Agronomy Research **15**(4), 1517–1529, 2017
https://doi.org/10.15159/AR.17.069

Theoretical research into the stability of motion of the ploughing tractor-implement unit operating on the 'push-pull' principle

V. Bulgakov[1], V. Adamchuk[2], V. Nadykto[3], O. Kistechok[3] and J. Olt[4,*]

[1]National University of Life and Environmental Sciences of Ukraine, 15, Heroyiv Oborony Str., UA 03041 Kyiv, Ukraine
[2]National Scientific Centre, Institute for Agricultural Engineering and Electrification, 11, Vokzalna Str., Glevaкha-1, Vasylkiv District, UA 08631 Kiev Region, Ukraine
[3]Tavria State Agrotechnological University, 18, B. Khmelnytsky Ave., UA 72312 Melitopol, Zaporozhye region, Ukraine
[4]Estonian University of Life Sciences, Institute of Technology, Kreutzwaldi 56, EE51014 Tartu, Estonia
[*]Correspondence: jyri.olt@emu.ee

Abstract. The reduction of power consumption in the ploughing operations can be achieved by way of improving the grip properties of the implement-carrying tractors, which is facilitated by setting up ploughing units for operation on the 'push-pull' principle. The aim of the current research is to substantiate the set-up and parameters of the ploughing unit with a front-mounted plough basing on the theoretical investigation of the stability of its motion in the horizontal plane. The methods of research include obtaining the amplitude- and phase-frequency response characteristics of the dynamic system in order to analyse the stability of its motion under the action of external statistically random perturbations. Also, the methods of software development and PC-based numerical computation are applied. The results of the study prove that the increase of the ploughing unit's travel speed results in the considerable rise of the gain of the amplitude of the tractor's heading angle oscillation in response to the oscillation of the angular displacement of the front-mounted plough in the horizontal plane. The phase-frequency response displays the same behaviour, changing substantially together with the unit's travel speed. Raising the coefficient of resistance to tyre slip of the tractor's rear wheels from 100 to 130 kPa and the front wheels from 140 to 175 kPa results in a minor decrease of the amplitude of oscillation of the tractor's heading angle φ. The ploughing unit has the best response to the input effect, i.e. the oscillation of the front-mounted plough's angular displacement β_p, when it operates using the 1+5 combination. The increase of the number of front-mounted plough bodies from 1 to 3 results in the substantial growth of the tractor's heading angle oscillation amplitude. Taking into account the way, in which the ploughing unit as a dynamic system responds to the input perturbance, the increase of its travel speed can be restricted not by the condition of its motion stability, but solely by the agrotechnical requirements applied to such a process operation as ploughing. The ploughing unit's motion stability substantially improves in case of higher air pressure in the tyres on the tractor's front and rear wheels. This effect is especially pronounced in the perturbance oscillation frequency range (0...1.5 s^{-1}) that covers the main part of its dispersion.

Key words: tractor, plough, push–pull, dynamic system, stability of motion.

INTRODUCTION

Ploughing as the main operation in the process of soil preparation requires considerable inputs of power, which stipulates the active search for ways to reduce them (Macmillan, 2002; Gil-Sierra et al., 2007; Mircea & Nicolae, 2014; Bulgakov et al., 2015; Nadykto et al., 2015). One of the steps towards this aim is the improvement of the combine tractor's grip properties by increasing its weight as part of the ploughing unit (hitched weight).

The numerous theoretical and experimental studies have shown that the needed increase of weight can be achieved, when setting up the ploughing tractor-implement unit, by combining ploughs with the tractor on the 'push-pull' principle. It has been found in the theoretical studies that the vertical component of the front-mounted plough's tractive resistance increases the weight acting on the front wheels, which means that the hitched weight of the tractor also increases. That results in the certain decrease of its wheel slipping as well as the reduction of the specific fuel consumption of the overall ploughing unit (Kopiks & Viesturs, 2010; Li et al., 2015; Xu et al., 2015; Bulgakov et al., 2016).

At the same time, the incorrect attachment of the front-mounted plough to the power unit can result in the situation, when instead of additionally loading the tractor's front wheels they will be partially released from load, leading to the inevitable loss of controllability and stability of the motion of the whole ploughing tractor-implement unit. It has been established by the theoretical studies (Kasymov & Zolotarev, 1988) that such a negative effect can be avoided, in case of a tractor with a nominal tractive effort rating of 30–32 kN, by using the front-mounted plough with two bodies and the rear-mounted plough with three bodies ('2+3' set-up). In this pattern, the tractor's right wheels roll in the furrow, the front-mounted plough is attached to the tractor in the horizontal plane and the carrier wheel of the ploughing implement is placed outside the furrow.

The ploughing tractor-implement unit under investigation operating on the 'push-pull' principle comprises a Class 3 combine tractor, a front-mounted two-body plough and a rear-mounted three-body plough (Fig. 1).

Figure 1. Tractor-implement unit operating on the 'push-pull' principle.

From among the possible alternatives for attaching the front-mounted ploughing implement to the wheeled combine tractor, the following two are taken into consideration: rigid and articulated attachment. For the analytical investigation we have chosen the second of the mentioned options. The preference for such a design solution can be explained by the fact that it allows the front-mounted ploughing implement to deflect aside in case of encountering a mechanical obstruction and thus prevent its damage. At the same time, in order to ensure the stable motion of the front-mounted implement, the instantaneous centre of turn of the combine tractor's front mounting mechanism shall be situated forward of the ploughing implement suspension axis (Bulgakov et al., 2016).

In this study, the physical object of investigation is a ploughing unit operating on the 'push-pull' principle with a front-mounted plough. Despite the fact that the motion of the combine tractor is performed with its right side wheels in the furrow, the articulated attachment of the ploughing implement at the front can cause stability problems not only with regard to its motion in the horizontal plane, but also to the ploughing tractor-implement unit overall (Larson et al., 1976; Molari et al., 2012; Kyurchev, 2014).

The paper is aimed at assessing the effect that the design and process parameters and the mode of operation of a ploughing tractor-implement unit operating on the 'push-pull' principle have on the stability of its motion just in case of the articulated attachment of the front-mounted plough to the combine tractor.

MATERIALS AND METHODS

The first step is the development of a mathematical model of functioning of the ploughing tractor-implement unit as a dynamic system, which will describe its response (reaction of the tractor's heading angle) to the input perturbing action. In the ploughing tractor-implement unit under consideration the said input is produced by the front-mounted plough, which deflects from the unit's line of travel through a certain angle, when it moves performing the work process of ploughing (Bulgakov et al., 2016).

For this purpose, at first an equivalent schematic model of the considered tractor-implement unit has to be developed. It is to be noted that the analytical mathematical model of a ploughing tractor-implement unit operating on the 'push-pull' principle, even if its linear version is assumed, will be represented by a system of rather complicated differential equations. For that reason, the following assumptions are made in order to simplify the generation of differential equations of motion for such a tractor-implement unit:

1) The field surface is horizontal, the ploughing unit has no fore-and-aft trim.

2) The redistribution of the vertical load between the sides of the combine tractor due to its tilt caused by the motion of its right wheels in the furrow is ignored.

3) The variation of the longitudinal components of the tractive resistance of the ploughing unit's front- and rear-mounted ploughs does not have any essential effect on its translational velocity, therefore, it is assumed constant.

4) The tyre slip angles of the combine tractor wheels situated on the same geometrical axis as well as the lateral forces applied to them are assumed sufficiently small.

5) It is supposed that the angular displacements of the combine tractor's steerable wheels are small enough to regard them as equal to each other.

Also, it is necessary to specify at the beginning the conditions of attachment of the front-mounted plough to the combine tractor. It is assumed that the front-mounted plough has an articulated connection to the tractor following the schematic model shown in Fig. 2. In this case, the 'Centre of Resistance' of the front-mounted ploughing implement is at point π, at which its tractive resistance force R_{pp} is also concentrated.

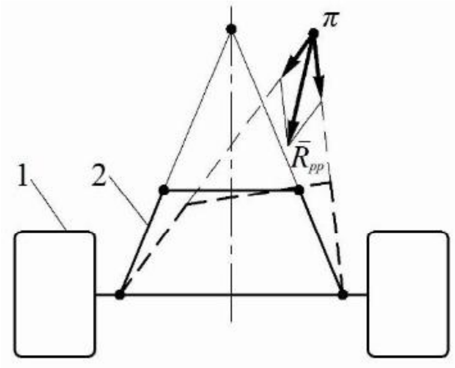


Figure 2. Schematic model of attachment of front-mounted plough to combine tractor: 1 – tractor's front axle; 2 – tractor's front mounting mechanism.

Taking into consideration the above-mentioned allowances, it is assumed that the ploughing tractor-implement unit performs uniform translational motion in the fixed plane $X_1O_1Y_1$ of the field surface at a velocity of V_o. The combine tractor's centre of mass (point S_T) is assigned the role of the origin for the moving frame of reference XOY. With respect to this frame, the power unit together with the ploughs mounted at its front and rear performs independent motions in the form of the translational displacement X_{S_T} and the course turning through an angle of φ.

The next step is to model the external forces acting on the ploughing tractor-implement unit under consideration in the horizontal plane. Apart from the already mentioned front-mounted plough resistance force R_{pp} deflected from the tractor's fore-and-aft axis S_TY_T through an angle of β_p, the following forces also act:

– The moving force F_B of the combine tractor's rear wheels applied at point B and making with its longitudinal symmetry axis a slip angle of δ_B;
– The moving force F_A of the power unit's front wheels applied at point A and deflected from the front running gear's direction of travel through a slip angle of δ_A;
– The lateral forces P_{BA} and P_{BB} applied at points A and B respectively;
– The rear-mounted plough resistance force R_{pz} deflected from the tractor's centreline through an angle of β_z.

Taking into account the above-stated considerations, the equivalent schematic model can be set up as shown in Fig. 3.

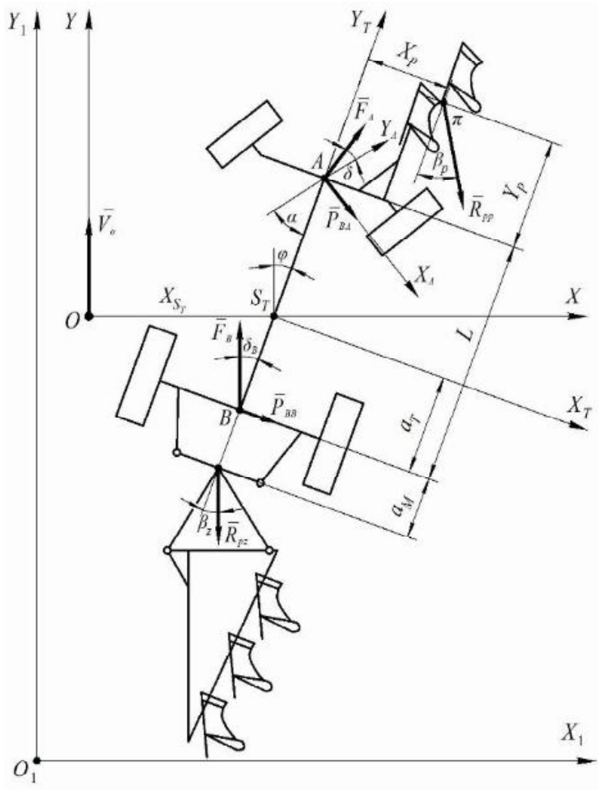


Figure 3. Equivalent schematic model of motion of ploughing tractor-implement unit operating on the 'push-pull' principle in the horizontal plane: S_T – mass centre of the combibe tractor; L – distance between S_T and A; a_T – distance between S_T and B; a_M – distance between B and hitching point.

In order to generate the differential equations of motion of the dynamic system under consideration, the original equations in the form of the Lagrange equations of the second kind are used in accordance with the assumed generalized coordinates X_{S_T} and φ. The result is as follows:

$$\left.\begin{aligned} \frac{d}{dt}\frac{\partial T_T}{\partial \dot{X}_{S_T}} - \frac{\partial T_T}{\partial X_{S_T}} = Q_{X_{S_T}}, \\ \frac{d}{dt}\frac{\partial T_T}{\partial \dot{\varphi}} - \frac{\partial T_T}{\partial \varphi} = Q_{\varphi}, \end{aligned}\right\} \qquad (1)$$

where T_T – combine tractor's kinetic energy; $Q_{X_{S_T}}$, Q_φ – generalized forces on the respective generalized coordinates X_{S_T} and φ.

After determining the required values for the system (1), performing the necessary transformations and reductions, the analytical mathematical model of motion of a ploughing tractor-implement unit operating on the 'push-pull' principle in the horizontal plane becomes represented by the following system of two nonlinear differential equations:

$$\left.\begin{aligned} A_{11}\cdot\ddot{X}_{S_T} + A_{12}\cdot\dot{X}_{S_T} + A_{13}\cdot\dot{\varphi} + A_{14}\cdot\varphi &= f_{11}\cdot\alpha + f_{12}\cdot\beta_p + f_{13}\cdot\beta_z, \\ A_{21}\cdot\ddot{\varphi} + A_{22}\cdot\dot{\varphi} + A_{23}\cdot\varphi + A_{24}\cdot\dot{X}_{S_T} &= f_{21}\cdot\alpha + f_{22}\cdot\beta_p + f_{23}\cdot\beta_z + f_{24}, \end{aligned}\right\} \quad (2)$$

where

$A_{11} = M_A;$

$A_{12} = (k_A + k_B - F_A - F_B)\cdot V_o^{-1};$

$A_{13} = [(k_A - F_A)\cdot(L - a_T) - (k_B - F_B)\cdot a_T]\cdot V_o^{-1};$

$A_{14} = -A_{12}\cdot V_o;$

$A_{21} = J_a;$

$A_{22} = [(k_A - F_A)\cdot(L - a_T)^2 + (k_B - F_B)\cdot a_T^{\,2}]\cdot V_o^{-1};$

$A_{23} = -A_{13}\cdot V_o;$

$A_{24} = A_{13};$

$f_{11} = k_A;$

$f_{12(A,B)} = n_p\cdot b_k\cdot k_o\cdot h;$

$f_{12(C)} = 0;$

$f_{13} = n_z\cdot b_k\cdot k_o\cdot h;$

$f_{21} = (L - a_T)\cdot k_A;$

$f_{22(A)} = n_p\cdot b_k\cdot k_o\cdot h\cdot(L - a_T + Y_P);$

$f_{22(B)} = n_p\cdot b_k\cdot k_o\cdot h\cdot(L - a_T - Y_P);$

$f_{22(C)} = 0;$

$f_{23} = -n_z\cdot b_k\cdot k_o\cdot h\cdot(a_M + a_T);$

$f_{24} = n_p\cdot b_k\cdot k_o\cdot h\cdot X_P$

The following designations have been assigned in the system of differential equations (2): α – angle of turn of the combine tractor's steerable wheels, rad; M_A – mass of the unit, kg; k_A, k_B – coefficients of resistance to tyre slip of the combine tractor's front and rear wheels respectively, kN rad^{-1}; J_a – tractor-implement unit's moment of inertia about the vertical axis that passes through the point S_T, Nm s^2; n_p, n_z – number of bodies in the front- and rear-mounted ploughs respectively; b_k – width of plough body, m; k_o – coefficient of specific resistance of the ploughing unit, kN m^{-2}; h – ploughing depth, m; X_p, Y_p – coordinates of the front-mounted plough's centre of resistance, m; L, a_T, a_M – design parameters, m (Fig. 3).

One of the perturbations for the straight-line motion of the ploughing tractor-implement unit is the front-mounted plough's angular displacement β_p. Response of the dynamic system to the said perturbation is assumed to be represented by the change of the combine tractor's heading angle φ.

In order to simplify the solution of the differential equation system (2) with the use of a PC, the Laplace transformations are used, which imply changing from the original function to its respective transforms by introducing the special operator $p = \frac{d}{dt}$. That enables switching from the complex system of differential equations to a relatively simple system of algebraic equations.

In this particular case, after performing the described transformations, the differential equation system (2), i.e. the analytical mathematical model of the ploughing tractor-implement unit operating on the 'push-pull' principle, in the operator format will appear as follows:

$$\left.\begin{aligned} K_{11}\cdot X_{S_T}(p)+K_{12}\cdot\varphi(p)=F_{11}\cdot\alpha(p)+F_{12}\cdot\beta_p(p)+F_{13}\cdot\beta_z(p)+F_{14}\cdot 1(p),\\ K_{21}\cdot X_{S_T}(p)+K_{22}\cdot\varphi(p)=F_{21}\cdot\alpha(p)+F_{22}\cdot\beta_p(p)+F_{23}\cdot\beta_z(p)+F_{24}\cdot 1(p),\end{aligned}\right\} \qquad (3)$$

where $K_{11} = A_{11} \cdot p^2 + A_{12} \cdot p$; $K_{12} = A_{13} \cdot p + A_{14}$; $K_{21} = A_{24} \cdot p$; $K_{22} = A_{21} \cdot p^2 + A_{22} \cdot p + A_{23}$; $F_{11} = f_{11}$; $F_{12} = f_{12}$; $F_{13} = f_{13}$; $F_{14} = 0$; $F_{21} = f_{21}$ $F_{22} = f_{22}$; $F_{23} = f_{23}$; $F_{24} = f_{24}$; $1(p)$ – unit step transform of the input parameter.

The following parameters act as the input variables in the system of equations (3):

– control input in the form of the combine tractor steerable wheel's turn angle (α);
– angular displacement β_p of front-mounted plough in the horizontal plane;
– angular displacement β_z of rear-mounted plough in the horizontal plane;
– turn-around moment $R_{pp} \cdot X_p$, represented by factor f_{24} in the differential equation system (2).

The following are the output parameters of functioning of the dynamic system under consideration, i.e. the discussed tractor-implement unit:

– transverse offset X_{S_T} of the combine tractor's centre of mass;
– combine tractor's heading angle φ.

For the analytical investigation of the dynamic system under consideration, i.e. the studied tractor-implement unit operating on the 'push-pull' principle, when it performs the work process of ploughing, the most important issue is the system's behaviour in the presence (or absence) of variation of the angular displacement (β_p) of the front-mounted plough in the horizontal plane.

Therefore, in order to analyse the variation of the mentioned angular displacement (β_p), it is necessary to generate the corresponding transfer function. In order to generate the mentioned transfer function, use will be made of the polynomial known from the automatic control theory, which in its most general form appears as follows:

$$W(p)=\frac{F_m\cdot p^m+F_{m-1}\cdot p^{m-1}+\ldots+F_1\cdot p+F_o}{C_n\cdot p^n+C_{n-1}\cdot p^{n-1}+\ldots+C_1\cdot p+C_o} \qquad (4)$$

where F_m, C_n – factors of the numerator and denominator of the transfer function defined by the kinematic and design parameters of automatic control system under study; m and n – indexes of the equations.

Later, when using the expression (4), the coefficients F_m and C_n will be derived from the analytical mathematical model of the ploughing tractor-implement unit under examination.

In order to assess the stability of motion of the ploughing tractor-implement unit operating on the 'push-pull' principle, the transfer function $W_{\beta p}(p)$ can be used, which represents the ratio between the front-mounted plough's angular displacement β_p and the combine tractor's heading angle φ. Expressly:

$$W_{\beta p}(p)=\frac{D_{\beta p}}{D} \qquad (5)$$

where $D_{\beta p}$ – determinant that defines the angular displacement β_p of the front-mounted plough in the horizontal plane; D – general determinant of the system of equations (3).

The components of the expression (5) have to be found. The determinants are obtained in the form of 2×2 matrices, which appear as follows:

$$D_{\beta p} = \begin{vmatrix} K_{11} & F_{12} \\ K_{21} & F_{22} \end{vmatrix} \tag{6}$$

$$D = \begin{vmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{vmatrix} \tag{7}$$

The elements of the determinants (6) and (7) are constituents of the system of equations (3). In this context, the determinant $D_{\beta p}$ defined by the expression (6) is obtained by replacing the column of the coefficients K_{12} and K_{22} related to the output variable φ with the column of the coefficients F_{12} and F_{22} related to the input variable β_p – angular displacement of the front-mounted plough in the horizontal plane. The determinant D defined by the expression (7) is composed of the coefficients in the left-hand side of the system of equations (3).

After expanding the determinants (6) and (7) and substituting their values into (5), the final representation of the transfer function [$W_{\beta p}$ (p)] is obtained:

$$W_{\beta p}\left(p\right) = \frac{F_1 \cdot p + F_o}{p\left(C_2 \cdot p^2 + C_1 \cdot p + C_o\right)}, \tag{8}$$

where $F_1 = f_{22} \cdot A_{11}$; $F_o = f_{22} \cdot A_{12} - f_{12} \cdot A_{24}$; $C_2 = A_{11} \cdot A_{21}$; $C_1 = A_{12} \cdot A_{21} + A_{11} \cdot A_{22}$; $C_o = A_{12} \cdot A_{22} + A_{11} \cdot A_{23} - A_{13} \cdot A_{24}$; p – differentiation operator.

The next step is to determine the perturbations in the form of a unit step function. For this purpose, it is necessary to form its transfer function W_1 (p), and, in order to achieve that, again, the initial expression (4) will be used. In the discussed case the transfer function W_1 (p) is a representation of the sudden and instantaneous change of the turn-around moment created by the front-mounted plough's resistance force R_{pp}. The analysis will be based on the conditions of the said turn-around moment's effect on the combine tractor's heading angle φ. The transfer function in this case appears as follows:

$$W_1\left(p\right) = \frac{D_M}{D}. \tag{9}$$

where

$$D_M = \begin{vmatrix} K_{11} & 0 \\ K_{21} & F_{24} \end{vmatrix} \tag{10}$$

And the determinant D that appears in the expression (9) is the determinant defined by the expression (7). After expanding the determinant (10), applying and expanding the determinant (7) and substituting their values into (9), the final representation of the transfer function W_1 (p) of the following form is obtained:

$$W_1\left(p\right) = \frac{F_1 \cdot p + F_o}{C_2 \cdot p^2 + C_1 \cdot p + C_o}. \tag{11}$$

Meanwhile, it is to be noted that the automatic control theory states that the dynamic system responds to the input action in the form of a unit step function as a simple amplifying circuit. Since $p = i \cdot \omega = 0$ in this case, then the final result is the transfer function of perturbations W_1 (p) of the following appearance:

$$W_1(p) = \frac{F_o'}{C_o}, \tag{12}$$

where $F_o' = f_{24} \cdot A_{12}$; $C_o = A_{12} \cdot A_{22} + A_{11} \cdot A_{23} - A_{13} \cdot A_{24}$.

Meanwhile, F_o' is the solution of the determinant D_M, which was defined by the expression (10). The next phase is the determining of the respective amplitude-frequency response and phase-frequency response characteristics. If in (8) the value $i \cdot \omega$ (where $i = \sqrt{-1}$ and ω – frequency of the perturbing action) is substituted for the differentiation operator *p*, then it becomes possible to obtain expressions for the real and imaginary frequency responses of the dynamic system under consideration. That is:

$$U(\omega) = \frac{M \cdot M_1 + N \cdot N_1}{M_1^2 + N_1^2} = R_e, \tag{13}$$

$$V(\omega) = \frac{M_1 \cdot N - M \cdot N_1}{M_1^2 + N_1^2} = I_m, \tag{14}$$

where $M = F_o - F_2 \cdot \omega^2 + F_4 \cdot \omega^4 - \ldots$; $N = F_1 \cdot \omega - F_3 \cdot \omega^3 + F_5 \cdot \omega^5 - \ldots$;

$M_1 = C_o - C_2 \cdot \omega^2 + C_4 \cdot \omega^4 - \ldots$; $N_1 = C_1 \cdot \omega - C_3 \cdot \omega^3 + C_5 \cdot \omega^5 - \ldots$.

The coefficients of the real and imaginary components of the transfer function *M*, *N* and M_1, N_1 are presented in their general form. The specific values of these coefficients are determined after expanding the transfer functions (8) and (12).

In the case of the ploughing tractor-implement unit under study, the mentioned coefficients for the transfer function (8) are as follows: $M = F_o - F_2 \cdot \omega^2$; $N = F_1 \cdot \omega - F_3 \cdot \omega^3$; $M_1 = C_o - C_2 \cdot \omega^2$; $N_1 = C_1 \cdot \omega - C_3 \cdot \omega^3$. For the transfer function (12) the coefficients have the following values: $M = F_o$; $N = 0$; $M_1 = C_o$; $N_1 = 0$. Hence, the imaginary part of the frequency response (12) is equal to null.

At the same time, the values F_o, F_1, F_3, C_o, C_1, C_2 and C_3 represent in this expression the combination of the coefficients appearing in the transfer function (8) and found from the system of equations (3).

Thus, the desired amplitude-frequency response characteristic can be found from the following expression:

$$A(\omega) = \left[U(\omega)^2 + V(\omega)^2 \right]^{\frac{1}{2}}. \tag{15}$$

Applying the expression (15) with the use of the standard computer programmes on a PC, the computation of the amplitude-frequency response characteristics of the ploughing tractor-implement unit under consideration has been performed.

The phase-frequency response characteristic can be estimated using the methods described in (Klujev, 1986). It should be noted that this analytical study analyses the response of the dynamic system, i.e. the ploughing tractor-implement unit, to the perturbing actions. In accordance with the automatic control theory, the most desirable amplitude-frequency response in the case under consideration is the smallest possible response. On the other hand, the phase shift or else phase-frequency response has to have as high a value as possible. Ideally, the phase-frequency response characteristic has to go to infinity.

RESULTS AND DISCUSSION

The assessed indices of the motion stability of the dynamic system under consideration, i.e. the ploughing tractor-implement unit, are the characteristics of its amplitude- and phase-frequency response to the external perturbing action (amplitude of oscillation of the angle β_p) obtained from the numerical calculations carried out by us with the use of a PC.

The analysis of the derived amplitude-frequency response characteristics shows that the increase of the velocity V_o of the working motion of the ploughing tractor-implement unit is followed by a significant rise of the factor of the power unit heading angle φ oscillation amplitude gain under the effect of the oscillation of the angular displacement β_p of the front-mounted ploughing implement in the horizontal plane, which is undesirable. Moreover, this rise becomes more intensive at $V_o = 2.0$ m s^{-1} (Fig. 4).

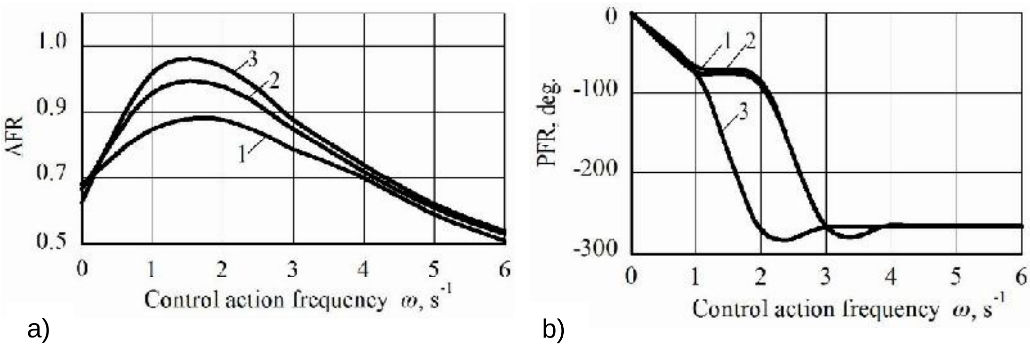


Figure 4. Amplitude- (AFR) – a) and phase- (PFR) – b) frequency response of dynamic system to oscillation of angular displacement β_p of front-mounted plough at different velocities V_o of working motion of ploughing tractor-implement unit: 1) 1.5 m s^{-1}; 2) 2.0 m s^{-1}; 3) 2.5 m s^{-1}.

The same intensity of change in relation to the velocity V_o of the ploughing unit's working motion is also observed in the phase-frequency response characteristics of the dynamic system. Only at $V_o = 2.5$ m s^{-1} they are completely different from those obtained for the velocity ratings up to 2.0 m s^{-1} (Fig. 4).

On the other hand, this difference is observed only within a very limited range of frequencies of the perturbing action: 1...3 s^{-1}. Moreover, when a follow-up dynamic system (the dynamic system under consideration falls exactly into that category of systems) responds to a perturbation, it is more important how it responds to the amplitude of the input action rather than to its phase. Analysing the obtained amplitude-frequency response characteristics from that point of view, it can be concluded that the undesirable increase of the amplitude of the combine tractor's heading angle φ takes place in the same range of perturbation frequencies: 1...3 s^{-1}. The following analysis will show, whether the growth of the amplitude-frequency response is substantial or not.

The process velocity of the working motion of up-to-date ploughing tractor-implement units reaches 2.0...2.5 m s^{-1}. It can be seen from the diagrams in Fig. 4 that, when V_o increases from 2.0 to 2.5 m s^{-1}, the amplitude-frequency response of the ploughing unit to the perturbing action rises maximum by mere 5% at $\omega = 1.5$ s^{-1}.

Generally, such a rise is insignificant, therefore, the increase of the travel speed V_o of the ploughing unit under consideration may be restricted not by the requirements of its motion stability, but solely by the agrotechnical requirements applied to such a process operation as ploughing.

The following issue to be analysed is how the amplitude-frequency response and phase-frequency response of the dynamic system depend on the air pressure in the combine tractor's tyres. The calculations show that the increase of this parameter in the power unit's rear running gear from 100 to 130 kPa, in the front running gear – from 140 to 175 kPa results in the reduction of its heading angle (φ) oscillation amplitude. The effect is especially pronounced in a perturbation frequency range of 0...1.5 s^{-1} (Fig. 5). According to the data obtained by us earlier, this is exactly the range of frequencies (0.5...1.5 s^{-1}), where the major part of the ploughs' angular displacement oscillation dispersion is concentrated.

The reached result is explained by the fact that the increase of the air pressure in the tyres results in the higher values of their slip resistance coefficients. That means that the tyres on the running wheels become stiffer and more resistant to the action of lateral forces.

The mentioned range of ω (i.e. 0...1.5 s^{-1}) features also the desirable behaviour of the phase-frequency response in case of increasing the parameters ρ_{sA} and ρ_{sB}. Moreover, when the tyre air pressure is maximum, the dynamic system under consideration changes from a minimum-phase element into a nonminimum-phase element, which follows from the appearance of its phase-frequency response characteristic (Curve 2, Fig. 5). The causes of such a result need a separate investigation.

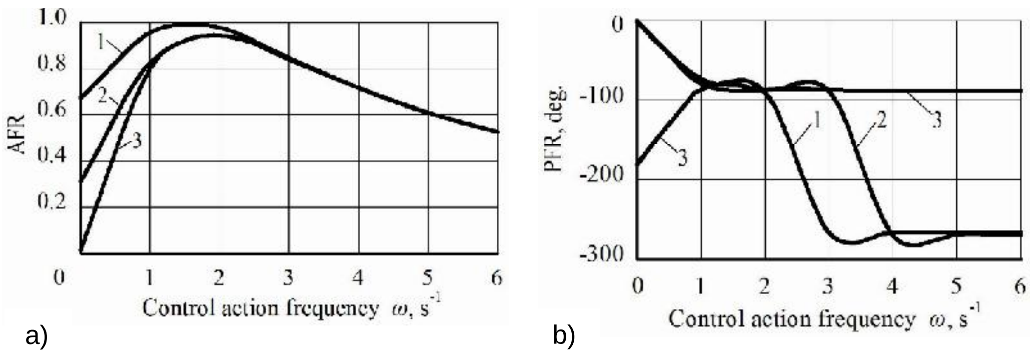
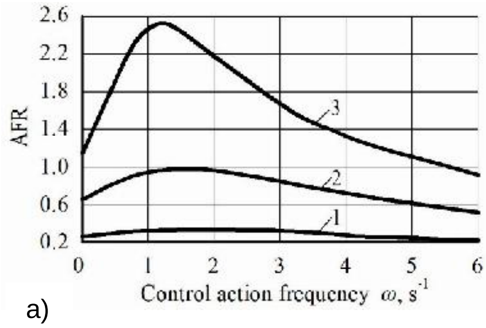


Figure 5. Amplitude- (AFR) – a) and phase- (PFR) – b) frequency response of dynamic system to oscillation of angular displacement of front-mounted plough at different pressures in front (ρ_{sB}) and rear (ρ_{sA}) wheel tyres of tractor: 1) ρ_{sB} = 100 kPa; ρ_{sA} = 140 kPa; 2) ρ_{sB} = 115 kPa; ρ_{sA} = 155 kPa; 3) ρ_{sB} = 130 kPa; ρ_{sA} = 175 kPa.

All the analysis above is concerned with the stability of motion of a ploughing tractor-implement unit under the condition of combining a front-mounted plough with two bodies (n_p = 2). A logical question arises, how the dynamic system's response to the perturbing action will change in case of n_p = 1 and n_z = 3.

The analysis of the mathematical modelling data shows that the ploughing tractor-implement unit has the best response to the input effect, i.e. the oscillation of the front-mounted plough's angular displacement β_ρ, when the combination pattern is '1+5'. The worst amplitude-frequency response is observed, when implementing the tractor-implement unit that operates with the use of the pattern '3+3'. The increase of the number of front-mounted ploughing implement bodies from 1 to 3 results in the significant growth of the amplitude of oscillation of the combine tractor's heading angle φ. Thus, at a frequency of $\omega = 1.5\ s^{-1}$ the perturbing action gain shows undesirable growth, increasing almost seven-fold (Curves 1 and 3, Fig. 6).



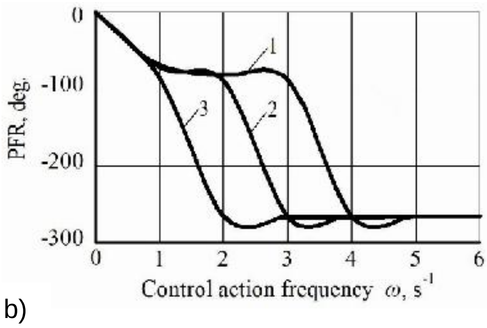


Figure 6. Amplitude- (AFR) – a) and phase- (PFR) – b) frequency response of dynamic system to oscillation of angular displacement of front-mounted plough at different combination patterns of plough bodies in front-mounted and rear-mounted ploughing implements: 1) '1+5'; 2) '2+4'; 3) '3+3'.

At the same time, in case of the ploughing unit that operates on the pattern '2+4', the gain of the dynamic system responding to the input perturbation within the whole range of its oscillation does not exceed unity (Curve 2, Fig. 6), which can be quite acceptable.

The phase-frequency response characteristics closest to the desirable ones have been found in the case of the ploughing tractor-implement unit that operates using the pattern '3+3'. But, firstly, that takes place only in a relatively narrow range of frequencies: 1.2 ... 2.8 s^{-1}. Secondly, the said range of frequencies contains an insignificant fraction of the dispersion of such a perturbing action as the plough's angular displacement in the horizontal plane. Overall, that indicates unequivocally the inexpediency of implementing a ploughing unit that operates on the pattern '3+3'.

CONCLUSIONS

When the velocity of motion of a ploughing unit, in which the instantaneous centre of turn of the tractor's front mounting mechanism is offset forward, increases, the amplitude frequency response of the dynamic system to the perturbing action represented by the front-mounted plough's angular displacement in the horizontal plane deteriorates, but the phase frequency response, contrarily, improves. However, since the undesirable change of the amplitude frequency response, in terms of quantity, is inessential, then the increase of the travel speed of the ploughing tractor-implement unit under consideration can be restricted not by the condition of its motion stability, but solely by the agrotechnical requirements to the performance of such a process operation as ploughing.

The stability of motion of the ploughing tractor-implement unit substantially improves, when the air pressure in the tyres on the combine tractor's front and rear running gear is made higher. This effect is especially pronounced in the perturbing action oscillation frequency range (0...1.5 s^{-1}) that covers the main part of its dispersion.

The increase of the number of bodies in the front-mounted plough from 1 to 3 causes the substantial growth of the amplitude of the combine tractor heading angle oscillation. That said, the implementation of a ploughing unit set up on the pattern '2+4' results in the dynamic system that responds to the input perturbing action within the whole range of its oscillation at a gain not exceeding 1, which is totally acceptable.

REFERENCES

Bulgakov, V., Adamchuk, V., Arak, M., Nadykto, V., Kyurchev, V. & Olt, J. 2016. Theory of vertical oscillations and dynamic stability of combined tractor-implement unit. *Agronomy Research* **14**(3), 689–710.

Bulgakov, V., Kyurchev, V., Nadykto, V. & Olt, J. 2015. Structure development and results of testing a novel modular power unit. *Agriculture and Agricultural Science Procedia* **7**, 40–44.

Gil-Sierra, J., Ortiz-Cañavate, J., Gil-Quiros, V. & Casanova-Kindelan, J. 2007. Energy efficiency in agricultural tractors: A methodology for their classification. *Applied Engineering in Agriculture* **23**(2), 145–150.

Kasymov, A.S. & Zolotarev, V.V. 1988. Steady rectilinear motion of arable unit with front and rear linkage. *Traktory i Selskohozjaistvennyie Machiny* **1**, 13–18.

Kopiks, N. & Viesturs, D. 2010. Research into models of choice of tractor aggregates. In: *Engineering for Rural Development*, Jelgava, 27.–28.05.2010.

Kyurchev, V. 2014. Technical change and financial efficiency: plowing–rotary tractors in Ukraine. *Journal of Finance and Accounting* **2**(1), 8–12.

Larson, D.L., Smith, D.W. & Liljedahl, J.B. 1976. Dynamics of three-dimensional tractor motion. *Transactions of the American Society of Agricultural Engineers* **19**(1), 195–200.

Li, Z., Mitsuoka, M., Inoue, E., Okayasu, T., Hirai, Y. & Zhu, Z. 2015. Modification of a tractor dynamic model considering the rotatable front end. *Journal of the Agriculture*, Kyushu University **60**(1), 219–224.

Macmillan, R.H. 2002. The Mechanics of Tractor Implement Performance. Theory and Worked Examples. University of Melbourne, 165 pp.

Mircea, N. & Nicolae, I. 2014. Study on the dynamic interaction between agricultural tractor and trailer during braking using Lagrange equation. *Applied Mechanic and Materials* **659**, 515– 520.

Molari, G., Bellentani, L., Guarnieri, A., Walker, M. & Sedoni, E. 2012. Performance of an agricultural tractor fitted with rubber tracks. *Biosystems Engineering* **111**(1), 57–63.

Nadykto, V., Arak, M. & Olt, J. 2015. Theoretical research into the frictional slipping of wheel-type propelling units taking into account the limitation of their impact on the soil. *Agronomy Research* **13**(1), 148–157.

Xu, H., Zhang, Y., Liu, H., Qi, S. & Li, W. 2015. Effects of configuration parameters on lateral dynamics of tractor-two trailer combination. *Advances in Mechanical Engineering* **7**(11).

Klujev, A. 1986. Automatic control. Publishing House: University, Moscow, 351 pp. (in Russian).