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Theoretical research into the motion of combined fertilising and sowing tractor-implement unit

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Abstract. A mathematical model has been developed representing the motion of a seed drill combination simultaneously performing the preceding banded placement of mineral fertilisers. Such a combined unit comprises the gang-up wheeled tractor, the fertiliser distribution module behind the tractor attached to it with the use of a hitch and intended for the banded placement of mineral fertilisers and the grain drill behind the fertiliser distribution module attached to it also with the use of a hitch. For the components of this dynamic system the coordinates of their centres, their masses as well as the external forces and the reactions of the soil surface applied to them have been determined. In order to use the original dynamic equations in the form of the Lagrange equations of the second kind, the generalised coordinates and kinetic energy relations have been determined. Following the necessary transformations, a system of six differential equations of motion has been generated, which characterises the behaviour of the combined machine unit during its plane-parallel motion. In this system, two line coordinates and one angular coordinate characterise the behaviour of the propulsion and power unit (wheeled tractor), while three angular coordinates characterise the rotations of the draft gear and the centres of the machines integrated with its use.

Key words: tractor, generalised force, kinetic energy, plane-parallel motion, modelling.

INTRODUCTION

The methodology of generating analytical mathematical models of agricultural machines and machine units is rather comprehensively presented in the numerous works by P.M. Vasilenko (1996; 1980) and A. Vilde & A. Rucis (2012). It is to be noted that the main type of motion of just agricultural machines (towed, direct-mounted and self-propelled) is their plane-parallel motion, because this type of motion determines the quality of performance of the aimed work processes. Many studies have been published about the research into the operation of combined agricultural machine units (Endrerud, 1999; Macmillan, 2002; Kutkov, 2004; Karayel & Özmerzi, 2008; Schreiber & Kutzbach, 2008; Jingling et al., 2011; Xin et al., 2012. Altikat et al., 2013; Fleischmann

et al., 2013; Šarauskis & Vaitauskiene, 2014; Valainis et al., 2014; Nadykto et al., 2015; Bulgakov et al., 2016).

It should be stressed that the agro-technical and performance data of combined machine and tractor units as well as their productivity depend to a considerable extent on the nature of just their plane-parallel motion. Therefore, research into the plane-parallel motion of various machine units is needed both for the comparative assessment of the existing ones and the design of new concepts. The basic method of such research is the generation and solution of differential equations of the motion of machine combinations (Vasilenko, 1996).

The aim of this study was optimising the kinematic and design parameters of the combined fertilising and sowing tractor-implement unit that comprises a wheeled combine tractor with a fertiliser distributor for strip fertilisation and a grain drill trained behind the tractor, on the basis of the computational solution of the derived differential equations of its plane parallel motion.

MATERIALS AND METHODS

The methods of generating analytical mathematical models for machines and machine units, based on the use of the theoretical mechanics, the higher mathematics, the theory of tractor, programming and numerical calculations with the use of the PC have been used in the study.

The completed numerous agronomical experimental field studies have shown that the application of fertilisers together with the planting of grain and other agricultural crops, when the starter doses of fertilisers are applied on the seed bed and the main doses of fertilisers are applied below the seeding-down level with an offset in the horizontal plane, allows to achieve the substantial saving of fertilisers, 30…45% on average. Thus, the combined performance of the grain and other agricultural crop seeding operation simultaneously with the main fertiliser application to the soil proves to be an efficient resource-saving measure. Thereby, it becomes necessary to arrange and study such combined tractor-implement units, which could implement simultaneously both the seeding-down and the application of the starter and main doses of fertilisers.

In order to achieve that aim, the analytical mathematical model of the said combined tractor-implement unit need to be generated. The unit includes the wheeled combine tractor, to which first the fertilising unit is hitched with the use of an implement-attaching linkage, then follows the seeding unit kinematically connected, also with the use of an implement-attaching linkage, to the fertilising unit.

In order to generate the analytical mathematical model of such a combined fertilising and sowing unit, certain provisions generally applied in modelling will be used. We will begin with designing the equivalent schematic model of the combined unit under consideration, which requires first making certain assumptions.

For example, it is necessary to take into account only the main elements of the combined tractor-implement unit, which effect various motions, while being parts of a dynamic system. Since the dynamic system under consideration is a multi-mass system, the calculations can be simplified by taking into consideration only the motions that have an effect on the quality of the work process performance. The machine unit (dynamic system) will be referred to the fixed Cartesian coordinate system Oxyz. It is also assumed

that, during the progression of the combined unit on the surface of the field, all its points move in the planes that are parallel to plane xOy (Fig. 1).

In order to generate the system of differential equations of motion of the mechanical system under consideration, it will be taken in its current position, then its position during its motion on the plane will be described with the use of six independent generalised coordinates. Also, it is assumed that at the initial instant $t = 0$ the mechanical system was aligned along the axis Ox and started moving from the quiescent state.

Because of that, the motion of the mechanical system under consideration will be described by six differential equations of second order with reference to the mentioned independent generalised coordinates. Hence, the mathematical model of the tractor-implement unit will also be the model of a mechanical system with six degrees of freedom.

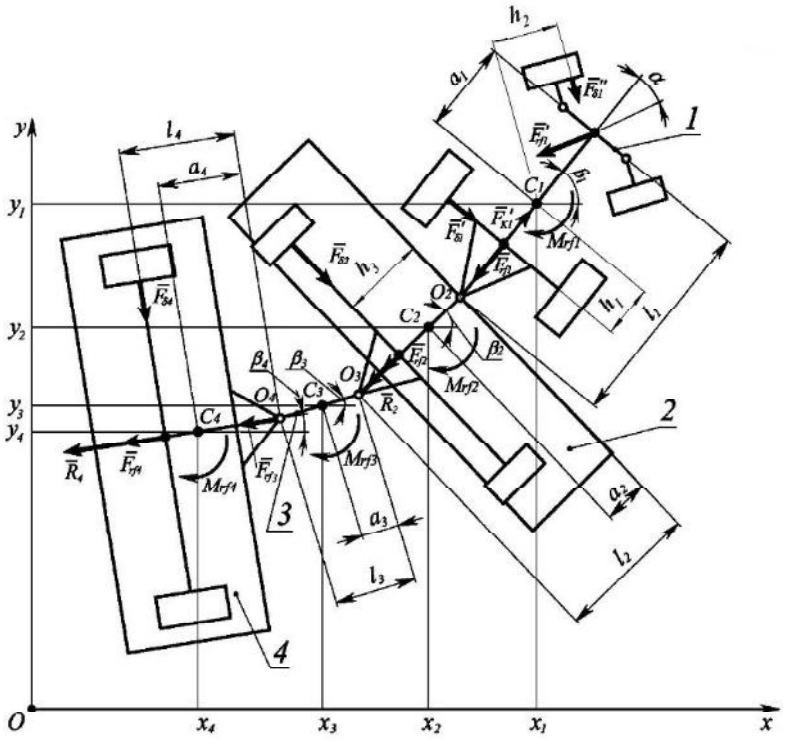

Figure 1. Equivalent schematic model of combined fertilising and sowing tractor-implement unit: 1 – tractor; 2 – fertilising unit; 3 – trailing arm; 4 – seeding unit.

The mechanical system under study will be referred to the fixed Cartesian coordinate system Oxyz. The axes Ox and Oy will be situated in the horizontal plane (i.e. in the field surface plane), while the axis Oz will point vertically up.

In order to generate the differential equations of motion of the obtained mechanical system, it will be put in arbitrary motion in the positive direction and its position during the motion will be characterised with six independent generalised coordinates: x_1, y_1, where x_1, y_1 – coordinates of the tractor's centre of mass; β_1, β_2, β_3, β_4 – respective angles between the longitudinal axes of the mechanical system's members and the axis Ox;

$m_i\left(i=\overline{1.4}\right)$ – masses of the mechanical system's members; $C_i(x_i,\ y_i)$ – centre of mass of the *i*-th member of the system, $\left(i=\overline{1.4}\right)$; a_i – distance from the member's centre of mass to its front articulation joint; l_i – distance between the adjacent articulation joint axes.

Let the mechanical system at the initial instant (t = 0) be aligned along the axis Ox and start moving from the quiescent state.

The motion of the obtained mechanical system will be described following the established method with the use of Lagrange equations of the second kind:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_s}\right)-\frac{\partial T}{\partial q_s}=Q_s,\ \ \left(s=\overline{1.6}\right), \tag{1}$$

where T – kinetic energy of the mechanical system; q_s – generalised coordinate; s – number of the coordinate; Q_s – generalised force that corresponds to the generalised coordinate q_s.

After determining all the components needed for substitution into the original equations (1) and performing all necessary transformations in each of the mentioned generalised coordinates (Adamchuk et al., 2015), the following system of differential equations of the plane parallel motion of the combined fertilising and sowing tractor-implement unit is obtained (each of the six equations in the said system of differential equations is assigned its own reference number from (2) to (7)):

$$\begin{aligned}&\left(F'_{\delta 1}-F_{\delta 2}\right)\beta_1-F_{\delta 2}\beta_2-F_{\delta 4}\beta_4=\\&=F_{k1}-F'_{rf1}-F_{\delta 2}\alpha-F_{rf1}-F_{rf2}-R_2-F_{rf3}-F_{rf4}-R_4;\end{aligned} \tag{2}$$

$$\begin{aligned}&\left(m_1+m_2+m_3+m_4\right)\ddot{y}_1-\left(m_2+m_3+m_4\right)\left(l_1-a_1\right)\ddot{\beta}_1-\left(m_2a_2+m_3l_2+m_4l_2\right)\ddot{\beta}_2-\\&-\left(m_3a_3+m_4l_3\right)\ddot{\beta}_3-m_4a_4\ddot{\beta}_4+\left(m_2+m_3+m_4\right)\left(l_1-a_1\right)\beta_1\dot{\beta}_1^2+\\&+\left(m_2a_2+m_3l_2+m_4l_2\right)\beta_2\dot{\beta}_2^2+\left(m_3a_3+m_4l_3\right)\beta_3\dot{\beta}_3^2+m_4a_4\beta_4\dot{\beta}_4^2-\\&-\left(F'_{k1}-F'_{rf1}-F_{rf1}\right)\beta_1+\left(F_{rf2}+R_2\right)\beta_2+F_{rf3}\beta_3+\left(F_{rf4}+R_4\right)\beta_4=\\&=-F'_{\delta 1}+F'_{rf1}\alpha-F'_{\delta 2}-F_{\delta 2}-F_{\delta 4};\end{aligned} \tag{3}$$

$$\begin{aligned}&-\left(m_2+m_3+m_4\right)\left(l_1-a_1\right)\ddot{y}_1+\left[I_1+m_2\left(l_1-a_1\right)^2+m_3\left(l_1-a_1\right)^2+m_4\left(l_1-a_1\right)^2+\right.\\&\left.+2I_{k1}\left(\frac{d_1}{r_{k1}}\right)^2+\frac{2I'_{k1}\left(d_0+d_1\right)\left[\left(d_0+d_1\right)+a_1\alpha\right]}{\left(r'_{k1}\right)^2}\right]\ddot{\beta}_1+\\&+\left[m_2a_2+m_3l_2+m_4l_2\right]\left(l_1-a_1\right)\ddot{\beta}_2+\left(m_3a_3+m_4l_3\right)\left(l_1-a_1\right)\ddot{\beta}_3+m_4\left(l_1-a_1\right)a_4\ddot{\beta}_4-\\&-\left(m_2+m_3+m_4\right)\left(l_1-a_1\right)^2\beta_1\dot{\beta}_1^2-\left(m_2a_2+m_3l_2+m_4l_2\right)\left(l_1-a_1\right)\beta_2\dot{\beta}_2^2-\\&-\left(m_3a_3+m_4l_3\right)\left(l_1-a_1\right)\beta_3\dot{\beta}_3^2-m_4\left(l_1-a_1\right)a_4\beta_4\dot{\beta}_4^2+\frac{2I'_{k1}\left(d_0+d_1\right)d_0\alpha}{\left(r'_{k1}\right)^2}\dot{\beta}_1^2-\\&-\left(-F_{rf2}-R_2-F_{rf3}-F_{rf4}-R_4\right)\left(l_1-a_1\right)\beta_1-\left(F_{rf2}+R_2\right)\left(l_1-a_1\right)\beta_2-F_{rf3}\left(l_1-a_1\right)\beta_3-\\&-\left(F_{rf4}+R_4\right)\left(l_1-a_1\right)\beta_4=M_{C1}+\left(F_{\delta 2}+F_{\delta 4}\right)\left(l_1-a_1\right);\end{aligned} \tag{4}$$

$$
\begin{aligned}
&-\left(m_2a_2+m_3l_2+m_4l_2\right)\ddot{y}_1+\left(m_2a_2+m_3l_2+m_4l_2\right)\left(l_1-a_1\right)\ddot{\beta}_1+\\
&+\left[I_2+m_2a_2^2+m_3l_2^2+m_4l_2^2+\frac{I_{k2}}{r_{k2}^2}\left(d_{l_2}^2+d_{r_2}^2\right)\right]\ddot{\beta}_2+\left(m_3l_2a_3+m_4l_2l_3\right)\ddot{\beta}_3+\\
&+m_4l_2a_4\ddot{\beta}_4-\left(m_2a_2+m_3l_2+m_4l_2\right)\left(l_1-a_1\right)\beta_1\dot{\beta}_1^2-\left(m_2a_2^2+m_3l_2^2+m_4l_2^2\right)\beta_2\dot{\beta}_2^2-\\
&-\left(m_3l_2a_3+m_4l_2l_3\right)\beta_3\dot{\beta}_3^2-m_4a_4l_2\beta_4\dot{\beta}_4^2-\left(-F_{rf3}l_2-F_{rf4}l_2-R_4l_2\right)\beta_2-\\
&-F_{rf3}l_2\beta_3-\left(F_{rf4}l_2+R_4l_2\right)\beta_4=M_{O2}+F_{\delta4}l_2;
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
&-\left(m_3a_3+m_4l_3\right)\ddot{y}_1+\left(m_3a_3+m_4l_3\right)\left(l_1-a_1\right)\ddot{\beta}_1+\left(m_3a_3l_2+m_4l_3l_2\right)\ddot{\beta}_2+\\
&+\left[I_3+m_4l_3^2+\frac{I_{k3}}{r_{k3}^2}\left(d_{l_3}^2+d_{r_3}^2\right)\right]\ddot{\beta}_3+m_4l_3a_4\ddot{\beta}_4-\left(m_3a_3+m_4l_3\right)\left(l_1-a_1\right)\beta_1\dot{\beta}_1^2-\\
&-\left(m_3a_3l_2+m_4l_3l_2\right)\beta_2\dot{\beta}_2^2-m_4l_3^2\beta_3\dot{\beta}_3^2-m_4l_3a_4\beta_4\dot{\beta}_4^2-\\
&-\left(-F_{rf4}l_3-R_4l_3\right)\beta_3-\left(F_{rf4}l_3+R_4l_3\right)\beta_4=M_{O3}+F_{\delta4}l_3;
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
&-m_4a_4\ddot{y}_1+m_4a_4\left(l_1-a_1\right)\ddot{\beta}_1+m_4a_4l_2\ddot{\beta}_2+m_4a_4l_3\ddot{\beta}_3+\\
&+\left[I_4+m_4a_4^2+\frac{I_{k4}}{r_{k4}^2}\left(d_{l_4}^2+d_{r_4}^2\right)\right]\ddot{\beta}_4-m_4a_4\left(l_1-a_1\right)\beta_1\dot{\beta}_1^2-\\
&-m_4a_4l_2\beta_2\dot{\beta}_2^2-m_4a_4l_3\beta_3\dot{\beta}_3^2-m_4a_4^2\beta_4\dot{\beta}_4^2=M_{O4};
\end{aligned}
\tag{7}
$$

Respectively, the above equations of the obtained system, expressed in terms of constant coefficients, will appear as follows:

$$
\left.
\begin{aligned}
&A_{11}\ddot{y}_1+A_{12}\ddot{\beta}_1+A_{13}\ddot{\beta}_2+A_{14}\ddot{\beta}_3+A_{15}\ddot{\beta}_4+A_{16}\beta_1\dot{\beta}_1^2+A_{17}\beta_2\dot{\beta}_2^2+A_{18}\beta_3\dot{\beta}_3^2+\\
&+A_{19}\beta_4\dot{\beta}_4^2+A_{110}\beta_1+A_{111}\beta_2+A_{112}\beta_3+A_{113}\beta_4+A_{114}\dot{\beta}_1^2=B_1,\\
&A_{21}\ddot{y}_1+A_{22}\ddot{\beta}_1+A_{23}\ddot{\beta}_2+A_{24}\ddot{\beta}_3+A_{25}\ddot{\beta}_4+A_{26}\beta_1\dot{\beta}_1^2+A_{27}\beta_2\dot{\beta}_2^2+A_{28}\beta_3\dot{\beta}_3^2+\\
&+A_{29}\beta_4\dot{\beta}_4^2+A_{210}\beta_1+A_{211}\beta_2+A_{212}\beta_3+A_{213}\beta_4+A_{214}\dot{\beta}_1^2=B_2,\\
&A_{31}\ddot{y}_1+A_{32}\ddot{\beta}_1+A_{33}\ddot{\beta}_2+A_{34}\ddot{\beta}_3+A_{35}\ddot{\beta}_4+A_{36}\beta_1\dot{\beta}_1^2+A_{37}\beta_2\dot{\beta}_2^2+A_{38}\beta_3\dot{\beta}_3^2+\\
&+A_{39}\beta_4\dot{\beta}_4^2+A_{310}\beta_1+A_{311}\beta_2+A_{312}\beta_3+A_{313}\beta_4+A_{314}\dot{\beta}_1^2=B_3,\\
&A_{41}\ddot{y}_1+A_{42}\ddot{\beta}_1+A_{43}\ddot{\beta}_2+A_{44}\ddot{\beta}_3+A_{45}\ddot{\beta}_4+A_{46}\beta_1\dot{\beta}_1^2+A_{47}\beta_2\dot{\beta}_2^2+A_{48}\beta_3\dot{\beta}_3^2+\\
&+A_{49}\beta_4\dot{\beta}_4^2+A_{410}\beta_1+A_{411}\beta_2+A_{412}\beta_3+A_{413}\beta_4+A_{414}\dot{\beta}_1^2=B_4,\\
&A_{51}\ddot{y}_1+A_{52}\ddot{\beta}_1+A_{53}\ddot{\beta}_2+A_{54}\ddot{\beta}_3+A_{55}\ddot{\beta}_4+A_{56}\beta_1\dot{\beta}_1^2+A_{57}\beta_2\dot{\beta}_2^2+A_{58}\beta_3\dot{\beta}_3^2+\\
&+A_{59}\beta_4\dot{\beta}_4^2+A_{510}\beta_1+A_{511}\beta_2+A_{512}\beta_3+A_{513}\beta_4+A_{514}\dot{\beta}_1^2=B_5,\\
&A_{61}\ddot{y}_1+A_{62}\ddot{\beta}_1+A_{63}\ddot{\beta}_2+A_{64}\ddot{\beta}_3+A_{65}\ddot{\beta}_4+A_{66}\beta_1\dot{\beta}_1^2+A_{67}\beta_2\dot{\beta}_2^2+A_{68}\beta_3\dot{\beta}_3^2+\\
&+A_{69}\beta_4\dot{\beta}_4^2+A_{610}\beta_1+A_{611}\beta_2+A_{612}\beta_3+A_{613}\beta_4+A_{614}\dot{\beta}_1^2=B_6.
\end{aligned}
\right\}
\tag{8}
$$

where

$A_{11}=0;\ A_{12}=0; A_{13}=0; A_{14}=0; A_{15}=0; A_{16}=0; A_{17}=0; A_{18}=0; A_{19}=0;$

$A_{110}=F'_{\delta 1}-F_{\delta 2}; A_{111}=-F_{\delta 2}; A_{112}=0; A_{113}=-F_{\delta 4}; A_{114}=0;$

$A_{21}=m_1+m_2+m_3+m_4;\ A_{22}=-(m_2+m_3+m_4)(l_1-a_1);$

$A_{23}=-(m_2a_2+m_3l_2+m_4l_2);\ A_{24}=-(m_3a_3+m_4l_3);\ A_{25}=-m_4a_4;$

$A_{26}=(m_2+m_3+m_4)(l_1-a_1);\ A_{27}=(m_2a_2+m_3l_2+m_4l_2);$

$A_{28}=(m_3a_3+m_4l_3);\ A_{29}=m_4a_4;\ A_{210}=-(F'_{k1}-F'_{rf1}-F_{rf1});$

$A_{211}=F_{rf2}+R_2;\ A_{212}=F_{rf3};\ A_{213}=F_{rf3}+R_4;\ A_{214}=0;$

$A_{31}=-(m_2+m_3+m_4)(l_1-a_1);\ A_{32}=I_1+(m_2+m_3+m_4)(l_1-a_1)^2+$

$+2I_{k1}\left(\frac{d_1}{r_{k1}}\right)^2+\frac{2I'_{k1}(d_0+d_1)\left[(d_0+d_1)+a_1\alpha\right]}{(r'_{k1})^2};$

$A_{33}=\left[m_2a_2+m_3l_2+m_4l_2\right](l_1-a_1);\ A_{34}=(m_3a_3+m_4l_3)(l_1-a_1);$

$A_{35}=m_4(l_1-a_1)a_4;\ A_{36}=-(m_2+m_3+m_4)(l_1-a_1)^2;$

$A_{37}=-(m_2a_2+m_3l_2+m_4l_2)(l_1-a_1);\ A_{38}=-(m_3a_3+m_4l_3)(l_1-a_1);$

$A_{39}=-m_4(l_1-a_1)a_4;\ A_{310}=-(-F_{rf2}-R_2-F_{rf3}-F_{rf4}-R_4)(l_1-a_1);$

$A_{311}=-(F_{rf2}+R_2)(l_1-a_1);\ A_{312}=F_{rf3}(l_1-a_1);$

$A_{313}=-(F_{rf4}+R_4)(l_1-a_1);\ A_{314}=\frac{2I'_{k1}(d_0+d_1)d_0\alpha}{(r'_{k1})^2};$

$A_{41}=-(m_2a_2+m_3l_2+m_4l_2);\ A_{42}=(m_2a_2+m_3l_2+m_4l_2)(l_1-a_1);$

$A_{43}=I_2+m_2a_2^2+m_3l_2^2+m_4l_2^2+\frac{I_{k2}}{r_{k2}^2}(d_{l_2}^2+d_{r_2}^2);$

$A_{44}=m_3l_2a_3+m_4l_2l_3;\ A_{45}=m_4l_2a_4;\ A_{46}=-(m_2a_2+m_3l_2+m_4l_2)(l_1-a_1);$

$A_{47}=-(m_2a_2^2+m_3l_2^2+m_4l_2^2);\ A_{48}=-(m_3l_2a_3+m_4l_2l_3);$

$A_{49}=-m_4a_4l_2;\ A_{410}=0;\ A_{411}=-(-F_{rf3}l_2-F_{rf4}l_2-R_4l_2);\ A_{412}=-F_{rf3}l_2;$

$A_{413}=-(F_{rf4}l_2+R_4l_2);\ A_{414}=0;\ A_{51}=-(m_3a_3+m_4l_3);$

$A_{52}=(m_3a_3+m_4l_3)(l_1-a_1);\ A_{53}=(m_3a_3+m_4l_3)l_2;$

$A_{54}=I_3+m_4l_3^2+\frac{I_{k3}}{r_{k3}^2}(d_{l_3}^2+d_{r_3}^2);\ A_{55}=m_4l_3a_4;$

$A_{56}=-(m_3a_3+m_4l_3)(l_1-a_1);\ A_{57}=-(m_3a_3+m_4l_3)l_2;\ A_{58}=-m_4l_3^2;$

$A_{59}=-m_4l_3a_4;\ A_{510}=0;\ A_{511}=0;\ A_{512}=-(-F_{rf4}-R_4)l_3;$

$A_{513}=-(F_{rf4}+R_4)l_3;\ A_{514}=0;\ A_{61}=-m_4a_4;\ A_{62}=m_4a_4(l_1-a_1);$

$A_{63}=m_4a_4l_2;\ A_{64}=m_4a_4l_3;\ A_{65}=I_4+m_4a_4^2+\frac{I_{k4}}{r_{k4}^2}(d_{l_4}^2+d_{r_4}^2);$

$A_{66}=-m_4a_4(l_1-a_1);\ A_{67}=-m_4a_4l_2;\ A_{68}=-m_4a_4l_3;\ A_{69}=-m_4a_4^2;$

$A_{610}=0;\ A_{611}=0;\ A_{612}=0;\ A_{613}=0;\ A_{614}=0;$

$B_1=F_{k1}-F'_{rf1}-F_{\delta 2}\alpha-F_{rf1}-F_{rf2}-R_2-F_{rf3}-F_{rf4}-R_4;$

$B_2=-F'_{\delta 1}+F'_{rf1}\alpha-F'_{\delta 2}-F_{\delta 2}-F_{\delta 4};\ B_3=M_{C1}+(F_{\delta 2}+F_{\delta 4})(l_1-a_1);$

$B_4=M_{O2}+F_{\delta 4}l_2;\ B_5=M_{O3}+F_{\delta 4}l_3;\ B_6=M_{O4};$

$M_{C1}=F'_{\delta 1}\cdot h_1-F'_{\delta 2}\cdot h_2;\ M_{O2}=F_{\delta 2}\cdot h_3;\ M_{O3}=0;\ M_{O4}=F_{\delta 4}\cdot l_4.$

Further, it is assumed that, when the angles β_1, β_2, β_3, β_4 are small, the velocities $\dot{\beta}_1$, $\dot{\beta}_2$, $\dot{\beta}_3$, $\dot{\beta}_4$ will also be small. The assumption is based on the sufficiently great inertia of the unit's constituent masses and the actual conditions of the unit movement on the field surface (during small displacements the components of the unit are unable to accelerate to high velocities).

Under the made assumption, especially at a first approximation, the products $\beta_1 \cdot \dot{\beta}_1^2$, $\beta_2 \cdot \dot{\beta}_2^2$, $\beta_3 \cdot \dot{\beta}_3^2$, $\beta_4 \cdot \dot{\beta}_4^2$, can be regarded as sufficiently small. Therefore, the terms of the equations in the system that contain the mentioned products can be discarded, which will result in the considerable simplification of the system of differential equations, the latter acquiring the form of a linear system of differential equations, which will appear as follows:

$$\begin{aligned}&\left(F'_{\delta 1}-F_{\delta 2}\right)\beta_1-F_{\delta 2}\beta_2-F_{\delta 4}\beta_4=\\&=F_{k1}-F'_{rf1}-F_{\delta 2}\alpha-F_{rf1}-F_{rf2}-R_2-F_{rf3}-F_{rf4}-R_4;\end{aligned} \tag{9}$$

$$\begin{aligned}&\left(m_1+m_2+m_3+m_4\right)\ddot{y}_1-\left(m_2+m_3+m_4\right)\left(l_1-a_1\right)\ddot{\beta}_1-\\&-\left(m_2a_2+m_3l_2+m_4l_2\right)\ddot{\beta}_2-\left(m_3a_3+m_4l_3\right)\ddot{\beta}_3-\\&-m_4a_4\ddot{\beta}_4-\left(F'_{k1}-F'_{rf1}-F_{rf1}\right)\beta_1+\left(F_{rf2}+R_2\right)\beta_2+\\&+F_{rf3}\beta_3+\left(F_{rf4}+R_4\right)\beta_4=-F'_{\delta 1}+F'_{rf1}\alpha-F'_{\delta 2}-F_{\delta 2}-F_{\delta 4};\end{aligned} \tag{10}$$

$$\begin{aligned}&-\left(m_2+m_3+m_4\right)\left(l_1-a_1\right)\ddot{y}_1+\left[I_1+m_2\left(l_1-a_1\right)^2+m_3\left(l_1-a_1\right)^2+m_4\left(l_1-a_1\right)^2+\right.\\&\left.+2I_{k1}\left(\frac{d_1}{r_{k1}}\right)^2+\frac{2I'_{k1}\left(d_0+d_1\right)\left[\left(d_0+d_1\right)+a_1\alpha\right]}{\left(r'_{k1}\right)^2}\right]\ddot{\beta}_1+\\&+\left[m_2a_2+m_3l_2+m_4l_2\right]\left(l_1-a_1\right)\ddot{\beta}_2+\left(m_3a_3+m_4l_3\right)\left(l_1-a_1\right)\ddot{\beta}_3+\\&+m_4\left(l_1-a_1\right)a_4\ddot{\beta}_4-\left(-F_{rf2}-R_2-F_{rf3}-F_{rf4}-R_4\right)\left(l_1-a_1\right)\beta_1-\\&-\left(F_{rf2}+R_2\right)\left(l_1-a_1\right)\beta_2-F_{rf3}\left(l_1-a_1\right)\beta_3-\\&-\left(F_{rf4}+R_4\right)\left(l_1-a_1\right)\beta_4=M_{C1}+\left(F_{\delta 2}+F_{\delta 4}\right)\left(l_1-a_1\right);\end{aligned} \tag{11}$$

$$\begin{aligned}&-\left(m_2a_2+m_3l_2+m_4l_2\right)\ddot{y}_1+\left(m_2a_2+m_3l_2+m_4l_2\right)\left(l_1-a_1\right)\ddot{\beta}_1+\\&+\left[I_2+m_2a_2^2+m_3l_2^2+m_4l_2^2+\frac{I_{k2}}{r_{k2}^2}\left(d_{l_2}^2+d_{r_2}^2\right)\right]\ddot{\beta}_2+\\&+\left(m_3l_2a_3+m_4l_2l_3\right)\ddot{\beta}_3+m_4l_2a_4\ddot{\beta}_4-\left(-F_{rf3}l_2-F_{rf4}l_2-R_4l_2\right)\beta_2-\\&-F_{rf3}l_2\beta_3-\left(F_{rf4}l_2+R_4l_2\right)\beta_4=M_{O2}+F_{\delta 4}l_2;\end{aligned} \tag{12}$$

$$
\begin{aligned}
&-\left(m_3a_3+m_4l_3\right)\ddot{y}_1+\left(m_3a_3+m_4l_3\right)\left(l_1-a_1\right)\ddot{\beta}_1+\left(m_3a_3l_2+m_4l_3l_2\right)\ddot{\beta}_2+\\
&+\left[I_3+m_4l_3^2+\frac{I_{k3}}{r_{k3}^2}\left(d_{l_3}^2+d_{r_3}^2\right)\right]\ddot{\beta}_3+m_4l_3a_4\ddot{\beta}_4-\left(-F_{rf4}l_3-R_4l_3\right)\beta_3-\\
&-\left(F_{rf4}l_3+R_4l_3\right)\beta_4=M_{O3}+F_{\delta4}l_3;
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
&-m_4a_4\ddot{y}_1+m_4a_4\left(l_1-a_1\right)\ddot{\beta}_1+m_4a_4l_2\ddot{\beta}_2+m_4a_4l_3\ddot{\beta}_3+\\
&+\left[I_4+m_4a_4^2+\frac{I_{k4}}{r_{k4}^2}\left(d_{l_4}^2+d_{r_4}^2\right)\right]\ddot{\beta}_4=M_{O4}.
\end{aligned}
\tag{14}
$$

It can be seen that only two equations of the system are identical: (2) and (9). Respectively, the equations (9) – (14) expressed in terms of constant coefficients will appear as follows:

$$
\left.
\begin{aligned}
&A_{11}\ddot{y}_1+A_{12}\ddot{\beta}_1+A_{13}\ddot{\beta}_2+A_{14}\ddot{\beta}_3+A_{15}\ddot{\beta}_4+\\
&+A_{16}\beta_1+A_{17}\beta_2+A_{18}\beta_3+A_{19}\beta_4=B_1,\\
&\\
&A_{21}\ddot{y}_1+A_{22}\ddot{\beta}_1+A_{23}\ddot{\beta}_2+A_{24}\ddot{\beta}_3+A_{25}\ddot{\beta}_4+\\
&+A_{26}\beta_1+A_{27}\beta_2+A_{28}\beta_3+A_{29}\beta_4=B_2,\\
&\\
&A_{31}\ddot{y}_1+A_{32}\ddot{\beta}_1+A_{33}\ddot{\beta}_2+A_{34}\ddot{\beta}_3+A_{35}\ddot{\beta}_4+\\
&+A_{36}\beta_1+A_{37}\beta_2+A_{38}\beta_3+A_{39}\beta_4=B_3,\\
&\\
&A_{41}\ddot{y}_1+A_{42}\ddot{\beta}_1+A_{43}\ddot{\beta}_2+A_{44}\ddot{\beta}_3+A_{45}\ddot{\beta}_4+\\
&+A_{46}\beta_1+A_{47}\beta_2+A_{48}\beta_3+A_{49}\beta_4=B_4,\\
&\\
&A_{51}\ddot{y}_1+A_{52}\ddot{\beta}_1+A_{53}\ddot{\beta}_2+A_{54}\ddot{\beta}_3+A_{55}\ddot{\beta}_4+\\
&+A_{56}\beta_1+A_{57}\beta_2+A_{58}\beta_3+A_{59}\beta_4=B_5,\\
&\\
&A_{61}\ddot{y}_1+A_{62}\ddot{\beta}_1+A_{63}\ddot{\beta}_2+A_{64}\ddot{\beta}_3+A_{65}\ddot{\beta}_4+\\
&+A_{66}\beta_1+A_{67}\beta_2+A_{68}\beta_3+A_{69}\beta_4=B_6.
\end{aligned}
\right\}
\tag{15}
$$

where

$A_{11}=0;\ A_{12}=0;\ A_{13}=0;\ A_{14}=0;\ A_{15}=0;$

$A_{16}=F'_{\delta 1}-F_{\delta 2};\ A_{17}=-F_{\delta 2};\ A_{18}=0;\ A_{19}=-F_{\delta 4};$

$A_{21}=m_1+m_2+m_3+m_4;\ A_{22}=-\left(m_2+m_3+m_4\right)\left(l_1-a_1\right);$

$A_{23}=-\left(m_2a_2+m_3l_2+m_4l_2\right);\ A_{24}=-\left(m_3a_3+m_4l_3\right);$

$A_{25}=-m_4a_4;\ A_{26}=-\left(F'_{k1}-F'_{rf1}-F_{rf1}\right);$

$A_{27}=F_{rf2}+R_2;\ A_{28}=F_{rf3};\ A_{29}=F_{rf4}+R_4;$

$A_{31}=-\left(m_2+m_3+m_4\right)\left(l_1-a_1\right);\ A_{32}=I_1+\left(m_2+m_3+m_4\right)\left(l_1-a_1\right)^2+$

$+2I_{k1}\left(\frac{d_1}{r_{k1}}\right)^2+\frac{2I'_{k1}\left(d_0+d_1\right)\left[\left(d_0+d_1\right)+a_1\alpha\right]}{\left(r'_{k1}\right)^2};$

$A_{33}=\left[m_2a_2+m_3l_2+m_4l_2\right]\left(l_1-a_1\right);\ A_{34}=\left(m_3a_3+m_4l_3\right)\left(l_1-a_1\right);$

$A_{35}=m_4\left(l_1-a_1\right)a_4;\ A_{36}=-\left(-F_{rf2}-R_2-F_{rf3}-F_{rf4}-R_4\right)\left(l_1-a_1\right);$

$A_{37}=-\left(F_{rf2}+R_2\right)\left(l_1-a_1\right);\ A_{38}=F_{rf3}\left(l_1-a_1\right);$

$A_{39}=-\left(F_{rf4}+R_4\right)\left(l_1-a_1\right);$

$A_{41}=-\left(m_2a_2+m_3l_2+m_4l_2\right);\ A_{42}=\left(m_2a_2+m_3l_2+m_4l_2\right)\left(l_1-a_1\right);$

$A_{43}=I_2+m_2a_2^2+m_3l_2^2+m_4l_2^2+\frac{I_{k2}}{r_{k2}^2}\left(d_{l_2}^2+d_{r_2}^2\right);$

$A_{44}=m_3l_2a_3+m_4l_2l_3;\ A_{45}=m_4l_2a_4;$

$A_{46}=0;\ A_{47}=-\left(-F_{rf3}l_2-F_{rf4}l_2-R_4l_2\right);\ A_{48}=-F_{rf3}l_2;$

$A_{49}=-\left(F_{rf4}l_2+R_4l_2\right);$

$A_{51}=-\left(m_3a_3+m_4l_3\right);\ A_{52}=\left(m_3a_3+m_4l_3\right)\left(l_1-a_1\right);$

$A_{53}=\left(m_3a_3+m_4l_3\right)l_2;\ A_{54}=I_3+m_4l_3^2+\frac{I_{k3}}{r_{k3}^2}\left(d_{l_3}^2+d_{r_3}^2\right);$

$A_{55}=m_4l_3a_4;\ A_{56}=0;\ A_{57}=0;\ A_{58}=-\left(-F_{rf4}-R_4\right)l_3;$

$A_{59}=-\left(F_{rf4}+R_4\right)l_3;$

$A_{61}=-m_4a_4;\ A_{62}=m_4a_4\left(l_1-a_1\right);$

$A_{63}=m_4a_4l_2;\ A_{64}=m_4a_4l_3;\ A_{65}=I_4+m_4a_4^2+\frac{I_{k4}}{r_{k4}^2}\left(d_{l_4}^2+d_{r_4}^2\right);$

$A_{66}=0;\ A_{67}=0;\ A_{68}=0;\ A_{69}=0;$

$B_1=F_{k1}-F'_{rf1}-F_{\delta 2}\alpha-F_{rf1}-F_{rf2}-R_2-F_{rf3}-F_{rf4}-R_4;$

$B_2=-F'_{\delta 1}+F'_{rf1}\alpha-F'_{\delta 2}-F_{\delta 2}-F_{\delta 4};\ B_3=M_{C1}+\left(F_{\delta 2}+F_{\delta 4}\right)\left(l_1-a_1\right);$

$B_4=M_{O2}+F_{\delta 4}l_2;\ B_5=M_{O3}+F_{\delta 4}l_3;\ B_6=M_{O4};$

$M_{C1}=F'_{\delta 1}\cdot h_1-F'_{\delta 2}\cdot h_2;\ M_{O2}=F_{\delta 2}\cdot h_3;\ M_{O3}=0;\ M_{O4}=F_{\delta 4}\cdot l_4.$

Further, the numerical analysis of the obtained system of equations (15) is to be carried out with the use of the PC and the software programmes developed by the authors.

Under the condition that $A_{11}\ldots A_{15} = 0$, the first equation of the system (15) becomes static, i.e. equal to zero, therefore, it is omitted in the following considerations.

The examined mathematical model of the combined fertilising and sowing tractor-implement unit represents its inertia properties. This is indicated by the differential equations of the analytical mathematical model, which contain only the second derivatives of the independent coordinates (i.e. $\ddot{y}_1$, $\ddot{\beta}_1$, $\ddot{\beta}_2$, $\ddot{\beta}_3$ and $\ddot{\beta}_4$).

It is to be noted that the inertia of the third member of the combined tractor-implement unit under consideration can be ignored on account of its relatively small mass. Under the assumption that $m_3 = l_3 = \alpha_3 = 0$, the system of equations (15) will assume the following form:

$$\left.\begin{aligned}
&A_{21}\ddot{y}_1 + A_{22}\ddot{\beta}_1 + A_{26}\beta_1 + A_{23}\ddot{\beta}_2 + A_{27}\beta_2 + A_{25}\ddot{\beta}_4 + A_{29}\beta_4 = K\alpha + K_1,\\
&A_{31}\ddot{y}_1 + A_{32}\ddot{\beta}_1 + A_{36}\beta_1 + A_{33}\ddot{\beta}_2 + A_{37}\beta_2 + A_{35}\ddot{\beta}_4 + A_{39}\beta_4 = B_3,\\
&A_{41}\ddot{y}_1 + A_{42}\ddot{\beta}_1 + A_{46}\beta_1 + A_{43}\ddot{\beta}_2 + A_{47}\beta_2 + A_{45}\ddot{\beta}_4 + A_{49}\beta_4 = B_4,\\
&A_{61}\ddot{y}_1 + A_{62}\ddot{\beta}_1 + A_{66}\beta_1 + A_{63}\ddot{\beta}_2 + A_{67}\beta_2 + A_{65}\ddot{\beta}_4 + A_{69}\beta_4 = B_6,
\end{aligned}\right\} \qquad (16)$$

where $K = F'_{rf1}$, $K_1 = -F'_{\delta 1} - F'_{\delta 2} - F_{\delta 2} - F_{\delta 4}$.

In order to simplify the process of solving the system of differential equations (16), the Laplace transformation will be applied. It implies, as is known, the transition from the original function to its mapping via the introduction of a special operator – complex variable $p = \frac{d}{dt}$. It provides, as a result, the possibility to change from the complicated system of differential equations to a relatively simple system of algebraic equations. The following will be obtained:

$$\left.\begin{aligned}
&K_{21}\cdot y_1(p) + K_{22}\cdot \beta_1(p) + K_{23}\cdot \beta_2(p) + K_{25}\cdot \beta_4(p) = K\cdot\alpha(p) + K_1\cdot 1(p),\\
&K_{31}\cdot y_1(p) + K_{32}\cdot \beta_1(p) + K_{33}\cdot \beta_2(p) + K_{35}\cdot \beta_4(p) = B_3\cdot 1(p),\\
&K_{41}\cdot y_1(p) + K_{42}\cdot \beta_1(p) + K_{43}\cdot \beta_2(p) + K_{45}\cdot \beta_4(p) = B_4\cdot 1(p),\\
&K_{61}\cdot y_1(p) + K_{62}\cdot \beta_1(p) + K_{63}\cdot \beta_2(p) + K_{65}\cdot \beta_4(p) = B_6\cdot 1(p),
\end{aligned}\right\} \qquad (17)$$

where

$$\begin{aligned}
&K_{21} = A_{21}\cdot p^2, && K_{31} = A_{31}\cdot p^2, && K_{41} = A_{41}\cdot p^2, && K_{61} = A_{61}\cdot p,\\
&K_{22} = A_{22}\cdot p^2 + A_{26}, && K_{32} = A_{32}\cdot p^2 + A_{36}, && K_{42} = A_{42}\cdot p^2, && K_{62} = A_{62}\cdot p,\\
&K_{23} = A_{23}\cdot p^2 + A_{27}, && K_{33} = A_{33}\cdot p^2 + A_{37}, && K_{43} = A_{43}\cdot p^2 + A_{47}, && K_{63} = A_{63}\cdot p,\\
&K_{25} = A_{25}\cdot p^2 + A_{29}, && K_{35} = A_{35}\cdot p^2 + A_{39}, && K_{45} = A_{45}\cdot p^2 + A_{49}, && K_{65} = A_{65}\cdot p.
\end{aligned}$$

The system of equations (17) represents the dynamic state of the combined fertilising and sowing tractor-implement unit under the effect of the controlling $[\alpha(p)]$ and perturbing input variables. The latter ones include the singular exposures $K_0\cdot 1(p)$ and $B_3 =\cdot 1(p)$, $B_4 =\cdot 1(p)$, and $B_6 =\cdot 1(p)$. The coordinate y_1 and the angles β_1, β_2 and β_4 are the original variables of this system of equations.

The transfer function (W_α) of the combined fertilising and sowing tractor-implement unit under consideration, which represents the controllability of its motion, is expressed in the form of the ratio of two determinants:

$$W_\alpha = \frac{D_\alpha}{D} \cdot \tag{18}$$

This function characterises the reaction of the combine tractor in terms of the change of its course angle (β_1) under the effect of the control action, which is represented by the angular displacement of the front wheels of the power unit (combine tractor) – α.

The principal determinant of the system (17) comprising the coefficients in its left part appears as follows:

$$D = \begin{vmatrix} K_{21} & K_{22} & K_{23} & K_{25} \\ K_{31} & K_{32} & K_{33} & K_{35} \\ K_{41} & K_{42} & K_{43} & K_{45} \\ K_{61} & K_{62} & K_{63} & K_{65} \end{vmatrix} \tag{19}$$

In order to generate the determinant D_α, the second column in the principal determinant D, which represents the course angle β_1 of the turning combine tractor, will be replaced by the column comprising those coefficients in the right part of the system of equations (17), which are related to the element $\alpha(p)$. It can be seen that the described condition is met by a column with the coefficient K in its first row and zeroes in the rest of the rows. That results in the following:

$$D_\alpha = \begin{vmatrix} K_{21} & K & K_{23} & K_{25} \\ K_{31} & 0 & K_{33} & K_{35} \\ K_{41} & 0 & K_{43} & K_{45} \\ K_{61} & 0 & K_{63} & K_{65} \end{vmatrix} \tag{20}$$

Taking into account what was stated above, the transfer function of the response of the combined fertilising and sowing tractor-implement unit to the control action will eventually take the following form:

$$W_\alpha = \frac{p^2\left(F_4 \cdot p^4 + F_2 \cdot p^2 + F_0\right)}{p^2\left(C_6 \cdot p^6 + C_4 \cdot p^4 + C_2 \cdot p^2 + C_0\right)} \tag{21}$$

where

$$F_4 = K\left(A_{31} \cdot A_{45} \cdot A_{63} - A_{31} \cdot A_{43} \cdot A_{65} + A_{41} \cdot A_{33} \cdot A_{65} - A_{41} \cdot A_{35} \cdot A_{63} - A_{33} \cdot A_{61} \cdot A_{45} + A_{43} \cdot A_{61} \cdot A_{35}\right),$$

$$F_2 = K\left(A_{31} \cdot A_{63} \cdot A_{49} - A_{31} \cdot A_{47} \cdot A_{65} - A_{41} \cdot A_{63} \cdot A_{39} + A_{41} \cdot A_{37} \cdot A_{65} - A_{33} \cdot A_{61} \cdot A_{49} + \right.$$
$$\left. + A_{43} \cdot A_{61} \cdot A_{39} + A_{61} \cdot A_{35} \cdot A_{47} - A_{61} \cdot A_{45} \cdot A_{37}\right),$$

$$F_0 = K\left(A_{61} \cdot A_{37} \cdot A_{49} - A_{61} \cdot A_{37} \cdot A_{49}\right),$$

$$C_6 = A_{21}\cdot A_{32}\cdot A_{43}\cdot A_{65} - A_{21}\cdot A_{32}\cdot A_{45}\cdot A_{63} - A_{21}\cdot A_{33}\cdot A_{42}\cdot A_{65} + A_{21}\cdot A_{33}\cdot A_{62}\cdot A_{45} +$$
$$+ A_{21}\cdot A_{42}\cdot A_{35}\cdot A_{63} - A_{21}\cdot A_{43}\cdot A_{35}\cdot A_{62} - A_{22}\cdot A_{31}\cdot A_{43}\cdot A_{65} + A_{22}\cdot A_{31}\cdot A_{45}\cdot A_{63} +$$
$$+ A_{22}\cdot A_{41}\cdot A_{33}\cdot A_{65} - A_{22}\cdot A_{41}\cdot A_{35}\cdot A_{63} - A_{22}\cdot A_{33}\cdot A_{61}\cdot A_{45} + A_{22}\cdot A_{43}\cdot A_{61}\cdot A_{35} +$$
$$+ A_{31}\cdot A_{23}\cdot A_{43}\cdot A_{65} - A_{31}\cdot A_{23}\cdot A_{62}\cdot A_{45} - A_{31}\cdot A_{42}\cdot A_{25}\cdot A_{63} + A_{31}\cdot A_{25}\cdot A_{43}\cdot A_{62} -$$
$$- A_{23}\cdot A_{32}\cdot A_{41}\cdot A_{65} + A_{23}\cdot A_{32}\cdot A_{61}\cdot A_{45} + A_{23}\cdot A_{41}\cdot A_{35}\cdot A_{62} - A_{23}\cdot A_{42}\cdot A_{61}\cdot A_{35} +$$
$$+ A_{32}\cdot A_{41}\cdot A_{25}\cdot A_{63} - A_{32}\cdot A_{25}\cdot A_{43}\cdot A_{61} - A_{41}\cdot A_{33}\cdot A_{25}\cdot A_{62} + A_{33}\cdot A_{42}\cdot A_{25}\cdot A_{61} -$$
$$- A_{21}\cdot A_{32}\cdot A_{63}\cdot A_{49},$$

$$C_4 = A_{21}\cdot A_{32}\cdot A_{47}\cdot A_{65} + A_{21}\cdot A_{33}\cdot A_{62}\cdot A_{49} + A_{21}\cdot A_{42}\cdot A_{63}\cdot A_{39} - A_{21}\cdot A_{42}\cdot A_{37}\cdot A_{65} -$$
$$- A_{21}\cdot A_{43}\cdot A_{62}\cdot A_{39} + A_{21}\cdot A_{43}\cdot A_{36}\cdot A_{65} - A_{21}\cdot A_{35}\cdot A_{62}\cdot A_{47} + A_{21}\cdot A_{62}\cdot A_{45}\cdot A_{37} -$$
$$- A_{21}\cdot A_{36}\cdot A_{45}\cdot A_{63} + A_{22}\cdot A_{31}\cdot A_{63}\cdot A_{49} - A_{22}\cdot A_{31}\cdot A_{47}\cdot A_{65} - A_{22}\cdot A_{41}\cdot A_{63}\cdot A_{39} -$$
$$+ A_{22}\cdot A_{41}\cdot A_{37}\cdot A_{65} - A_{22}\cdot A_{33}\cdot A_{61}\cdot A_{49} + A_{22}\cdot A_{43}\cdot A_{61}\cdot A_{39} + A_{22}\cdot A_{61}\cdot A_{35}\cdot A_{47} -$$
$$- A_{22}\cdot A_{61}\cdot A_{45}\cdot A_{37} - A_{31}\cdot A_{23}\cdot A_{62}\cdot A_{49} + A_{31}\cdot A_{42}\cdot A_{27}\cdot A_{65} - A_{31}\cdot A_{42}\cdot A_{63}\cdot A_{29} +$$
$$+ A_{31}\cdot A_{25}\cdot A_{62}\cdot A_{47} - A_{31}\cdot A_{43}\cdot A_{26}\cdot A_{65} + A_{31}\cdot A_{43}\cdot A_{29}\cdot A_{29} + A_{31}\cdot A_{26}\cdot A_{46}\cdot A_{63} -$$
$$- A_{31}\cdot A_{62}\cdot A_{27}\cdot A_{45} + A_{23}\cdot A_{32}\cdot A_{61}\cdot A_{49} + A_{23}\cdot A_{41}\cdot A_{62}\cdot A_{39} - A_{23}\cdot A_{41}\cdot A_{36}\cdot A_{65} -$$
$$- A_{23}\cdot A_{42}\cdot A_{61}\cdot A_{39} + A_{23}\cdot A_{61}\cdot A_{36}\cdot A_{45} - A_{32}\cdot A_{41}\cdot A_{27}\cdot A_{65} + A_{32}\cdot A_{41}\cdot A_{63}\cdot A_{29} -$$
$$- A_{32}\cdot A_{25}\cdot A_{61}\cdot A_{47} - A_{32}\cdot A_{43}\cdot A_{61}\cdot A_{29} + A_{32}\cdot A_{61}\cdot A_{27}\cdot A_{45} + A_{41}\cdot A_{33}\cdot A_{26}\cdot A_{65} -$$
$$- A_{41}\cdot A_{33}\cdot A_{62}\cdot A_{29} - A_{41}\cdot A_{25}\cdot A_{62}\cdot A_{37} + A_{41}\cdot A_{25}\cdot A_{36}\cdot A_{63} - A_{41}\cdot A_{26}\cdot A_{35}\cdot A_{63} +$$
$$+ A_{41}\cdot A_{35}\cdot A_{62}\cdot A_{27} + A_{33}\cdot A_{42}\cdot A_{61}\cdot A_{29} - A_{33}\cdot A_{61}\cdot A_{26}\cdot A_{45} + A_{42}\cdot A_{25}\cdot A_{61}\cdot A_{37} -$$
$$- A_{42}\cdot A_{61}\cdot A_{35}\cdot A_{27} - A_{25}\cdot A_{43}\cdot A_{61}\cdot A_{36} + A_{43}\cdot A_{61}\cdot A_{26}\cdot A_{35},$$

$$C_2 = A_{21}\cdot A_{62}\cdot A_{39}\cdot A_{49} - A_{21}\cdot A_{62}\cdot A_{47}\cdot A_{39} - A_{21}\cdot A_{36}\cdot A_{63}\cdot A_{49} + A_{21}\cdot A_{36}\cdot A_{47}\cdot A_{65} -$$
$$- A_{22}\cdot A_{61}\cdot A_{37}\cdot A_{49} + A_{22}\cdot A_{61}\cdot A_{47}\cdot A_{39} + A_{31}\cdot A_{26}\cdot A_{63}\cdot A_{49} - A_{31}\cdot A_{26}\cdot A_{47}\cdot A_{65} -$$
$$- A_{31}\cdot A_{62}\cdot A_{27}\cdot A_{49} + A_{31}\cdot A_{62}\cdot A_{29}\cdot A_{47} + A_{23}\cdot A_{61}\cdot A_{36}\cdot A_{49} + A_{32}\cdot A_{61}\cdot A_{27}\cdot A_{49} -$$
$$- A_{32}\cdot A_{61}\cdot A_{29}\cdot A_{47} - A_{41}\cdot A_{26}\cdot A_{63}\cdot A_{39} + A_{41}\cdot A_{26}\cdot A_{37}\cdot A_{65} + A_{41}\cdot A_{62}\cdot A_{27}\cdot A_{39} -$$
$$- A_{41}\cdot A_{62}\cdot A_{37}\cdot A_{29} - A_{41}\cdot A_{27}\cdot A_{36}\cdot A_{65} + A_{41}\cdot A_{36}\cdot A_{63}\cdot A_{29} - A_{33}\cdot A_{61}\cdot A_{26}\cdot A_{49} -$$
$$- A_{42}\cdot A_{61}\cdot A_{27}\cdot A_{39} + A_{42}\cdot A_{61}\cdot A_{37}\cdot A_{29} - A_{25}\cdot A_{61}\cdot A_{36}\cdot A_{47} + A_{43}\cdot A_{61}\cdot A_{26}\cdot A_{39} -$$
$$- A_{43}\cdot A_{61}\cdot A_{36}\cdot A_{29} + A_{61}\cdot A_{26}\cdot A_{35}\cdot A_{47} - A_{61}\cdot A_{26}\cdot A_{45}\cdot A_{37} + A_{61}\cdot A_{27}\cdot A_{36}\cdot A_{45},$$

$$C_0 = A_{61}\cdot A_{26}\cdot A_{47}\cdot A_{39} - A_{61}\cdot A_{26}\cdot A_{37}\cdot A_{49} + A_{61}\cdot A_{27}\cdot A_{36}\cdot A_{49} - A_{61}\cdot A_{36}\cdot A_{29}\cdot A_{47}.$$

The denominator of the transfer function (21), which represents the natural oscillations of the system via its determinant *D*, has two zero roots. And it indicates unequivocally that the non-isolated dynamic system under consideration (i.e. the combined tractor-implement unit) is unstable. Therefore, it makes no sense to discuss the stability criteria of Routh-Hurwitz, Mikhailov or Nyquist. Only after complementing the system of differential equations of motion of the said system (19) with a mathematical model of the driver, the stability of its motion can be taken into consideration.

The analysis of the controllability of motion of the studied tractor-implement unit will be carried out with the use of the following algorithm. Basing on the transfer function (21) and using the generally accepted methods of the dynamic system self-control theory, the respective characteristics of the amplitude-frequency (AFR) and phase-frequency (PFR) response to control inputs by the fertilising and sowing tractor-implement unit under consideration can be calculated. The former describes the rate of amplification of the input signal by the dynamic system, the latter represents the lag of its response to the said signal.

Since the examined tractor-implement unit, regarding its physical nature, is effectively a dynamic servo-system, its desirable (ideal) amplitude-frequency response (AFR) and phase-frequency response (PFR) are known a priori. Provided that the system responds to the oscillating control input within the working range of its frequencies, the mentioned characteristics must be as follows:

$$\text{AFR} = 1$$
$$\text{PFR} = 0$$

It has been shown by the previous fundamental scientific research that the working range of control input frequencies in case of agricultural tractor-implement units as dynamic servo-systems (designated ω) usually does not exceed 0.5 Hz (or 3.14 s^{-1}). Hence, the desirable (ideal) amplitude-frequency response of the tractor-implement unit must be equal to 1, when the frequency of oscillations of the angular displacement of the tractor's steering wheels stays within a range of up to 3.14 s^{-1}, and it must be equal to 0 outside that range. Physically, this implies that the heading angle of the power unit (as a response to the input control action) must be equal to the angular displacement of its steering wheels, i.e. $\varphi = \alpha$, when the frequency of oscillations of the input parameter α changes from $\omega = 0$ to $\omega = 3.14$ s^{-1}. In case of $\varphi > \alpha$ or $\varphi < \alpha$ we have, accordingly, the overcontrol or undercontrol of the dynamic system by the input signal, both of which are equally undesirable.

It is to be noted that the real amplitude-frequency response in most instances can be different from the desirable (ideal) one. But, the algorithm of mathematical modelling in those instances remains unequivocal and invariable. That is to say the diagrammatical design of the tractor-implement unit or the design-and-process property value, which delivers the actual amplitude-frequency response and phase-frequency response that are close to the ideal ones, will prevail.

With the use of the analytical mathematical model of the plane parallel motion of the combined fertilising and sowing tractor-implement unit developed by us, the controllability of the combined unit's motion can be estimated with regard to the influence of any if its parameters contained in the coefficients K and A_{ij} of the system of equations (19). But, at this stage of research the following parameters will be examined:

- coefficient of rolling resistance f of the running gear of the combine tractor as well as the fertiliser distributor and the grain drill. In the analytical mathematical model of the plane parallel motion of the combined fertilising and sowing tractor-implement unit the mentioned coefficient represents the forces that resist the rolling of its members: F'_{rf1}, F_{rf1}, F_{rf2} and F_{rf4};
- α_2, α_4 – distances from the hitch points of the fertiliser distributor and the grain drill to their centres of mass (Figs 4, 6, 5);
- l_2, l_4 – lengths of the hitch frames of the fertiliser distributor and the grain drill.

As the masses of the towed implements (fertiliser distributor and grain drill) m_2 and m_4 are correlated with the traction force category of the employed tractor (in the considered case it is traction force category 1.4; Nadykto et al., 2015), this phase of research does not provide for the assessment of the effect that the change of these parameters has on the controllability of motion of the combined tractor-implement unit.

The analysis of the obtained phase-frequency response characteristics of the studied dynamic system has shown that the system's response lagging behind the control action is constant and equal to -180° or -3.14 rad (Fig. 2).

In general, such behaviour of the phase of the response to the control action is characteristic of conservative dynamic systems with virtually absent dissipative processes. Formally, the dynamic system under consideration is just such kind of system, since the effect of dissipative forces on it has been assumed to be insignificant.

The analysis of the obtained amplitude-frequency response calculation results proves the following. At the same control action oscillation frequency, the higher the coefficient of rolling resistance f is, the greater the rate of amplification of the said control input by the dynamic system will be (Fig. 3).

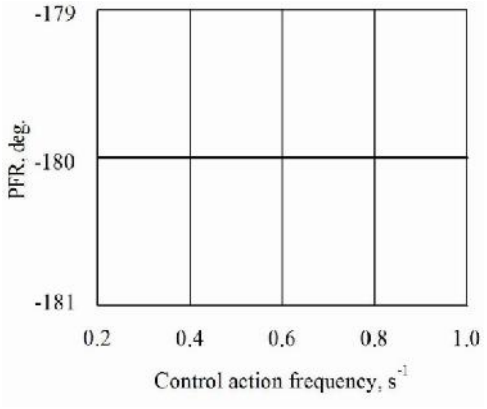

Figure 2. Phase-frequency characteristic of dynamic system's response to control action.

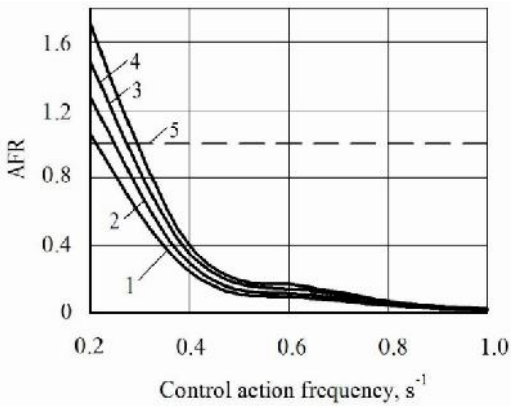

Figure 3. Amplitude-frequency characteristic of dynamic system's response to control action at different values of coefficient of rolling resistance: 1) 0.10; 2) 0.12; 3) 0.14; 4) 0.16; 5) desirable (ideal) amplitude-frequency characteristic.

For instance, when the combined fertilising and sowing tractor-implement unit advances on relatively firm agricultural background (f = 0.10, Graph 1 in Fig. 3), the angular displacement of the tractor's front steering wheels with a frequency of 0.2 s^{-1} gives rise to its response in the form of the heading angle change with a gain rate of 1.1. Meanwhile, when the tractor-implement unit in question operates on broken background (f = 0.16, Graph 4, Fig. 3, the rate of amplification of the examined input signal that has the same frequency (i.e. 0.2 s^{-1}) by the dynamic system (tractor) increases to a level of 1.7, thus more and more departing from the ideal state (Graph 5, Fig. 3).

With the increase of the frequency of angular displacement of the combine tractor steering wheels, the influence of the agricultural background, on which the combined tractor-implement unit travels, decreases. Under a condition of $\omega > 0.3$ s^{-1} the actual

amplitude-frequency characteristics become lower than 1. The dynamic system under consideration shifts to the input signal undercontrol mode, which is undesirable.

At the same time, for each condition of the agricultural background represented by its value of the coefficient of rolling resistance *f*, such a desirable frequency of angular displacement of the combine tractor steering wheels (ω_0) exists, which provides for an actual amplitude-frequency characteristic meeting the requirements to the ideal one. The graphical interpretation of this relation is shown in Fig. 4.

It can be seen from the analysis of the graphically derived function $\omega_0 = f(f)$ that the control action oscillation frequency will vary within a range of 0.210…0.295 s^{-1}, depending on the agricultural background. It is to be stressed that the lower limit of the range (i.e. 0.21 s^{-1}) coincides with the frequency of angular displacement recommended by the researchers for the steering wheels of the combine tractor in the agricultural tractor-implement unit during its travel on the headland.

Obviously, it is rather problematic to maintain the required frequency ω_0 under the real practical conditions, if operating in the manual mode of power unit (tractor) control. At the present time, it is more reasonable to apply a GPS-navigator complete with an automatic manoeuvring system of the UniDrive type or some other one.

As it was already pointed out earlier, the controllability of motion of the combined tractor-implement unit under consideration can be to a certain extent influenced by its such design parameters as α_2 and l_2 (Figs 5, 4, 3). The first of them is essentially the length of the hitch frame, provided that the centre of mass of hitched fertiliser distributor 2 is situated close to its running gear axle. The second one defines the longitudinal coordinate of the point of connection of grain drill 4 to the combine tractor (Figs 1, 2, 3, 4).

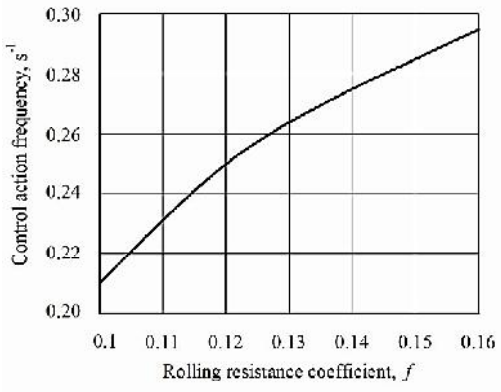

Figure 4. Relation between the desirable frequency of angular displacement of steering wheels of combine tractor in combined fertilising and sowing tractor-implement unit and the conditions of its motion (coefficient of rolling resistance *f*).

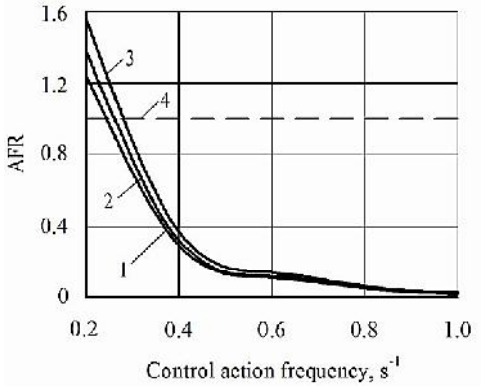

Figure 5. Amplitude-frequency characteristic of dynamic system's response to control action at different values of the design parameter α_2: 1) 1.15 m; 2) 2.15 m; 3) 3.15 m; 4) desirable (ideal) amplitude-frequency characteristic.

It is known from the towed machine dynamics theory that the best stability of the machine's plane parallel motion in the horizontal plane is achieved, when the parameters α_2 and l_2 are as great as possible. At the same time, the accordingly increased kinematic length of the combined unit implies the growth of the unit's non-productive time consumption during its manoeuvring on the headland. Taking that into account, a compromise solution with regard to the parameters α_2 and l_2 can be arrived at only following the assessment of their influence on the controllability of motion of the studied combined fertilising and sowing tractor-implement unit.

The analysis of the mathematical modelling results proves that the rise of the parameter α_2 from 1.15 m to 3.15 m produces: at control action oscillation frequencies of $\omega < 0.24$ s^{-1} – desirable, but at frequencies of $\omega \geq 0.3$ s^{-1} – undesirable decrease of the actual amplitude-frequency response (Fig. 5).

Thus, at $\omega = 0.2$ s^{-1} and $\alpha_2 = 3.15$ m the amplitude-frequency characteristic of the dynamic system's response to the control action is equal to 1.58 (Graph 1, Fig. 2). In practice this means that the heading angle β_1 of the tractor in the combined fertilising and sowing tractor-implement unit under consideration (Figs 2, 3, 4) will change with respect to the angular displacement α of its steering wheels with a gain rate of 1.58. In other words, the dynamic system will operate with an over-response to (i.e. excessive amplification of) the input signal at a surplus of 58%, which is also undesirable, as is known from the theory of dynamic servo-system self-control.

On the other hand, after reducing the parameter α_2 to 1.15 m, the indicated undesirable over-response will become more than two times smaller, as the amplitude-frequency characteristic of the dynamic system under consideration decreases to a level of 1.23 (Graph 3, Fig. 5).

When the frequency of oscillation of the angular displacement of the steering wheels of the power unit (tractor) is set at a level of $\omega = 0.3$ s^{-1}, the amplitude-frequency characteristic of the tractor-implement unit's response to the control action at $\alpha_2 = 1.15$ m is altogether ideal, i.e. equal to 1 (Graph 1, Fig. 5). Increasing the design parameter under consideration α_2 to 3.15 m at the same frequency ω will decrease the amplitude frequency characteristic to a level of 0.7 (Graph 3, Fig. 5). In this case the dynamic system replicates the control action with an under-response at a deficit of 30%, which is undesirable as well.

Only for the combine tractor steering wheel angular displacement oscillation frequencies ω in a range from 0.24 s^{-1} to 0.30 s^{-1} (Fig. 5) it is possible to select such a value of the design parameter α_2, which will facilitate the virtually ideal controllability of motion of the combined fertilising and sowing tractor-implement unit under consideration.

A similar, by its nature, conclusion can be reached with respect to the selection of the design parameter l_2 as well. What will be different is that the control action oscillation frequency range, within which the actual amplitude frequency response characteristics of the dynamic system (i.e. the combined fertilising and sowing tractor-implement unit under consideration) match the ideal ones, will be narrower. Analysing the curves in Fig. 6, it is possible to conclude that this range will span approximately from 0.23 to 0.26 s^{-1}.

While the increase of the design parameter α_2 results in the uniform behaviour of the respective amplitude frequency response characteristics of the dynamic system under consideration, the consequences of the same variation of the parameter l_2 are different. Thus, at a frequency of $\omega = 0.2$ s^{-1}, for example, the amplitude frequency response characteristic f(α_2) is inversely correlating and almost linear (Graph 1, Fig. 7).

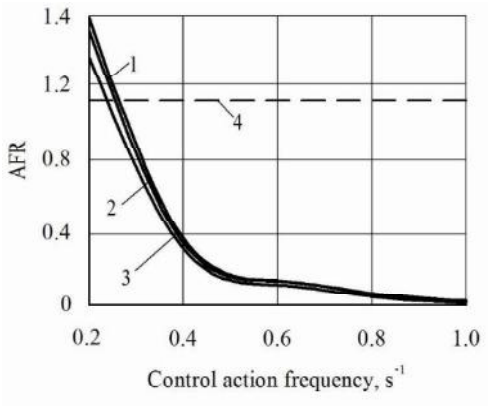

Figure 6. Amplitude-frequency characteristic of dynamic system's response to control action at different values of the design parameter l_2: 1) 3.15 m; 2) 2.15 m; 3) 4.15 m; 4) desirable (ideal) amplitude-frequency characteristic.

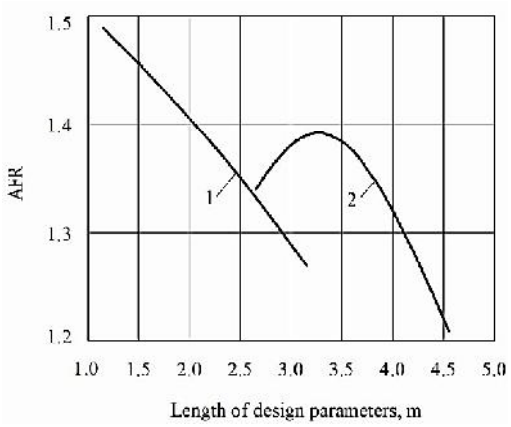

Figure 7. Amplitude-frequency characteristics of dynamic system's response to control action at a frequency of 0.2 s^{-1} at different values of the design parameters α_2 (1) and l_2 (2).

At the same time, the amplitude frequency response function f(l_2) at a frequency of $\omega = 0.2$ s^{-1} is curvilinear and to a certain extent approximates a parabolic curve (Graph 2, Fig. 7). This type of relation between the amplitude frequency response and the parameter l_2 continues, as it follows from the analysis of Fig. 6, with the increase of the combine tractor steering wheel angular displacement oscillation frequency until it reaches a level of at least 0.5 s^{-1}.

The behaviour of Graph 2 (Fig. 7) suggests that the preference in the selection of the parameter l_2 should be given to its greater values. In that event, the values of the actual amplitude frequency response of the dynamic system are closer to 1.

At the same time, the increased parameter l_2 results in the turn of grain drill 4 (Fig. 1) about fertiliser distributor 2 (Figs 1, 2, 3, 5) without their collision, when the combined tractor-implement unit travels on the headland. That eventually implies that raising the parameter l_2 is limited by the values that ensure the accident-free turning ability of the said combined tractor-implement unit.

Following the analysis of the results obtained by modelling the controlled motion of the combined fertilising and sowing tractor-implement unit in a horizontal plane, it becomes evident that the effect the design parameters α_4 and l_4 have on this process (Figs 2, 3, 4) is similar, in terms of both quality and quantity, to that of the parameters α_2 and l_2.

CONCLUSIONS

Thereby, after solving the obtained system of differential equations of the plane parallel motion of the combined fertilising and sowing tractor-implement unit with the use of the PC, the following final conclusions have been reached:

1. Depending on the cultivated field surface condition, the oscillation frequency of the control action, i.e. the angular displacement of the steering wheels of the combined tractor-implement unit under consideration, has to stay within a range of $\omega = 0.210\ldots0.295\ s^{-1}$. At the same time, the greater values will be more appropriate for the operation of the tractor-implement unit on a looser agricultural background, the lower ones will better suit firmer backgrounds.

2. In order to provide for the better controllability of motion of the combined fertilising and sowing tractor-implement unit, the preference has to be given to greater values of the design parameters α_2, l_2 and l_4 (Figs 2, 3, 4). The limits for these values are stipulated by the requirement to ensure the accident-free turning ability of the fertiliser distributor with respect to both the combine tractor and the hitched grain drill.

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