Agronomy Research **14**(3), 689–710, 2016

Theory of vertical oscillations and dynamic stability of combined tractor-implement unit

V. Bulgakov[1], V. Adamchuk[2], M. Arak[3], V. Nadykto[4], V. Kyurchev[4] and J. Olt[3,*]

[1]National University of Life and Environmental Sciences of Ukraine, 15, Heroyiv Oborony Str., UK 03041 Kyiv, Ukraine
[2]National Scientific Centre, Institute for Agricultural Engineering and Electrification, 11, Vokzalna Str., Glevaкha-1, Vasylkiv District, UK 08631 Kiev Region, Ukraine
[3]Estonian University of Life Sciences, Kreutzwaldi 56, EE51014 Tartu, Estonia
[4]Tavria State Agrotechnological University of Ukraine, Khmelnytskoho pr. 18, Melitopol, UK 72312 Zaporozhye region, Ukraine
[*]Correspondence: jyri.olt@emu.ee

Abstract. Currently, throughout the world quite extensive use is made of combined tractor-implement units, which are capable of performing several process operations in the same pass. At the same time, the state-of-the-art ploughing and general-purpose tractors that can carry as front- so rear-mounted implements and accordingly feature both the front and rear PTOs, also able to travel efficiently as forward so in reverse gear, are most suited for the performance of such operations. Authors developed and successfully tested a combined tractor-implement unit on the basis of a wheeled ploughing and general-purpose tractor, which can in one pass efficiently chop the after harvesting crop residues with a front-mounted rotary chopper and simultaneously perform tillage with a rear-mounted plough. The aim of this study is the elaboration of the theoretical basis for the process of vertical oscillation of the combined ploughing and chopping tractor-implement unit and the validation of its dynamic stability in the longitudinal and vertical plane. The research has been performed with the use of the methods of designing the analytical mathematical models of functioning of agricultural machines and machine assembly units based on the theory of tractor, the vibration theory, the theory of automatic control and dynamic stability and the methods of computer programme construction and PC-assisted numerical computation. The dynamics of the said unit have been studied basing on the analysis of the amplitude frequency characteristics of the unit as a dynamic system responding to external perturbations appearing in the form of soil surface irregularities. Following the results of the undertaken analytical study, first the equivalent schematic model of the discussed combined tractor-implement unit in the longitudinal and vertical plane was developed, the unit's characteristic points were defined, the linear and angular displacements specified and acting forces applied. Each pneumatic-tyre wheel of the unit represented by its elastically damping model had point contacts with the soil surface irregularities defined by the respective elevations. Using the original dynamic equations in the form of the Lagrange equations of the second kind, first we defined the generalised coordinates and the formulae for the kinetic and potential energy, dissipation functions and generalised forces, then, after performing the necessary transformations, we set up the system of four differential equations, which described the motion of the dynamic system under consideration. Further, we applied the Laplace transformations to the obtained differential equation system, which provided for obtaining the system of equations in the operator form and preparing them for the representation suitable for PC-assisted numerical calculations with the use of the developed computer programme. In accordance with the numerical computation results, graphs were plotted for the amplitude and phase frequency response characteristics of the tractor's vertical oscillations at different stiffness coefficients of its steering wheels, the amplitude frequency response characteristics of the chopper's oscillations depending on its mass and its support wheel tyres' stiffness coefficient as well as the characteristics of the plough's oscillations at different stiffness coefficients of its pneumatic-tyre ground support wheel.

Key words: tractor-implement unit, dynamic system, elastically damping model, oscillation, modelling.

INTRODUCTION

The wide application of multi-purpose combined tractor-implement units (Li et al., 2015; Xu et al., 2015) capable of performing several process operations in the same pass is stipulated by their apparent advantages as regards the significant reduction of costs, shorter running times, lower soil compacting effect, improvement of the quality indicators etc. (Nadykto et al., 2015). The most appropriate power units for such combined tractor-implement assemblies are state-of-the-art ploughing and general-purpose tractors that can carry as front- so rear-mounted implements and accordingly feature both the front and rear PTOs, also able to travel efficiently as forward so in reverse gear.

On the basis of the wheeled ploughing and general-purpose tractor we developed and then successfully tested a combined tractor-implement unit, which can in one pass efficiently capture, chop and spread after harvesting crop residues (dead and laid, up to one metre tall) using the front-mounted rotary chopper and simultaneously perform tillage with the rear-mounted plough, i.e. Plough the said residues down to the required depth. It is exactly these two process operations performed just in the described order that are used most often and feature the highest efficiency in their application.

Authors also developed another combined unit, which is also of current interest and achieves equally high efficiency in operation. It is a combined unit, which in the same pass strews mineral fertilisers over the field with the use of its front-mounted fertiliser spreader and ploughs them under with the rear-mounted plough.

At the same time, the construction and efficient operation of state-of-the-art wide-span combined tractor-implement units calls accordingly for the development of the new research and technology fundamentals of their combining. That includes first of all analysing their dynamic properties and finding the parameters that provide for their stable motion, when different work processes are performed simultaneously.

The research into the dynamics of agricultural tractor-implement units has been the subject of quite a number of studies, including the classical works by P. Vasilenko (1962; 1968; 1996), L. Gyachev (1981), M. Karkee (2009), Larson et al. (1976), Mircae & Nicolae (2014), Rabbani et al. (2011) and others. But, the objects of research in the above-mentioned works were agricultural tractor-implement units combining only one agricultural machine, either front-mounted or towed behind (rear-mounted). Meanwhile, the analytical investigation of the said tractor-implement units as dynamic systems was made in horizontal or vertical plains and the differential equations were most often derived with the use of the original dynamic equations in the form of the Lagrange equations of the second kind. The research into the dynamics of combined agricultural tractor-implement units (featuring front-mounted and trail-behind machines) is the subject of works (Mitsuoka et al., 2008; Pădureanu et al., 2013). Nevertheless, in the mentioned works the analysis of the said units' oscillations in the longitudinal and vertical plane not always takes into consideration the case, when the support and gauge wheels of the agricultural machines have pneumatic tyres and have to be represented by elastically damping models (i.e. They have to feature the respective stiffness and damping coefficients), instead they are considered as rigid structures. Moreover, currently the up-to-date plough designs provide for the use of pneumatic-tyre wheels as the ground support wheels and also several such wheels can be installed in the rear-mounted plough. This implies the need to update the mathematical models of motion of combined tractor-implement units by developing their equivalent schematic models that are more accurate and at the same time take into account the real operating conditions to the maximum possible extent, thereafter setting up the differential equations of motion. Meanwhile, it is the research into the combined tractor-implement units that involves the examination of their oscillatory motions just in the longitudinal and vertical plane, which to a considerable extent determine the traction and operation properties of the units and quality performance of the combined implements, that is of the greatest interest.

The aim of this study is to develop the theoretical fundamentals of the vertical oscillation of the combined tractor-implement unit comprising a wheeled ploughing and general-purpose tractor, a rear-mounted plough and a front-mounted vegetation chopper and to substantiate its dynamic stability in the longitudinal and vertical plane.

MATERIALS AND METHODS

The research has been carried out with the application of the methodology for the modelling of the functioning of agricultural implements and implement units; the theory of tractor; the higher mathematics; the theoretical mechanics, in particular, the use of the original equations in the form of the Lagrange equations of the second kind, the Laplace transformation; as well as the principles of computer programme construction and pc-assisted numerical computation.

THEORY AND MODELLING

When the mentioned chopping and ploughing tractor-implement unit travels in operation, the front-mounted vegetation residue chopper and the rear-mounted plough impart to the carrying wheeled tractor vibration (jolts, hits) caused by the soil surface profile irregularities, the varying resistance of the tilling tool, the intermittent loads on the chopper and so on. In general, all these three segments of the tractor-implement unit are in this case subject to translational vertical and angular displacements in the longitudinal and vertical plane.

To develop the analytical mathematical model of the combined tractor-implement unit under consideration, we have first of all to devise its equivalent schematic model, taking into consideration only the motions in the longitudinal and vertical plane (Fig. 1). It is to be noted in advance that the detailed description of this equivalent schematic model will be provided step by step during the detailed discussion and modelling of the components of the tractor-implement unit.

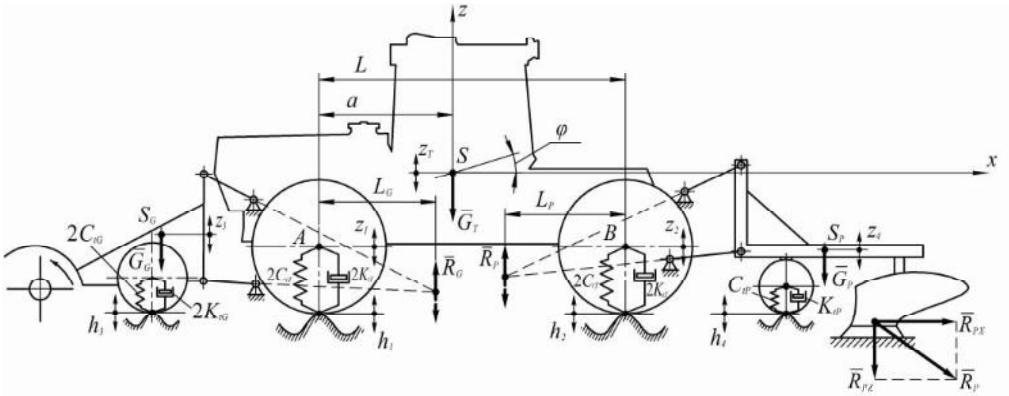


Figure 1. Equivalent schematic model of the combined chopping and ploughing tractor-implement unit in the longitudinal and vertical plane.

The interaction between the carrying tractor and the front- and rear-mounted agricultural implements is through the lower and central arms of the tractor's rear and front implement suspension mechanisms. When the unit travels in operation, the main (working) position of these mechanisms is floating. Therefore, when the tractor-implement unit moves performing the work process, we have every reason to ignore the angular oscillations of the front-mounted vegetation residue chopper and the rear-mounted plough. Their most notable rotation in the longitudinal and vertical plane could take place only in case the combined unit was negotiating rather high soil surface irregularities occurring at a sufficiently high rate. But, the probability of meeting such irregularities is minimal, because the macrorelief of the fields operated in the present-day agriculture is in most cases sufficiently even.

In view of the above, let's lay down the main assumptions to be used in the development of the analytical mathematical model of a combined tractor-implement unit designed on the basis of a ploughing and general-purpose tractor:

1. Since the angular oscillations of the process-related parts of the chopping and ploughing tractor-implement unit are insignificant, we assume that the sine and tangent of the small argument are approximately equal to the argument itself, while the cosine is equal to unity.

2. In order to simplify the solving of the set problem, it is most reasonable to set up the differential equations of the vertical oscillations of the chopping and ploughing tractor-implement unit individually for each of its segments (i.e. The tractor, the vegetation residue chopper and the plough). Their mutual influence on each other will be represented by forces that have equal magnitudes and opposite directions and are applied at the points of the instantaneous centres of turn of the tractor's front and rear implement suspension mechanisms.

3. The movement of the tractor as part of the chopping and ploughing unit is created by the right wheels in the furrow. Meanwhile, the inclination of the tractor in the longitudinal and transverse plane is taken into account by its positioning, with respect to the front-mounted vegetation residue chopper and the rear-mounted plough, on a horizontal plane, below the field surface by a half of the tilling depth. In that case, it is

possible to consider the vertical loads applied to the wheels on the same axle of the tractor as equal.

4. Further, we assume that during the working movement of the tractor-implement unit under consideration, the wheeled tractor retains its constant point contact with the soil surface, i.e. with the surface of the agricultural background.

5. Meanwhile, the variation of the soil surface irregularities is described by a stochastic stationary and ergodic function of the distance.

6. If the amplitude of the vertical variation of the longitudinal profile of the soil surface irregularities is insignificant, then it is possible to assume that the resisting forces in the pneumatic wheel tyres are proportionate to the variation velocity, while their resilient members have linear characteristics.

Following the made assumptions, we will first design the analytical mathematical model of vertical oscillations only for the carrying tractor. To solve this problem, we will examine a ploughing and general-purpose tractor as part of a chopping and ploughing tractor-implement unit, representing it by a separate equivalent schematic model (Fig. 2). The wheels of the tractor are represented by elastically damping models with the stiffness coefficients C_{tT} of its tyres and the coefficients of resistance to deformation (damping) K_{tT} of the tyres. Since the equivalent schematic model (Fig. 2) represents two wheels on each axle of the tractor, the said coefficients are doubled accordingly. Each of the tractor's wheels (front and rear ones) has a point contact with the soil surface and travels over its irregularities, the heights of which are denoted as follows: h_1 – for the irregularities under the front wheels of the tractor and h_2 – under the rear wheels of the tractor.

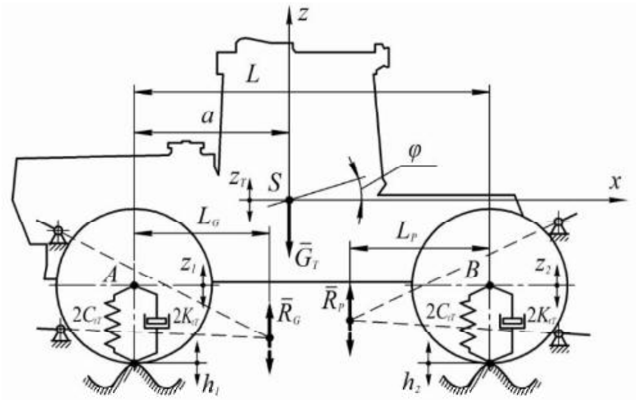


Figure 2. Equivalent schematic model of vertical oscillations of carrying tractor.

Now we are going to show in the equivalent schematic model all the forces that act on the tractor during its movement. In accordance with the assumptions described earlier, the action of the front-mounted vegetation residue chopper and the rear-mounted plough on the tractor will be represented by the reactions $\bar{R}_G$ and $\bar{R}_P$, localised at the points of instantaneous centre of turn of the front and rear implement suspension mechanisms of the tractor, respectively. The said reactions $\bar{R}_G$ and $\bar{R}_P$ are situated at a distance of L_G and L_P from the centre line of the front and rear axles of the tractor, respectively. The tractor is also subjected to the action of the force of its weight $\bar{G}_T$, localised at its centre of mass (point S). The longitudinal base length of the tractor is denoted by L, while a is the distance from the tractor's front axle to its centre of mass.

Let's define now the Cartesian coordinate system xSz in the equivalent schematic model, the origin of which coincides with the tractor's centre of mass (point S), the axis x is directed horizontally towards the tractor's traction wheels, the axis z – vertically upwards.

In the described representation, the analytical mathematical model of the tractor as part of the combined chopping and ploughing unit features two degrees of freedom: vertical oscillations z_T of its centre of mass (point S) and angular oscillations φ of the frame.

The differential equations of motion (oscillation) of the tractor in the longitudinal and vertical plane will be set up in the form of the Lagrange equations of the second kind as follows (Dreizler & Lüdde, 2010):

$$\frac{d}{dt}\left(\frac{\partial T_T}{\partial \dot{q}_i}\right)-\frac{\partial T_T}{\partial q_i}+\frac{\partial E_T}{\partial q_i}+\frac{\partial D_T}{\partial \dot{q}_i}=Q_i, \tag{1}$$

where: q_i – generalised coordinate (i = 1, 2); T_T – the tractor's kinetic energy; E_T – the tractor's potential energy; D_T – the tractor's energy dissipation function; Q_i – generalised force.

Now we will determine the components of the expression (1).

First of all, the formula will be determined for the kinetic energy T_T of the vertical oscillations of the tractor, and it will have the following form:

$$T_T=\frac{M_T\dot{z}_T^2+J_T\dot{\varphi}^2}{2}, \tag{2}$$

where: M_T – the tractor's mass (kg); J_T – the tractor's moment of inertia about the axis, which runs through its centre of mass (point S) and is normal to the longitudinal and vertical plane (kg m^2).

Further, let's determine the components of the expression (2). First of all, the generalised coordinates z_T and φ are in a certain manner related to the vertical displacements of the tractor's front and rear axles, i.e. to z_1 (point A) and z_2 (point B). Therefore, the said relation can be analytically represented by the following two functions:

$$z_T=\frac{z_1(L-a)+z_2a}{L}, \tag{3}$$

$$tan\varphi=\frac{z_2-z_1}{L}, \tag{4}$$

where L and a – longitudinal base length and longitudinal coordinate of the tractor's centre of mass (point S) (m).

Since we have $\tan\varphi\approx\varphi$ for small angular displacements, which was described earlier, the expression (4) can be written down in a simplified form:

$$\varphi=\frac{z_2-z_1}{L}, \tag{5}$$

Further, after differentiating the expressions (3) and (5), we will obtain:

$$\dot{z}_T = \frac{\dot{z}_1 (L-a) + \dot{z}_2 a}{L}, \tag{6}$$

$$\dot{\varphi} = \frac{\dot{z}_2 - \dot{z}_1}{L}. \tag{71}$$

After substituting the derivative values from (6) and (7) in expression (2), then carrying out the relevant transformations and denoting the coefficients D_1, D_2 and D_3 , we will come to the expression for the tractor's kinetic energy in the following form:

$$T_T = \frac{D_1 \dot{z}_1^2 + 2D_2 \dot{z}_1 \dot{z}_2 + D_3 \dot{z}_2^2}{2}, \tag{8}$$

where: $D_1 = \frac{M_T (L-a)^2 + J_T}{L^2}$; $D_2 = \frac{M_T a(L-a) - J_T}{L^2}$; $D_3 = \frac{M_T a^2 + J_T}{L^2}$.

Now, let's perform operations in accordance with the original equation (1). As the tractor's kinetic energy T_T depends only on the speed and does not depend on the generalised coordinate, so:

$$\frac{\partial T_T}{\partial q_i} = 0. \tag{9}$$

Thereafter, we will find the partial derivatives of the kinetic energy T_T with respect to the velocities on the generalised coordinates, which will appear in the following form:

$$\frac{\partial T_T}{\partial \dot{z}_1} = D_1 \dot{z}_1 + D_2 \dot{z}_2, \tag{10}$$

$$\frac{\partial T_T}{\partial \dot{z}_2} = D_2 \dot{z}_1 + D_3 \dot{z}_2, \tag{11}$$

The partial time derivatives of the expressions (10) and (11) are determined as follows:

$$\frac{d}{dt}\frac{\partial T_T}{\partial \dot{z}_1} = D_1 \ddot{z}_1 + D_2 \ddot{z}_2, \tag{12}$$

$$\frac{d}{dt}\frac{\partial T_T}{\partial \dot{z}_2} = D_2 \ddot{z}_1 + D_3 \ddot{z}_2. \tag{13}$$

Further, we will find the tractor's potential energy E_T. It will be equal to the work of the elastic forces on the tractor's front and rear axles. The said elastic forces are functions of the deflection of the respective elastic members, i.e. the tyres of the wheeled tractor's running gear. If we denote the deflection of the front wheel by z_{fw}, and of the rear wheel – z_{rw}, measuring these values from the static state of equilibrium of the dynamic system under consideration, then their magnitudes can be determined as follows:

$$\left. \begin{aligned} z_{fw} &= z_1 - h_1, \\ z_{rw} &= z_2 - h_2, \end{aligned} \right\} \tag{14}$$

where: h_1, h_2 – heights of the soil surface irregularities under the front and rear wheels of the tractor, respectively (m).

The time variable soil surface irregularity magnitudes h_1 and h_2 are just that perturbing factor originating from the agricultural background, which is the actual cause of the vertical oscillations of all segments of the tractor-implement unit under consideration.

The front and rear axle wheels of a ploughing and general-purpose tractor are equipped with identical wheels and also the tyres of each of the axles shown in the equivalent schematic model are in reality duplicated. Taking into account the abovesaid, the formula for finding the potential energy E_T of the tractor will appear as follows:

$$E_T = C_{tT}\ z_{fw}^2 + C_{tT}\ z_{rw}^2 = C_{tT}\left(z_{fw}^2 + z_{rw}^2\right). \tag{15}$$

Taking into account the expressions (14), the potential energy E_T of the tractor will have the following final representation:

$$E_T = C_{tT}\left(z_1^2 - 2z_1h_1 + h_1^2 + z_2^2 - 2z_2h_2 + h_2^2\right). \tag{16}$$

The partial derivatives of the potential energy E_T will be as follows:

$$\frac{\partial E_T}{\partial z_1} = 2C_{tT}\left(z_1 - h_1\right), \tag{17}$$

$$\frac{\partial E_T}{\partial z_2} = 2C_{tT}\left(z_2 - h_2\right). \tag{18}$$

The tractor's energy dissipation function D_T will be determined in terms of the resistance forces, which are proportionate to the displacement velocities. The said resistance forces are also caused by the wheel tyres of the tractor's running gear. As already mentioned for the case under consideration, i.e. for a ploughing and general-purpose tractor, the wheels on the front and rear axles have identical tyres shown doubled in the equivalent schematic model, hence the tractor's energy dissipation function D_T will have the following form:

$$D_T = K_{tT}\ \dot{z}_{fw}^2 + K_{tT}\ \dot{z}_{rw}^2 = K_{tT}\left(\dot{z}_{fw}^2 + \dot{z}_{rw}^2\right). \tag{19}$$

In view of the system of equations (14), the expression (19) determining the dissipation function D_T will finally appear as follows:

$$D_T = K_{tT}\left(\dot{z}_1^2 - 2\dot{z}_1\dot{h}_1 + \dot{h}_1^2 + \dot{z}_2^2 - 2\dot{z}_2\dot{h}_2 + \dot{h}_2^2\right). \tag{20}$$

Then, the partial derivatives for the dissipation function D_T will be represented by the following expressions:

$$\frac{\partial D_T}{\partial \dot{z}_1} = 2K_{tT}\left(\dot{z}_1 - \dot{h}_1\right), \tag{21}$$

$$\frac{\partial D_T}{\partial \dot{z}_2} = 2K_{tT}\left(\dot{z}_2 - \dot{h}_2\right). \tag{22}$$

Now the only component that remains undefined in the expression (1) is the generalised forces Q_i. Since the analytical mathematical model of the tractor as part of the combined unit under consideration has two degrees of freedom, such generalised forces will also be two.

In order to determine them, we are going to impart to the dynamic system the virtual displacement δz_1. At the same time, the displacement of the tractor's rear axle will be held at the zero level, hence $\delta z_2 = 0$. Then we have the following active forces that perform work on the mentioned virtual displacement of the system: $\overline{R}_G$ and $\overline{R}_P$.

Now we compute the sum of the amounts of work δA by these forces on the virtual displacement of point A. It will be equal to:

$$\delta A = R_G \delta z_{(R_G)} + R_P \delta z_{(R_P)} - G_T \delta z_{(G_T)}, \tag{23}$$

where: $\delta z_{(R_G)}$, $\delta z_{(R_P)}$ and $\delta z_{(G_T)}$ – vertical displacements of the points of application of the forces $\overline{R}_G$, $\overline{R}_P$ and $\bar{G}_T$.

Taking into account the condition $\delta z_2 = 0$, we find from the expressions (6):

$$\delta z_{(G_T)} = \frac{\delta z_1 (L - a)}{L}. \tag{24}$$

Similar to that, we can write down:

$$\delta z_{(R_G)} = \frac{\delta z_1 (L - L_G)}{L}, \tag{25}$$

$$\delta z_{(R_P)} = \frac{\delta z_1 L_P}{L}. \tag{26}$$

As a result, the formula for determining the work by the forces R_G and R_P on the virtual displacement of the dynamic system δz_1 will appear as follows:

$$\delta A = \left[\frac{R_G (L - L_G) + R_P L_P - G_T (L - a)}{L} \right] \delta z_1 \tag{27}$$

From the expression (27) we can derive the generalised force Q_{z_1} causing the displacement δz_1. It will be equal to:

$$Q_{z_1} = \frac{R_G (L - L_G) + R_P L_P - G_T (L - a)}{L}. \tag{28}$$

Similarly, we determine also the second generalised force Q_{z_2}:

$$Q_{z_2} = \frac{R_G L_G + R_P (L - L_P) - G_T a}{L}. \tag{29}$$

Thus, we have found all components that constitute the expression (1), and it is possible to substitute them, perform the required transformations and obtain a system comprising the two differential equations of the forced oscillations of the ploughing and general-purpose tractor in the longitudinal and vertical plane, and this will just be the tractor's analytical mathematical model:

$$\left. \begin{aligned} A_{11}\ddot{z}_1 + A_{12}\dot{z}_1 + A_{13}z_1 + A_{14}\ddot{z}_2 &= f_{11}\dot{h}_1 + f_{12}h_1 + f_{13}, \\ A_{21}\ddot{z}_2 + A_{22}\dot{z}_2 + A_{23}z_2 + A_{24}\ddot{z}_1 &= f_{21}\dot{h}_2 + f_{22}h_2 + f_{23}, \end{aligned} \right\} \tag{30}$$

where

$$A_{11}=\frac{M_T\left(L-a\right)^2+J_T}{L^2};\quad A_{21}=\frac{M_Ta^2+J_T}{L^2};\quad A_{12}=2K_{tT};\quad A_{22}=A_{12};\quad A_{13}=2C_{tT};$$

$$A_{23}=A_{13};\quad A_{14}=\frac{M_Ta\left(L-a\right)-J_T}{L^2};\quad A_{24}=A_{14};\quad f_{11}=f_{21}=A_{12};\quad f_{12}=f_{22}=A_{13};$$

$$f_{13}=\frac{R_G\left(L-L_G\right)+R_PL_P-G_T\left(L-a\right)}{L};\; f_{23}=\frac{R_GL_G+R_P\left(L-L_P\right)-G_Ta}{L}.$$

Hereafter, we are going to develop, in accordance with the earlier made assumptions, the analytical mathematical model of the vegetation residue chopper that is front-mounted on the tractor.

In order to analyse the tractor front-mounted vegetation residue chopper as a dynamic model, we will use, the same as in the previous case, its equivalent schematic model (Fig. 3). The chopper's centre of mass is represented by the point S_G, which is made the point of origin of the orthogonal Cartesian coordinate system xS_Gz, in which the axis x is directed horizontally to the right, the axis z – vertically upwards. The two support and gauge pneumatic-tyre wheels of the chopper are represented by elastically damping models shown in the equivalent schematic model as one wheel with the doubled coefficients of: stiffness $2C_{tG}$ and damping $2K_{tG}$. Now we have to denote the forces applied to the chopper in the longitudinal and vertical plane. These forces are: weight force $\bar{G}_G$ applied at the point S_G and the force $\bar{R}_G$ generated by the implement-carrying tractor and applied at the point of the instantaneous centre of turn of its front implement suspension mechanism (the force has the same magnitude as the force already used in the consideration of the tractor's oscillations, but its direction is opposite). The vertical component of the force generated by the chopper's cutting unit when mowing the vegetation residues is ignored because of its insignificant magnitude.

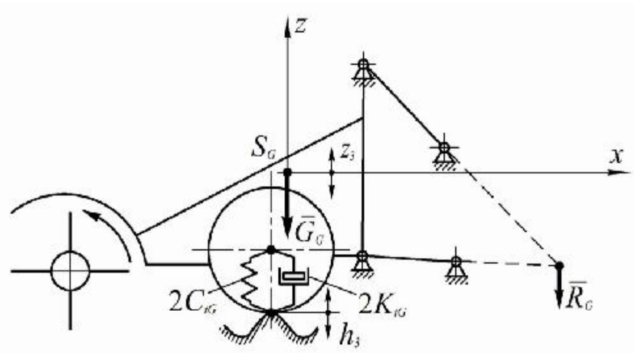


Figure 3. Equivalent schematic model of vertical oscillations of vegetation residue chopper front-mounted on tractor.

The chopper's gauge wheels also have point contacts with the soil surface irregularities, the height of which is denoted by h_3. The chopper has one degree of freedom in the longitudinal and vertical plane, which is the vertical displacement of its centre of mass (point S_G) – z_3. This vertical displacement can be regarded as the generalised coordinate q_3.

Using the original equations (1), we will set up the differential equations of motion (oscillation) of the vegetation residue chopper front-mounted on the carrying wheeled tractor. First of all, let's determine the chopper's kinetic energy T_G and potential energy E_G. They are described by the following expressions:

$$T_G = \frac{M_G \dot{z}_3^2}{2}, \tag{31}$$

$$E_G = C_{tG} \left(z_3 - h_3 \right)^2, \tag{32}$$

where M_G – mass of the vegetation residue chopper (kg).

The energy dissipation function D_G for the chopper, which is directly proportional to the velocity of the vertical displacement of its centre of mass, is represented by such an expression:

$$D_T = K_{tG} \left(\dot{z}_3 - \dot{h}_3 \right)^2. \tag{33}$$

Further, let's determine the generalised force. Since the analytical mathematical model of the vegetation residue chopper front-mounted on the tractor as part of the combined unit under consideration has one degree of freedom, we will have one generalised force Q_{z_3}.

First we will compute the sum of the amounts of work δA by all active forces effective on the virtual displacement of point S_G. It will be equal to:

$$\delta A = -\left(R_G \delta z_3 + G_G \delta z_3 \right). \tag{34}$$

The generalised force Q_{z_3} determined by the expression (34), which causes the vertical displacements of the centre of mass (point S_G) of the chopper, will be equal to:

$$Q_{z_3} = -\left(R_G + G_G \right). \tag{35}$$

Now we will use the derived expressions (31)-(33) and (35). We will carry out the necessary transformations of them, then substitute them in the equation (1) and obtain the analytical mathematical model of the forced vertical oscillations of the vegetation residue chopper front-mounted on the carrying tractor in the following form:

$$A_{31} \ddot{z}_3 + A_{32} \dot{z}_3 + A_{33} z_3 = f_{31} \dot{h}_3 + f_{32} h_3 + f_{33}, \tag{36}$$

where: $A_{31} = M_G$; $A_{32} = 2K_{tG}$; $A_{33} = 2C_{tG}$; $f_{31} = A_{32}$; $f_{32} = A_{33}$; $f_{33} = -\left(R_G + G_G \right)$.

Now, employing the earlier made assumptions, we are going to develop an analytical mathematical model for the rear-mounted plough.

Meanwhile, we should mention beforehand that the mounted plough as part of the combined unit can either feature a supporting ground wheel made in the form of a smooth steel wheel or it can be equipped with a supporting ground wheel that has a pneumatic tyre on its rim, the second of the options being currently widely used in the designs of state-of-the-art mounted ploughs (especially the gang / multi-furrow ones). In this latter case the rear-mounted plough of the discussed combined chopping and ploughing tractor-implement unit will during its operation have its own, independent vertical oscillations, which have to be taken into account as well. Therefore, we have developed for the rear-mounted plough an equivalent schematic model, which contains the pneumatic-tyre ground support wheel. As in the previous cases, the wheel is represented by the elastically damping model, which is shown in the equivalent schematic model in the form of coefficients of stiffness C_{tP} and damping K_{tP} (Fig. 4). And again, the plough's pneumatic-tyre support wheel has point contacts with the soil surface irregularities, the height of which is denoted by h_4.

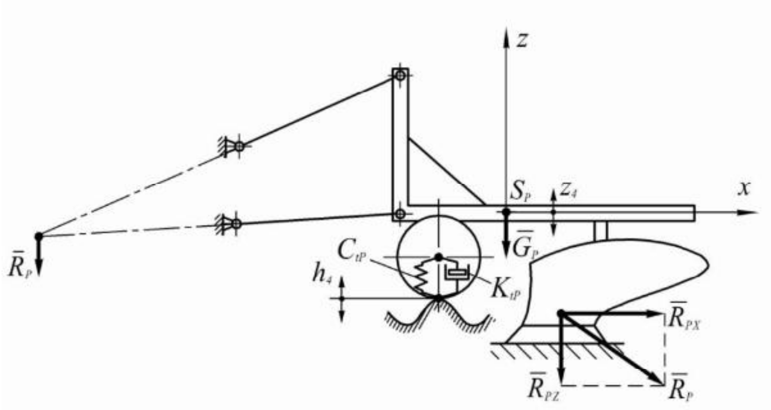


Figure 4. Equivalent schematic model of vertical oscillations of plough that is rear-mounted on tractor.

In the equivalent schematic model the centre of mass of the rear-mounted plough is represented by the point S_P, which is made the point of origin of the orthogonal Cartesian coordinate system xS_Pz, in which the axis x is horizontal and directed to the right, the axis z is directed upwards. The force applied at the mounted plough's centre of mass is the force of gravity $\bar{G}_P$. Another force, acting on the mounted plough in the longitudinal and vertical plane, is the force $\bar{R}_P$ – it is generated by the implement-carrying tractor and applied at the point of the instantaneous centre of turn of its rear implement suspension mechanism (the force has the same magnitude as the force already used in the consideration of the tractor's oscillations, but it has the opposite direction). Besides that, there is one more force acting in the said plane – the vertical component $\bar{R}_Z$ of the tractive resistance $\bar{R}_P$. When ploughs are tested, the horizontal component $\bar{R}_X$ of the tractive resistance is always determined. Since it is known that $\bar{R}_P = \bar{R}_X + \bar{R}_Z$ and according to the data in Macmillan (2002) $R_Z \approx 0,2R_X$.

In the longitudinal and vertical plane the rear-mounted plough also has one degree of freedom, which is the vertical displacement of its centre of mass (point S_P) – z_4. This vertical displacement of the plough's centre of mass can be regarded as the generalised coordinate q_4.

Now we are going to determine the plough's kinetic energy T_P and potential energy E_P, as well as the dissipation function D_P for the plough. They are described by the following expressions:

$$T_P = \frac{M_P \dot{z}_4^2}{2}, \tag{37}$$

$$E_P = C_{tP}\left(z_4 - h_4\right)^2, \tag{38}$$

$$D_P = K_{tP}\left(\dot{z}_4 - \dot{h}_4\right)^2, \tag{39}$$

where M_P – mass of the rear-mounted plough (kg).

To compute the generalised force Q_{z_4} we will use the expressions similar to the ones presented earlier. Thus, the sum of the amounts of work δA by all active forces effective on the virtual displacement of point S_P will be equal to:

$$\delta A = -\left(R_P \delta z_4 + G_P \delta z_4 + R_{PZ} \delta z_4\right). \tag{40}$$

The generalised force Q_{z_4} determined from the expression (40), which causes the vertical displacements of the plough's centre of mass (point S_P), will be equal to:

$$Q_{z_4} = -\left(R_P + G_P + R_{PZ}\right). \tag{41}$$

Using the derived expressions (37)–(39) and (41), after substituting them in the equation (1) and carrying out the necessary transformations, we will obtain the analytical mathematical model of the vertical oscillations of the rear-mounted plough in the following form:

$$A_{41}\ddot{z}_3 + A_{42}\dot{z}_4 + A_{43}z_4 = f_{41}\dot{h}_4 + f_{42}h_4 + f_{43}, \tag{42}$$

where

$A_{41} = M_P$; $A_{42} = K_{tP}$; $A_{43} = C_{tP}$; $f_{41} = A_{42}$; $f_{42} = A_{43}$; $f_{43} = -\left(R_P + G_P + R_{PZ}\right)$.

The differential equation (42) describes the oscillatory motion of the rear-mounted plough in the longitudinal and vertical plane.

Hence, if we consolidate the differential equations of the vertical oscillations of the carrying tractor (30), front-mounted vegetation residue chopper (36) and rear-mounted plough (42), we will obtain the analytical mathematical model of the chopping and ploughing tractor-implement unit in the longitudinal and vertical plane:

$$\left.\begin{aligned}
&A_{11}\ddot{z}_1 + A_{12}\dot{z}_1 + A_{13}z_1 + A_{14}\ddot{z}_2 = f_{11}\dot{h}_1 + f_{12}h_1 + f_{13},\\
&A_{21}\ddot{z}_2 + A_{22}\dot{z}_2 + A_{23}z_2 + A_{24}\ddot{z}_1 = f_{21}\dot{h}_2 + f_{22}h_2 + f_{23},\\
&A_{31}\ddot{z}_3 + A_{32}\dot{z}_3 + A_{33}z_3 = f_{31}\dot{h}_3 + f_{32}h_3 + f_{33},\\
&A_{41}\ddot{z}_3 + A_{42}\dot{z}_4 + A_{43}z_4 = f_{41}\dot{h}_4 + f_{42}h_4 + f_{43}.
\end{aligned}\right\} \tag{43}$$

The system of four differential equations (43) describes the process of vertical oscillations of the combined tractor-implement unit comprising a wheeled ploughing and general-purpose tractor, a rear-mounted plough and a front-mounted vegetation residue chopper, the constant coefficients of which were presented earlier.

The system of differential equations (43) in the presented form has the following input parameters:

1. Heights of the soil surface irregularities under the front h_1 and rear h_2 wheels of the carrying tractor, under the wheels of the vegetation residue chopper h_3 and under the wheel of the rear-mounted plough h_4;

2. The tractive resistance of the rear-mounted plough represented by its vertical component R_{PZ};

3. Some design parameters of the combined tractor-implement unit under consideration represented by the coefficients f_{14}, f_{24}, f_{34} and f_{44}.

The output parameters of the obtained system of differential equations (43) are the vertical displacements (amplitudes of oscillation) of the front z_1 and rear z_2 axles of the carrying tractor, the centre of mass of the front-mounted vegetation residue chopper z_3 and the centre of mass of the rear-mounted plough z_4.

If we carry out the Laplace transformation in the system of differential equations (43) by means of entering the operator $p = \frac{d}{dt}$, then we come to the presentation of the said system of differential equations in the operator form as follows:

$$\left.\begin{aligned} &K_{11}z_1(p) + K_{12}z_2(p) = F_{11}h_1(p) + F_{15}R_Z(p) + F_{16}, \\ &K_{21}z_1(p) + K_{22}z_2(p) = F_{22}h_2(p) + F_{25}R_Z(p) + F_{26}, \\ &K_{33}z_3(p) = F_{33}h_3(p) + F_{36}, \\ &K_{44}z_4(p) = F_{44}h_4(p) + F_{45}R_Z(p) + F_{46}. \end{aligned}\right\} \tag{44}$$

where

$K_{11} = A_{11}p^2 + A_{12}p + A_{13}$; $K_{12} = A_{14}p^2$; $K_{21} = A_{24}p^2$; $K_{22} = A_{21}p^2 + A_{22}p + A_{23}$; $K_{33} = A_{31}p^2 + A_{32}p + A_{33}$; $K_{44} = A_{41}p^2 + A_{42}p + A_{43}$; $F_{11} = f_{11}p + f_{12}$; $F_{15} = f_{13}$; $F_{16} = f_{14}$; $F_{22} = f_{21}p + f_{22}$; $F_{25} = f_{23}$; $F_{26} = f_{24}$; $F_{33} = f_{31}p + f_{32}$; $K_{36} = f_{34}$; $F_{44} = f_{41}p + f_{42}$; $F_{45} = f_{43}$; $F_{46} = f_{44}$.

Thus, the system of differential equations (44) in the presented form represents the analytical mathematical model of the combined chopping and ploughing tractor-implement unit.

RESULTS AND DISCUSSIONS

The effect that the arrangement and parameters of the chopping and ploughing tractor-implement unit have on the smoothness of its movements in the longitudinal and vertical plane can be evaluated with the use of the amplitude and phase frequency characteristics that describe the response of the dynamic system under consideration to external perturbations. In this case, such perturbations are:

1. Variation of the height of soil surface irregularities under the front wheels of the tractor – h_1;
2. Variation of the height of soil surface irregularities under the rear wheels of the tractor – h_2;
3. Variation of the height of soil surface irregularities under the wheels of the chopper – h_3;
4. Variation of the height of soil surface irregularities under the pneumatic-tyre support wheel of the plough – h_4;
5. Variation of the rear-mounted plough's tractive resistance represented by its vertical component – R_{PZ}.

After a computer programme was developed for the PC-assisted numerical computation of the obtained system of equations in the operator form (44), calculations were carried out, the results of which allowed plotting the graphs of the amplitude frequency response and phase frequency response characteristics. Subsequently, their values were compared with the most desired values. For this purpose, similar characteristics of the ideal follow-up dynamic systems were taken as the desired ones. It is to be pointed out that, when such ideal dynamic systems respond to external perturbations, the amplitude frequency response in the operating range shall tend to zero, while their phase frequency response vice versa – to infinity. Accordingly, those amplitude frequency response and phase frequency response characteristics obtained for the unit under consideration, which are the closest to the desired ones, will be the most suitable for the evaluation of the efficiency of the dynamics and the design.

Following the data obtained from the laboratory and field experimental investigations that we carried out earlier and the PC-assisted processing of its results with the use of statistical methods, it has been determined that the main spectrum of dispersions of the soil surface profile irregularity variation is located in a sufficiently wide frequency range of 0…15 m^{-1}. The argument of this normalised spectral density is the frequency ω in m^{-1} (Fig. 5, a). Further, we carry out the transition to the argument t (s), and as a result we obtain the normalised spectral density of the soil surface profile irregularity variation – its graph is presented in Fig. 5, b.

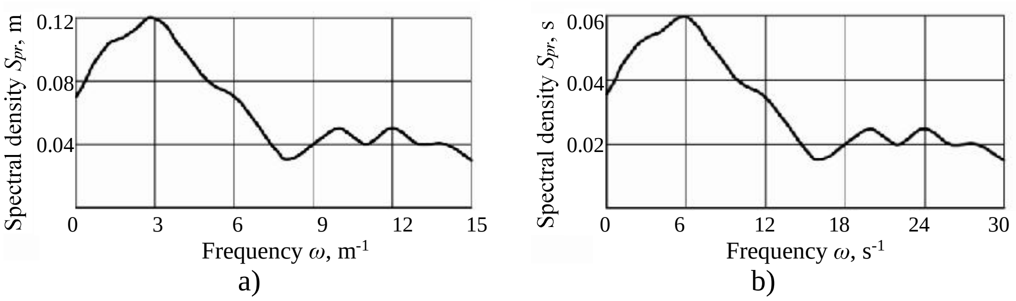


Figure 5. Normalised spectral density S_{pr} of variation of longitudinal soil surface irregularity profile as function of frequency ω (a) and time t (b).

Analysing the data in Fig. 5 b, one can see that the working range of frequencies for such an input parameter as the variation of soil surface irregularities is 0…30 s^{-1}, and this is the range that we are going to use in our subsequent analytical investigations.

First of all, let's investigate the dynamics of the vertical oscillations of the front axle of the tractor during its travelling as part of the chopping and ploughing unit. Doing that, the design amplitude and phase frequency response characteristics will be analysed, as we pointed out earlier, in that frequency range, where virtually all the dispersion of the agricultural background irregularity variation is located, i.e. within 0…30 s^{-1} (Fig. 5).

The results of the PC-assisted calculations have made it possible to determine the effect the elastic properties of the pneumatic tyres on the wheels of the carrying tractor, the support and gauge wheels of the vegetation residue chopper and the ground support wheels of the rear-mounted plough have on the smoothness of the movements of the combined tractor-implement unit under consideration.

The results of the performed calculations have shown that the increase of the stiffness coefficients of the pneumatic tyres on the wheels of the components of the combined tractor-implement unit improves the unit's response to the perturbing actions. This is most evident in the graph of the amplitude frequency response characteristic for the tractor's front axle (Fig. 6).

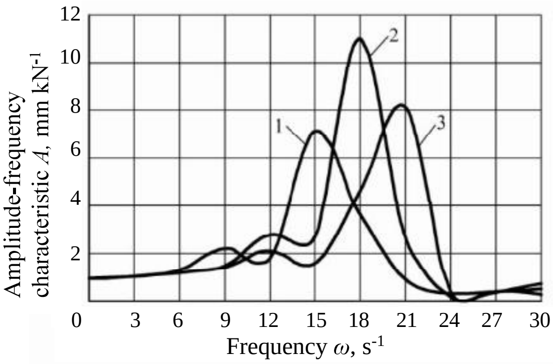


Figure 6. Amplitude frequency response characteristic of vertical oscillations of front axle of tractor, when it responds to variation of soil surface profile at different stiffness coefficients of tyres on its wheels C_{tT}: 1 – 250 kN m^{-1}; 2 – 350 kN m^{-1}; 3 – 450 kN m^{-1}.

It can be seen in the graphs in Fig. 6 that, when the value of C_{tT} increases from 250 kN m^{-1} to 450 kN m^{-1}, the amplitude frequency response characteristics decrease, which is the most desired effect, while the resonance peaks shift towards the higher frequencies of variation of the longitudinal soil surface profile irregularities. The effect is explained by the fact that the increase of the coefficient C_{tT} is followed by the decrease of the elastic properties of the pneumatic tyres on the wheels. As a result, this dynamic segment responds to the input signal with a smaller gain. But, it is apparent that this type of behaviour of the amplitude frequency response characteristics takes place, when $\omega > 12$ s^{-1} or almost 12 Hz.

The lag of the unit's response to the perturbing action depends little on the magnitudes of the stiffness coefficients of the tractor's wheel tyres. Within a perturbing action variation frequency range of 0…9 s^{-1} there is virtually no difference between the obtained phase shifts on the phase frequency response characteristics (Fig. 7).

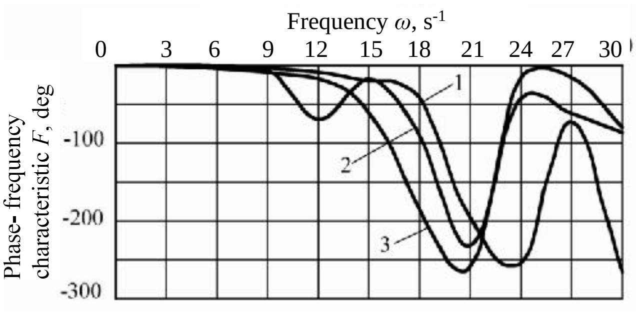


Figure 7. Phase frequency response characteristics of vertical oscillations of front axle of tractor, when it responds to variation of soil surface profile at different stiffness coefficients of tyres on its wheels C_{tT}: 1 – 250 kN m^{-1}; 2 – 350 kN m^{-1}; 3 – 450 kN m^{-1}.

At this point, it is also to be noted that, according to the results of our study, the stiffness coefficient C_{tT} of the pneumatic tyres on the rear wheels of the carrying tractor, the same as with its front wheels, has the similar effect on the dynamics of the vertical oscillations of the power unit of the combined tractor-implement unit under consideration.

However, it has been found that, unlike the stiffness of the front and rear wheels of the carrying tractor, the coefficients of resistance to deformation K_{tT} of the pneumatic tyres on its wheels have little effect on the smoothness of motion of the chopping and ploughing tractor-implement unit.

There is one very important point in the study – determination of the extent, to which the oscillations of the tractor's front and rear axles influence each other. The analytical amplitude frequency response characteristics show that the dynamics of their vertical displacements are independent. Thus, when the variation of the soil surface irregularity profile under the front wheels of the tractor stipulate the respective response of its front axle, the same variation has virtually no effect on the dynamics of the vertical displacements of the wheels on the rear axle (Fig. 8). Even in the resonance condition at $\omega = 12\ s^{-1}$ (Fig. 8) the size of the amplitude frequency response characteristic under consideration is so small that does not exceed a value of 0.04.

The nature of the vertical displacements of the front-mounted vegetation residue chopper, while being independent of the tractor's oscillations, depends on the implement's own certain design parameters. This includes, first of all, the stiffness coefficient C_{tG} of the support wheel tyres.

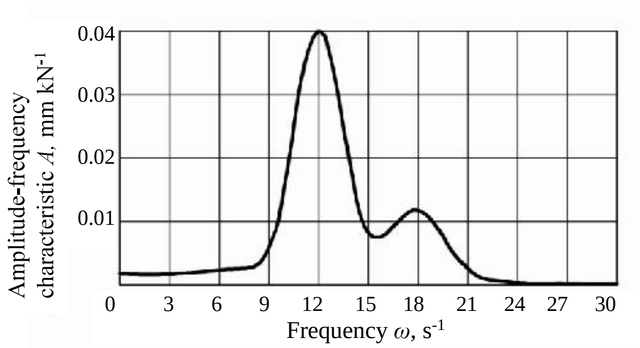


Figure 8. Amplitude frequency response characteristic of oscillations of tractor's rear axle, when it responds to variation of soil surface profile under tractor's front wheels.

For each examined value of this parameter, when we increase the frequency of soil surface irregularity profile variation, the amplitude frequency response characteristics of the chopper's vertical oscillations first grow, then, after reaching their maximum, they decrease, which is much desired (Fig. 9).

Within the range of frequencies $\omega = 0\ldots16\ s^{-1}$ (i.e. where the major share of the dispersion of soil surface profile variation is located, Fig. 5) this decrease is achieved through the increase of the coefficient C_{tG} value from 100 kN m^{-1} to 150 kN m^{-1}. In practice there is no need to apply greater values of C_{tG}, since the amplitude frequency response characteristic in a frequency range of $\omega = 0\ldots16\ s^{-1}$ decreases in that event insignificantly (curve 4, Fig. 9). At the same time, it is inexpedient to set the value of the coefficient C_{tG} below 100 kN m^{-1}, since in that case the respective amplitude frequency response characteristic increases, which is undesirable (curve 1, Fig. 9).

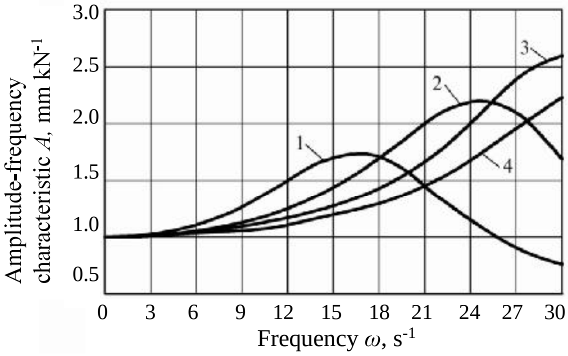


Figure 9. Amplitude frequency response characteristics of vertical oscillations of chopper's frame, when it responds to variation of soil surface profile at different tyre stiffness coefficients C_{tG}: 1 – 50 kN m^{-1}; 2 – 100 kN m^{-1}; 3 – 150 kN m^{-1}; 4 – 200 kN m^{-1}.

The second design parameter that has an effect on the dynamics of the vegetation residue chopper's vertical oscillations is its operating mass M_G. Its increase from 300 kg to 500 kg results in the undesirable increase of the amplitude frequency characteristic of the chopper's response to the field profile variation (curve 2, Fig. 10).

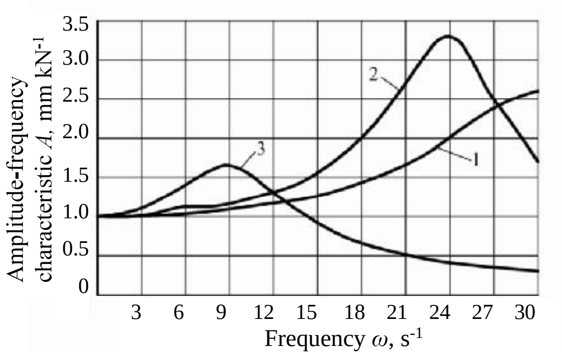


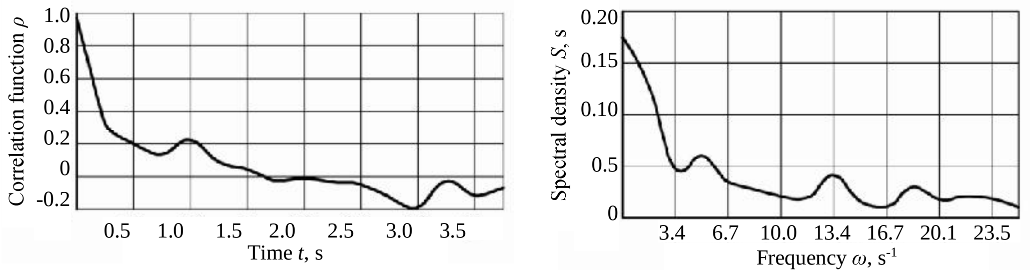
Figure 10. Amplitude frequency response characteristic of vertical oscillations of chopper's frame, when it responds to soil surface profile variation at different masses M_G and tyre stiffness coefficients C_{tG}: 1 – M_G = 300 kg; C_{tG} = 150 kN m^{-1}; 2 – M_G = 500 kg; C_{tG} = 150 kN m^{-1}; 3 – M_G = 300 kg; C_{tG} = 25 kN m^{-1}.

And that result cannot be remedied even by a substantial reduction of the stiffness coefficient C_{tG} of the pneumatic tyres of the front-mounted implement under consideration to a level of 25 kN m^{-1}. On the one hand, the amplitude frequency response characteristic decreases, at frequencies of ω > 14 s^{-1} it even falls below unity (curve 3, Fig. 10). But, on the other hand, at the soil surface profile variation frequencies, which are significant for the unit's operation, i.e. within ω = 0…9 s^{-1}, the magnitude of this characteristic exceeds to a considerable extent the value that is characteristic of the vertical oscillations of the chopper with a mass M_G equal to 300 kg (curve 1, Fig. 10).

Hence, we come to the conclusion that it is inadvisable to increase the operating mass M_G of the front-mounted vegetation residue chopper.

The next important point in the analytical study is to determine, how the chopping and ploughing unit's motion smoothness depends on the variation of the plough's tractive resistance $\bar{R}_{PX}$. To that end, we need first of all know the underlying structure of the force $\bar{R}_{PX}$ variation process. This information, as we know, is contained in its correlation function and spectral density.

The analysis of the experimental data, which we obtained earlier, shows that the variation of the plough's tractive resistance has non-periodic and relatively high frequency nature. The average duration of the correlation dependence for the process of variation of this parameter is approximately 1.6 s (Fig. 11, a).



a) b)

Figure 11. Normalised correlation function (a) and normalised spectral density (b) of variation of tractive resistance of plough as part of chopping and ploughing tractor-implement unit.

The frequency range of dispersion of the tilling tool tractive resistance variation is in this case equal to 0...25.0 s^{-1} (fig. 11, b). But, since the main part of this statistical characteristic falls within a frequency range of 0...15 s^{-1}, our further analysis will be done within just that range.

We start with the assessment of how the variation of the plough's tractive resistance affects the smoothness of motion of the tractor's rear axle at different values of its tyres' stiffness coefficient C_{tG}. Within a frequency range of $\omega = 0…6$ s^{-1} for the variation of the force $\bar{R}_{PX}$, the amplitude of the vertical displacements of the tractor's rear axle is virtually independent of the variation of the value C_{tG} (Fig. 12).

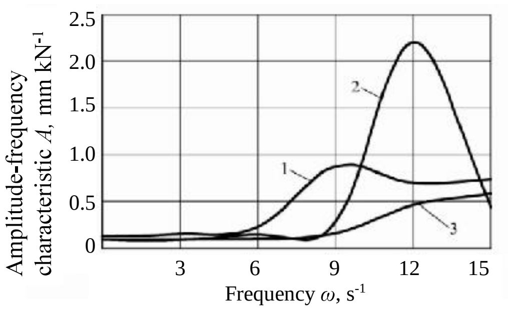


Figure 12. Amplitude frequency response characteristic of vertical oscillations of tractor's rear axle, when it responds to variation of rear-mounted plough's tractive resistance at different coefficients C_{tP}: 1 – 250 kN m^{-1}; 2 – 350 kN m^{-1}; 3 – 450 kN m^{-1}.

Outside the said frequency range, the following trend is observed: the greater the value of the coefficient C_{TP} is, the less the tractor's axle responds to the plough's tractive resistance variation. I.e. The harder the pneumatic tyre is, the lower its elastic properties are and, respectively, the lower the amplitude of its deflection in response to the impact of the perturbing factor, i.e. The variation of the force $\bar{P}_{PX}$, is.

Finally, we have investigated the response of the rear-mounted plough to the variation of its own tractive resistance. The analysis of amplitude frequency response characteristics (Fig. 13) has shown that the reduction of the stiffness C_{tP} of the pneumatic tyre on its ground support wheel results in the deterioration of the tractor's motion dynamics in the longitudinal and vertical plane. This is especially noticeable, when the coefficient of stiffness C_{tP} is equal to 100 kN m^{-1} (curve 1, Fig. 13).

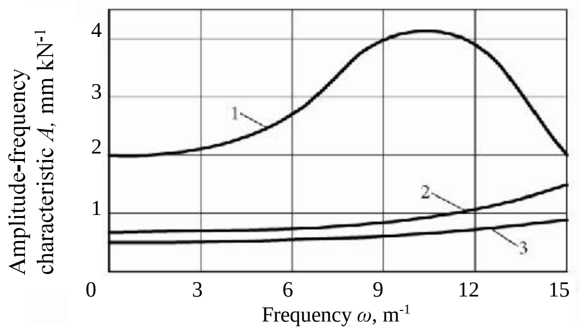


Figure 13. Amplitude frequency response characteristic of vertical oscillations of plough, when it responds to variation of its own tractive resistance at different coefficients of stiffness C_{tP} of tyre on its ground support wheel: 1 – 100 kN m^{-1}; 2 – 150 kN m^{-1}; 3 – 200 kN m^{-1}.

Following the results of the analysis of Fig. 13, the coefficient C_{tP} of the tyre on the plough's pneumatic-tyre support wheel shall be not less than 150 kN m^{-1}. At its lower values (curve 1, Fig. 13) an undesirable and, besides, considerable increase of the amplitude frequency response characteristic is observed, while this characteristic indicates the smoothness of the tractor's travel as function of its tractive resistance variation.

CONCLUSIONS

1. An equivalent schematic model has been developed for the new design of the chopping and ploughing tractor-implement unit, including three components of the dynamic system under consideration, to which the active forces are applied as well as the perturbing actions in the form of the given soil surface irregularities. The pneumatic-tyre wheels of the running gear have been approximated by elastically damping models, the linear and angular parameters have been defined.

2. Basing on the use of the original dynamic equations in the form of the Lagrange equations of the second kind, the defined generalised coordinates and the expressions for the kinetic energy, potential energy and dissipation function, a system of differential equations has been obtained, which describes the vertical oscillations of the combined tractor-implement unit under consideration.

3. The amplitude and phase frequency response characteristics of the unit have been computed on the basis of the obtained differential equations of its vertical

oscillations. Their analysis has shown that, in order to improve the smoothness (stability) of motion of the Class 3 tractor, the coefficient of stiffness of the tyres on its running wheels has to be increased to a value of 450 kN m^{-1}. In that case, the amplitude frequency response characteristics representing the response of the tractor to the perturbing actions, decrease, which is desired, while their maximum values shift towards higher frequencies, which means lower dispersions of the soil surface profile variation. At the same time, the phase frequency response characteristics representing the lag in the tractor's response to the external perturbing actions change only little.

4. Within a frequency range of $\omega = 0\ldots16$ s^{-1}, where the main spectrum of the field profile variation frequencies is located (and that is typical for most fields), the coefficient of stiffness of the tyres on the support wheels of the vegetation residue chopper and the tyre on the support wheel of the rear-mounted plough shall be at a level of 150 kN m^{-1}. It follows from the analysis of the amplitude frequency response characteristics that the compliance with this requirement ensures the lowest impact made by these implements on the dynamics of the tractor's vertical oscillations during its operation as part of the combined chopping and ploughing unit under consideration.

5. In order to achieve the sufficient smoothness (stability) of the tractor's motion, the operating mass of the front-mounted vegetation residue chopper should not be increased. Otherwise, the amplitude frequency response characteristics of the carrying tractor's vertical oscillations show undesirable increase to such an extent that this cannot be remedied even by a six-fold (from 150 to 25 kN m^{-1}) reduction of the coefficient of stiffness of the tyres on the support wheels of the front-mounted chopper.

6. The developed methodology of generating an analytical mathematical model for the combined tractor-implement unit under consideration can be used in the research into the dynamics of other agricultural implements and tractor-implement units.

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