed to the tool's travel direction) can be with good reasons considered an elastic medium.

To facilitate the analytical treatment of the process of digging the beet root out of the soil by the vibrating digging tool, we build first the equivalent schematic model, where the beet root is approximated by a regular cone, point O being its only (conventional) point of fixation in the ground. The vibrating digging tool comprises two digging shares, which can be represented by two conventional wedges $A_1B_1C_1$ and $A_2B_2C_2$, their wedged forward parts and lower parts moving in soil (Fig. 1). Each of the said wedges has angles of tilting in space designated α, β and γ, which are determined so that the share faces form an angled working passage, which necks in rearwards. Structurally, the vibrating digging tool is a usual share lifter, but kinematically it is connected to a drive (not shown in the schematic model), which induces its oscillations in the longitudinal vertical plane at preset frequency and amplitude. The digging tool as a whole advances linearly at a preset velocity $\overline{V}$, down the direction shown by an arrow.

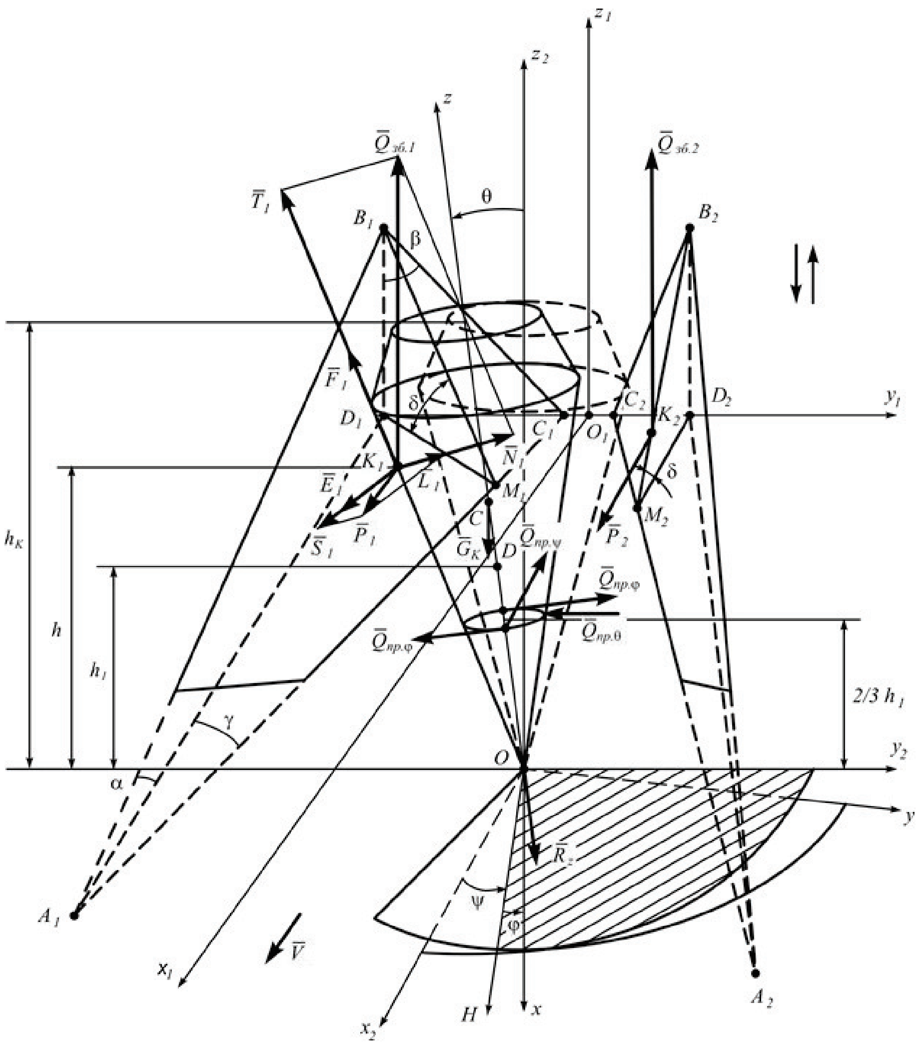

Figure 1. Equivalent schematic model of the force interaction between the vibrating digging tool and the beet root during the latter's gyration about the conventional point of fixation in the ground.

Further, we have to show in the equivalent schematic model the adopted coordinate systems. First, we relate to the vibrating digging tool the orthogonal Cartesian coordinate system $O_1x_1y_1z_1$ with the centre O_1 in the middle of its necked-in passage. In this system, the axis O_1x_1 is in line with the direction of the tool's translational motion, the axis O_1z_1 is vertically pointing up, and the axis O_1y_1 is pointing to the right (Fig. 1). The vibrating digging tool's oscillatory movements in the longitudinal vertical plane should be examined in reference to just that coordinate system $O_1x_1y_1z_1$.

We also introduce the moving coordinate system Oxyz rigidly connected with the beet root and having an origin at the point O, which is the point of fixation of the beet root in the ground, its axis Oz being aligned with the root's symmetry axis and pointing upwards, the axes Ox and Oy placed in the plane that is perpendicular to the axis Oz.

Besides, to characterise the gyration of the root about the fixation point O it is necessary to introduce one more orthogonal Cartesian coordinate system $O_2x_2y_2z_2$, which is shown in Fig. 1.

Since the vibrating digging tool at the moment of its getting in contact with the beet root advances linearly along the axis O_1x_1 (O_1x_2), the root deflects from its vertical position (effectively from the axis Oz_2) through some angle ψ unidirectionally with the motion of the tool. In the most general case, the initial contact between the beet root and the tool is asymmetric meaning that one of the digging shares gets into direct contact with the root body, while the other one makes contact through some thickness of the broken soil. This results, following the deformation of the said thickness of soil, in the beet root's deflection from its vertical position transversely through some angle θ. Moreover, the difference of the torques produced by the direct contact of the root with one of the shares and its contact with the other share through the thickness of soil can result in the rotation of the root through some angle φ about the axis Oz. Overall, summing up the above-mentioned physical conditions, we have good reasons to believe that the beet root in its interaction with the vibrating digging tool immediately during its lifting performs simultaneously the rotation about some line OH (nodal line) through the angle θ, the rotation about the axis Oz_2 through the angle ψ and the rotation about the axis Oz through the angle φ. Hence, the introduced angular displacements in space of the root during its lifting from the ground are Euler angles, the angle θ having a name of nutation angle, the angle ψ – precession angle, the angle φ – intrinsic rotation angle.

We also have to take into account that, since the root body has a conical shape, the direct contact between the digging shares and the root body is lost in case of the vibrating digging tool going down. As a result, the perturbing force stops acting on the beet root, therefore, the root under the effect of the elasticity of the soil surrounding it on the front and the root body's own elastic properties tends to return to the vertical equilibrium position. With the following upward motion the digging shares resume their contact with the root body, take hold of the root, impart to it first of all the perturbing force and the mentioned process of root's rotations recurs.

Thus, the beet root performs oscillations about the line of nodes OH, about the axis Oz_2 and about the axis Oz. Actually, the root's oscillations at the first stage of its lifting from the ground comprise the longitudinal linear oscillations of the point of the root's fixation in the ground O and the angular oscillations of the root relative to the point O characterised by the variation of the Euler angles θ, ψ and φ.

THEORY AND MODELLING

We have assumed that the beet root is in direct contact with only one of the work faces of vibrating digging tool, specifically $A_1B_1C_1$ at the point K_1, while the face $A_2B_2C_2$ acts on the root body surface via some thickness of the soil and this contact can occur at the point K_2 (Fig. 1). Certainly, the contact between the vibrating digging tool and the root body at the point K_2 is made throughout some area surrounding the point K_2, but in our further considerations we are going to assume that the point K_2 is the point of application of the forces imparted to the beet root.

Besides, the asymmetry of the contact with the beet root is also due to the fact that its symmetry axis (axis Oz) can be offset aside relative to the centre line of the sowing rows (due to the requirements of the agricultural sowing and plant handling technologies). We assume that prior to the commencement of the direct contact between the beet root and the digging tool, the axis Oz is parallel to the axis O_1z_1.

Also, we have to designate some representative points in the equivalent schematic model. Thus, the right lines drawn via the points B_1 and B_2 perpendicular to the wedge sides A_1C_1 and A_2C_2, respectively, generate at their intersections with the said wedge sides the respective points M_1 and M_2. Hence, δ is the dihedral angle ($\angle B_1M_1D_1$) between the first wedge's lower base $A_1D_1C_1$ and the work face $A_1B_1C_1$ and also the dihedral angle ($\angle B_2M_2D_2$) between the second wedge's lower base $A_2D_2C_2$ and the work face wedge $A_2D_2C_2$. Angle $2\gamma_k$ is the apical angle of the cone used as a model of the beet root. The meaning of other dimensions can be understood from the equivalent schematic model (Fig. 1).

Now, let's consider the forces originating from the interaction between the vibrating digging tool and the beet root.

Since the digging tool, as it has been stated, is a vibrational tool, it imparts the vertical perturbing force $\overline{Q}_{зб.}$, which varies under the following harmonic law:

$$Q_{зб.} = H \sin \omega t, \tag{1}$$

where: H – amplitude of the perturbing force; ω – frequency of the perturbing force.

That force plays the primary role in the process of soil breaking in the zone of the digging tool's work passage and the direct lifting of the beet root out of the ground. The perturbing force $\overline{Q}_{зб.}$ is applied to the beet root or the soil surrounding it on two sides, therefore, it is represented in the equivalent schematic model by two components $\overline{Q}_{зб.1}$ and $\overline{Q}_{зб.2}$, which apparently have the following values:

$$Q_{зб.1} = Q_{зб.2} = \frac{1}{2} H \sin \omega t \tag{2}$$

Further considerations require careful analysis of the relation between the oscillations of the vibrating digging tool and the concurrent action of the perturbing force $\overline{Q}_{зб.}$ on the root. It is sufficient to carry out the analysis only for one oscillation period, from $\omega t = 0$ to $\omega t = 2\pi$. In all other oscillation periods the process is repeated. As we already mentioned above, the perturbing force $\overline{Q}_{зб.}$ acts on the beet root only when the digging shares of the tool move up from their lowermost position to their uppermost position, making in this process contact with the conical root body.

For this reason, during the movement of the digging shares of the tool up on the interval $(0, \pi)$, the perturbing force $\overline{Q}_{зб.}$ acts on the beet root following the sinusoidal law

(1). In this process, on the interval $\left[0,\ \frac{\pi}{2}\right]$ it increases from the zero value $Q_{зб.}=0$ at the point $\omega t=0$ to the maximum value $Q_{зб.}=H$ at the point $\omega t=\frac{\pi}{2}$.

On the interval $\left[\frac{\pi}{2},\ \pi\right]$ it decreases from its maximum value $Q_{зб.}=H$ to the minimum one $Q_{зб.}=0$. On the interval (π, 2π) the digging shares of the tool move down, therefore, the perturbing force $\bar{Q}_{зб.}$ does not act on the root on this leg. On the interval (2π, 4π) everything recurs. Thus, in general on the intervals ($2k\pi$, $(2k+1)\pi$), $k = 0, 1, 2, \dots$, the perturbing force $\bar{Q}_{зб.}$ acts on the beet root following the sinusoidal law (1), and on the intervals ($(2k-1)\pi$, $2k\pi$), $k = 0, 1, 2, \dots$, it has no effect on the beet root, since it is equal to zero.

As the cutting edges A_1C_1 and A_2C_2 of the digging shares are located below the contact points K_1 and K_2, the soil in the area of the contact between the beet root and the vibrating digging tool is already sufficiently much broken, but the soil breaking occurs primarily in the front part of the tool's passage, while the direct contact between the beet root and the tool – in the middle and rear parts of the work passage. Therefore, in case of asymmetric contact with the beet root at the point K_1 the root is under the direct effect of the perturbing force $\bar{Q}_{зб.1}$, while at the contact point K_2 the perturbing force $\bar{Q}_{зб.2}$ acts only on the thickness of broken soil, which makes us assume that this latter force is virtually not imparted directly to the root body. Hence, at the first contact between the root body and the vibrating digging tool, the effect of perturbing force $\bar{Q}_{зб.2}$ on the beet root can be ignored and it can be assumed that the root is under the effect of only the perturbing force $\bar{Q}_{зб.1}$ acting from the side of the face $A_1B_1C_1$, i.e. only one digging share.

In addition, it is to be pointed out that the interesting aspect of the asymmetric contact with the beet root is that it makes possible the rotation of the root about its axis, promoting the intensive break-up of the bonds between the root and the soil (phenomenon of the root's spinning in the ground during its digging out). So, in case of asymmetric contact between the beet root and the vibrating digging tool we are going to take into the differential equations for the root's motion only the force action of the work face $A_1B_1C_1$ of one digging share. For this purpose we decompose the force $\bar{Q}_{зб.1}$ into two components: $\bar{N}_1$, normal to the face $A_1B_1C_1$, and $\bar{T}_1$, tangential to the same face, as shown in the equivalent schematic model in Fig. 1. This force is equal to:

$$\bar{Q}_{зб.1}=\bar{N}_1+\bar{T}_1 \qquad (3)$$

Apparently, the $\bar{T}_1$ force vector is parallel to the right line B_1M_1.

As the vibrating digging tool advances linearly along the axis O_1x_1 relative to the beet root fixed in the ground, we have a driving force $\overline{P_1}$ that acts along the course of the translational movement (along the axis O_1x_1) and also acts on the root along that axis at the moment, when contact is made between the beet root and digging tool. Now, let's decompose the force $\overline{P_1}$ also into two components: $\overline{L_1}$, normal to the wedge face $A_1B_1C_1$ and $\overline{S_1}$, tangential to the same face, i.e.:

$$\overline{P_1} = \overline{L_1} + \overline{S_1}. \tag{4}$$

Thus, at the contact point K_1 the beet root is under the effect of the force applied by the wedge $A_1B_1C_1$, which is equal to:

$$\overline{N}_{K_1} = \overline{N}_1 + \overline{L}_1, \tag{5}$$

and points along the normal to the surface of the wedge $A_1B_1C_1$.

Apparently, the magnitude of this force is:

$$N_{K_1} = N_1 + L_1 \tag{6}$$

Also, at the contact point K_1 the force of friction $\overline{F}_{K_1}$ is applied, which counteracts the slipping of the beet root on the work face of the wedge $A_1B_1C_1$ during its contact with the vibrating digging tool. The vector of this force is in opposition to the vector of the relative velocity of the wedge's slipping on the surface of the beet root. The root weight force $\overline{G}_k$ is applied vertically at the centre of mass of the beet root. Also, during the contact between the beet root and the vibrating digging tool, when the latter's shares move upwards, the root is under the effect of the soil's elastic deformation force acting along axis Oz, designated as $\overline{R}_z$ in the equivalent schematic model.

The tangential component $\overline{T_1}$ of the perturbing force $\overline{Q}_{зб.1}$ and the tangential component $\overline{S}_1$ of the driving force $\overline{P_1}$ do not act directly on the beet root, they only cause the breaking of the soil around the beet root, therefore, they are not included in the differential equations of the movement of the root as a solid body. Then, it is possible to derive from the equivalent schematic model (Fig. 1) the expressions for determining the normal $\overline{N}_1$ and tangential $\overline{T_1}$ components of the perturbing force $\overline{Q}_{зб.1}$, as well as the expressions for determining the normal $\overline{L}_1$ and tangential $\overline{S}_1$ components of the driving force $\overline{P_1}$.

Meanwhile, the forces, generated by the straining of the soil as an elastic medium during the displacement of the beet root in it, need to be determined.

The moment of the elastic soil deformation force due to the angular displacement of the beet root through the angle φ is equal to:

$$M_{np.\varphi} = -\int_0^{h_1}\int_0^{2\pi} \frac{c_1 z^2 dz \varphi \sin^2\gamma_k d\alpha}{2\pi\cos^3\gamma_k} = -\frac{c_1\, h_1^3 \varphi \sin^2\gamma_k}{3\cos^3\gamma_k}, \qquad (7)$$

where c_1 – elastic stiffness of the soil that determines the increase of the force acting on the contact surface in case of displacement of the contact surface for a contact area unit (N m^{-2}).

Similarly, it is possible to determine that the elastic forces in the soil resulting from the angular displacements of the beet root fixed in it about the axis Oz_2 through the angle ψ – $\overline{Q}_{np.\psi}$ and about the line of nodes OH through the angle θ – $\overline{Q}_{np.\theta}$ are equal to, respectively:

$$Q_{np.\psi} = \int_0^{h_1}\int_0^{\pi} \frac{c\sin\gamma_k \psi d\alpha\, z^2 dz}{\cos^3\gamma_k} = \frac{c\pi h_1^3 \sin\gamma_k \psi}{3\cos^3\gamma_k}, \qquad (8)$$

$$Q_{np.\theta} = \int_0^{h_1}\int_0^{\pi} \frac{c\sin\gamma_k \theta d\alpha\, z^2 dz}{\cos^3\gamma_k} = \frac{c\pi h_1^3 \sin\gamma_k \theta}{3\cos^3\gamma_k}, \qquad (9)$$

where c – elastic stiffness of the soil (ratio of the first coefficient of Winkler to the contact area) (N m^{-3}).

Apparently, the vectors $\overline{Q}_{np.\psi}$ and $\overline{Q}_{np.\theta}$ point along the normal to the surface of the beet root. The forces determined by the expressions (7, 8, 9) act as the restoring forces in the oscillatory process under consideration.

The force $\overline{R}_z$ is the resultant of the load distributed over the beet root's surface with an unbroken thickness of soil and the intensity vectors of this load point downwards parallel to the axis Oz. Therefore, the force $\overline{R}_z$ acts along the axis Oz and points downwards.

The magnitude of the force $\overline{R}_z$ is determined with the use of the following formula:

$$R_z = \int_0^{h_1}\int_0^{2\pi} \frac{c_1 z\, tg\gamma_k d\alpha\, dz}{h_1\cos\gamma_k} z_k = \frac{c_1 \pi h_1 \sin\gamma_k z_k}{\cos^2\gamma_k}. \qquad (10)$$

Further, we proceed to the generation of differential equations for the gyration about a point of the beet root as a solid body during its asymmetric contact with the vibrating digging tool. In this case, according to what was said earlier, the beet root moves as a solid body with one fixed point, the position of which is determined by the variation of the above-mentioned Euler angles φ, ψ and θ under the effect of the described forces and moments of forces acting on the root and characterised with the use of dynamic and kinematic equations of Euler.

In this case, if the moving coordinate system Oxyz is chosen in such a way that the coordinate axes are the principal axes of inertia for the point O, then the dynamic equations of Euler will take the following form (Dreizler & Lüdde, 2010):

$$\left.\begin{aligned} I_x \frac{d\omega_x}{dt} + \left(I_z - I_y\right)\omega_y\omega_z = M_x^e, \\ I_y \frac{d\omega_y}{dt} + \left(I_x - I_z\right)\omega_x\omega_z = M_y^e, \\ I_z \frac{d\omega_z}{dt} + \left(I_y - I_x\right)\omega_x\omega_y = M_z^e, \end{aligned}\right\} \tag{11}$$

where: ω_x, ω_y and ω_z – projections of the root's angular velocity during its angular displacement about the instantaneous axis of rotation on the axes of the moving coordinate system Oxyz; I_x, I_y and I_z – the moments of inertia of the beet root in reference to the coordinate axes Ox, Oy and Oz (principal axes of inertia of the root), respectively; M_x^e, M_y^e and M_z^e – principal moments of all external forces acting on the beet root in reference to the coordinate axes Ox, Oy and Oz, respectively.

As shown in the equivalent schematic model (Fig. 1), the axes of the moving coordinate system Oxyz are the principal axes of inertia of the beet root. Indeed, the axis Oz is the root's material symmetry axis. The axes Ox and Oy lie in the plane that is perpendicular to the axis Oz. According to (11), if a body has a material symmetry axis, it is a principal axis of inertia at all its points. The other two principal axes passing through any point of the symmetry axis (including the point O) lie in the planes that are perpendicular to that axis.

Further, to express the angular velocity projections ω_x, ω_y and ω_z in terms of the Euler angles and their derivatives, we have to add to the dynamic equations of Euler the kinematic Euler equations (11), which are of the following form:

$$\left.\begin{aligned} \omega_x &= \dot{\psi}\sin\theta\sin\varphi + \dot{\theta}\cos\varphi, \\ \omega_y &= \dot{\psi}\sin\theta\cos\varphi + \dot{\theta}\sin\varphi, \\ \omega_z &= \dot{\psi}\cos\theta + \dot{\varphi}. \end{aligned}\right\} \tag{12}$$

RESULTS AND DISCUSSION

It is possible to derive from the equivalent schematic model (Fig. 1) the moments of the external forces acting on the beet root during its contact with the vibrating digging tool about the axes Ox, Oy and Oz. After the substitution of the necessary axial moments of inertia and the derived magnitudes of the principal moments of all external forces into the system of differential equations, we obtain the following system of differential equations for the three-dimensional oscillations of the beet root fixed in the ground in the form of dynamic and kinematic equations of Euler:

$$
\left.
\begin{aligned}
&\left(0{,}48+0{,}15tg^2\gamma_k\right)m_k h_k^2\frac{d\omega_x}{dt}+\left(0{,}15tg^2\gamma_k+0{,}52\right)m_k h_k^2\omega_x\omega_z=\\
&=\Big[-P_1\left(h\,tg\gamma_k-h\theta\right)-f\left(0{,}5H\cos\delta\sin\omega t+P_1\sin\gamma\right)\cos\left(\alpha_{K_1\max}\sin\omega t-\gamma\right)\times\\
&\times\left(h\,tg\gamma_k-h\theta\right)+\frac{c\pi h_1^4\sin\gamma_k\theta\psi\left[\cos\left(\gamma_k-\theta\right)+\cos\left(\gamma_k+\psi\right)\right]}{4\cos^3\gamma_k}\Big]\sin\theta\sin\varphi+\\
&+\Big[-0{,}5H\,h\,tg\gamma_k\sin\omega t+hP_1\sin\psi+f\left(0{,}5H\cos\delta\sin\omega t+P_1\sin\gamma\right)\times\\
&\times\cos\left(\alpha_{K_1\max}\sin\omega t-\gamma\right)\sin\psi h+\frac{2}{3}G_k h_k\theta-\frac{c\pi h_1^4\sin\gamma_k\theta\cos\psi}{4\cos^3\gamma_k}\Big]\cos\varphi\,,\\
&\left(0{,}48+0{,}15tg^2\gamma_k\right)m_k h_k^2\frac{d\omega_y}{dt}+\left(0{,}48-0{,}15tg^2\gamma_k\right)m_k h_k^2\omega_z\omega_x=\\
&=\Big[-P_1\left(h\,tg\gamma_k-h\theta\right)-f\left(0{,}5H\cos\delta\sin\omega t+P_1\sin\gamma\right)\cos\left(\alpha_{K_1\max}\sin\omega t-\gamma\right)\times\\
&\times\left(h\,tg\gamma_k-h\theta\right)+\frac{c\pi h_1^4\sin\gamma_k\theta\psi\left[\cos\left(\gamma_k-\theta\right)+\cos\left(\gamma_k+\psi\right)\right]}{4\cos^3\gamma_k}\Big]\sin\theta\sin\varphi-\\
&-\Big[-0{,}5H\,h\,tg\gamma_k\sin\omega t+hP_1\sin\psi+f\left(0{,}5H\cos\delta\sin\omega t+P_1\sin\gamma\right)\times\\
&\times\cos\left(\alpha_{K_1\max}\sin\omega t-\gamma\right)\sin\psi h+\frac{2}{3}G_k h_k\theta-\frac{c\pi h_1^4\sin\gamma_k\theta\cos\psi}{4\cos^3\gamma_k}\Big]\sin\varphi\,,\\
&0{,}3m_k h_k^2 tg^2\gamma_k\frac{d\omega_z}{dt}=hP_1\cos\theta tg\gamma_k+f\left(0{,}5H\cos\delta\sin\omega t+P_1\sin\gamma\right)\times\\
&\times\cos\left(\alpha_{K_1\max}\sin\omega t-\gamma\right)\cos\theta\,tg\gamma_k h-\frac{c_1h_1^3\varphi\sin^2\gamma_k}{3\cos^3\gamma_k}+\Big[P_1\left(h\,tg\gamma_k-h\theta\right)+\\
&+f\left(0{,}5H\cos\delta\sin\omega t+P_1\sin\gamma\right)\cos\left(\alpha_{K_1\max}\sin\omega t-\gamma\right)\left(h tg\gamma_k-h\theta\right)-\\
&-\frac{c\pi h_1^4\sin\gamma_k\theta\psi\left[\cos\left(\gamma_k-\theta\right)+\cos\left(\gamma_k+\psi\right)\right]}{4\cos^3\gamma_k}\Big]\cos\theta,\\
&\omega_x=-\dot\psi\sin\theta\sin\varphi-\dot\theta\cos\varphi,\\
&\omega_y=-\dot\psi\sin\theta\cos\varphi+\dot\theta\sin\varphi,\\
&\omega_z=\dot\psi\cos\theta+\dot\varphi.
\end{aligned}
\right\}\qquad (13)
$$

We have obtained the system of differential equations (13) for the three-dimensional oscillations of the beet root caused by the perturbing force, which is a determined system that allows to carry out the multivariate modelling of the beet root digging process, because it contains the parameters of the vibrating digging tool, sugar beet root and the soil surrounding the root.

The solution of the differential equation system allows to determine first of all the law of the three-dimensional oscillations of the beet root about the conventional point of fixation in the ground, i.e. find the functions $\varphi = \varphi(t)$, $\psi = \psi(t)$ and $\theta = \theta(t)$. According to the results of calculations, if we assume the following averaged initial data: mass of root $m_k = 0.9$ kg; mass of soil surrounding root $m_{гр.} = 0.4$ kg; length of root $h_k = 0.25$ m; angles of trihedral wedges of vibrating digging tool $\gamma = 14°$, $\beta = 52°$; friction coefficient of steel on root body surface $f = 0.45$; amplitude of perturbing force $H = 500$ N; magnitude of lateral driving force $P_1 = 400$ N; maximum angle of deviation of friction force vector from the vector of this force at minimum magnitude $\alpha_{K_1 \max} = 30°$; frequency of oscillations of digging shares of vibrating digging tool $\nu = 10$ Hz; cone angle of the root $\gamma_k = 15°$; elastic stiffness of soil $c = 3 \cdot 10^5$ N m^{-3}, $c_1 = 2 \cdot 10^5$ N m^{-2}; current time $t = 0.025$ s, then the Euler angles obtain the following values: $\varphi = 10°$, $\psi = 9°$, $\theta = 7°$.

It is to be noted as well that the system of differential equations (13) characterises not only the three-dimensional oscillatory process, but also the angular displacement of the beet root about its own axis (spinning phenomenon), which has an especially notable effect on the process of breaking up the bonds between the root and the soil during the first stage of its lifting from the ground.

The results of the accomplished analytical treatment have been used in the design and engineering of new vibrating digging tools for state-of-the-art sugar beet harvesting machines.

CONCLUSIONS

1. The physical process of a sugar beet root standing in soil as an elastic medium and its interaction with a vibrating digging tool at the first stage of its lifting from the ground has been investigated.

2. It has been established that, for the purposes of analytical treatment of the process of sugar beet root lifting from the ground, the root can be represented by an elastic conical body residing in elastic medium and having one fixed point at its bottom. All necessary conditions for the use of kinematic and dynamic equations of Euler in the research into the root lifting have been obtained.

3. The new mathematical model of the vibration-assisted beet root lifting from the ground with the use of the theory of the body motion about a fixed point has been developed.

4. Using the original kinematic and dynamic equations of Euler, the system of differential equations has been set up for the oscillations of the beet root during its vibration-assisted lifting in the case, when the root interacts only with one digging share at one its point, i.e. when an asymmetric contact with the root body occurs.

5. Following the solution of the system of Euler differential equations, particular values have been obtained for the angular displacements of the beet root about the

coordinate axes, at which its efficient lifting from the ground is achieved. Specifically, for the average values of the parameters of the vibrating digging tool, root body and the soil surrounding it included in the equations, the Euler angles have the following numeric values: $\varphi = 10°$, $\psi = 9°$, $\theta = 7°$.

6. The obtained mathematical model allows to carry out the multivariate modelling of the process of vibration-assisted root digging out of the soil under various conditions of harvesting.

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