Mathematical model of cleaning potatoes on surface of spiral separator

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Abstract. Cleaning potato tubers from soil impurities and plant debris after digging them out of soil is a topical problem in the work process of potato production. Therefore, the engineering of new designs of potato heap separators necessitates the further studying of them and the optimisation of their kinematic and design parameters, which must not only ensure the high quality of cleaning, but also rule out the possibility of damaging the tubers. The aim of this study is to determine the design and kinematic parameters of the improved design of the spiral potato separator, which will ensure the high quality of cleaning and rule out the possibility of damaging the tubers, on the basis of the development of the new theory of potato tuber's motion on the surface of the separator. An analytical study has been carried out resulting in the construction of the equivalent schematic model of the interaction between the potato tuber and separator, the tuber being approximated by a material point on the surface formed by the two cantilevered spirals, which are the separator's tools. The separator's spirals are driven to rotate and at the same time they can perform oscillations in the vertical and axial plane under the action of the varying load generated by the continuous feeding of the potato heap for separation. In the model, the forces acting on the potato tuber's body are applied to it, the coordinate axes that have been selected and appropriately oriented are shown. A system of equations has been set up for the constructed equivalent schematic model, comprising three differential equations of the potato tuber body's motion on the surface of the trough formed by the two cantilevered spirals. The determined kinematic and design parameters will allow to raise the quality of cleaning potato tubers from soil impurities and plant debris.

Key words: potato, tuber, separation, impact interaction, impact impulse, rational parameters.

INTRODUCTION

Improving the quality of the cleaning of potatoes from soil impurities and plant debris directly after their digging out from the soil allows considerably improving the main indicators of the potato harvesting work process. In order to determine the optimum design and kinematic parameters of the engineered design, it is necessary to investigate analytically the process of interaction between the potato tuber and the helical cleaning surface of the separator, i.e. to construct the analytical mathematical model of the said process. The subsequent numerical modelling of the process under consideration with

the use of the PC will allow determining the optimum values of the mentioned parameters subject to the condition that the potato tubers are not damaged, when they are on the surface of the spiral separator during their cleaning.

The application of various separators and cleaning machines for cleaning the potato heap immediately after its extraction from the soil has stipulated the publication of considerable numbers of books and papers in various scientists (Peters, 1997; Petrov, 2004; Veerman & Wustman, 2005; Bishop et al., 2012; Wang et al., 2017). That being the case, significant part of the published papers are concerned with the conditions that ensure the stable motion of potato tubers on the separating surfaces of the cleaning machines and the guaranteed sifting (removal) of soil impurities and plant debris from the separation zone (Feller et al., 1987; Misener & McLeod, 1989; Ichiki et al., 2013; Bulgakov et al., 2018a; 2018b).

Authors developed a design of the spiral potato separator (Fig. 1), which includes the assembly of three spirals mounted each on its drive shaft with the other end free (cantilevered). The potato heap to be separated is fed on them from above, which results in a significant part of soil impurities being immediately sifted down. Meanwhile, the potato tubers are entrained by the coils of spirals and transported along their axes, provided that the coils of spiral springs do not entrain impurities, while the said coils are capable of self-cleaning from the stuck wet soil in the process of their operation. The field testing of the potato heap separator under consideration has shown positive results Bulgakov et al., 2017), which provides the basis for further investigation of the described process with the aim of optimising the design and kinematic parameters of the new separating appliance.
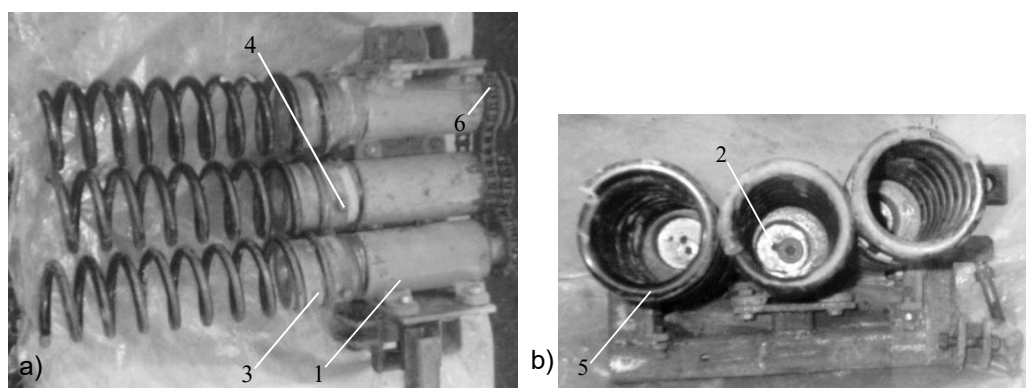

Figure 1. The spiral potato heap separator: a) top view; b) side view. 1 – bearing pedestal; 2 – hub; 3 – bearing ring; 4 – fastener; 5 – cleaning spiral; 6 – sprocket wheel.

The aim of the study is to determine the design and kinematic parameters of the improved design of the spiral potato separator, which ensure the high quality of cleaning and rule out any damage to the tubers, basing on the development of a new theory of the potato tuber's motion on the separator's surface.

MATERIALS AND METHODS

In order to construct the analytical mathematical model of the potato tuber's motion on the spiral separator's surface, it is necessary first to identify the said tuber with a material point, which is approximated by a solid sphere with a mass of m and a radius of r_b. A schematic model will be constructed basing on the analysis of various modes of the relative motion of a single potato tuber, i.e. a material point, on the surface of the spiral separator (Fig. 2). The latter is represented by the two cantilevered driven spirals 1 and 2 rotating about their longitudinal axes at equal angular velocities of ω. Thereby, one end of each spiral is attached rigidly to its drive shaft, the second end is free. The winding senses of the spirals 1 and 2 are shown by the arrows. The identical pitch S of both windings is shown in Fig. 1. Initially, the potato tuber's body in the form of the material point designated by the letter C, which has, as it was indicated earlier, a radius of r_b, arrives onto the surface of the first spiral winding 1, which has a specified radius of R and a pitch angle of γ. The spiral winding 1 starts entraining the material point C, i.e. the potato tuber, by its helical line into their joint motion, i.e. in the rotational motion together with the spiral winding itself, and the translational motion along the spiral's axis, i.e. along the direction of its winding.
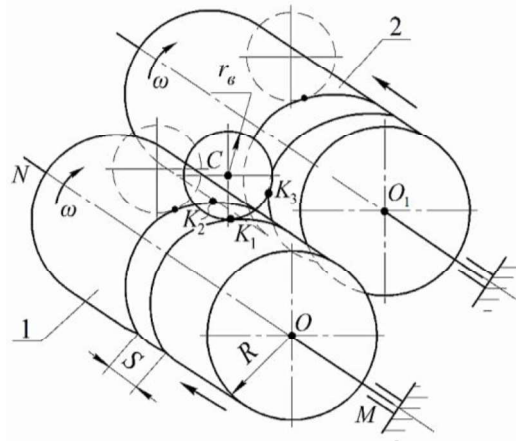

Figure 2. Schematic model of potato tuber's relative motion on surface of spiral separator.

However, as a result of the action of the force of gravity on the potato tuber and the rotation of the spiral 1, the potato tuber will in a very short time reach the trough between the neighbouring spirals 1 and 2. Obviously, within this short time, after its small displacement along the helical line (spiral), the potato tuber will virtually not move along the spiral's longitudinal axis, i.e. the axis MN in this instance. That means that the potato tuber, while residing on the upper side of the spiral 1, cannot be transported to any appreciable distance by the coils of this spiral alone. Therefore, the main advancement of the potato tuber along the MN axis can take place, only when the tuber is in the trough formed by the two neighbouring spirals 1 and 2. Moreover, the duration of the single potato tuber staying at the top, on the coil of only one spiral 1, will be negligibly small, because under the effect of its own weight or by the action of the continuously fed flow of the potato heap to be cleaned the tuber will certainly be pushed forward and down. Hence, detailed analysis is needed for the transportation of potato tubers just in the case, when the tuber has fallen into the trough formed by the two adjacent spirals 1 and 2 of the separator.

At the same time, after the potato tuber falls into the space between the two neighbouring spirals 1 and 2 (Fig. 2), i.e. into the trough, it will surely stay in it and it is unlikely that the tuber immediately ascends to the top of the spiral 2, although, in principle, such a situation cannot be ruled out in view of the rotation of the spiral 2 and

1592

the presence of the force of friction between the tuber and the coils of the spiral 2. Nevertheless, it is fair to say that the more probable scenario is, when the potato tuber under the action of the friction force, despite being possibly captured by the coils of the spiral 2 and drawn into the joint motion, very soon rolls or slides down either back into the same trough or into the next trough between the spirals 2 and 3. Summing up, the potato tuber's motion along the trough between the coils of the spirals 1 and 2 without any breakaways will be considered.

It is obvious that, upon reaching the certain critical angular velocity of ω in the rotation of the spiral 1, the potato tuber can take off from the surface of the spiral 1 and fly over all the three spirals, but that case ought to be regarded as a sufficiently rare one and it should be investigated separately.

Whereas in case of the potato tuber's motion on the coils of the spiral 1 without breaking away followed by its falling down into the trough between the spirals 1 and 2, the spiral 2 serves as the thrust surface that prevents the potato tuber from moving (perpendicular to the MN axis (effectively, on the surface of the spiral 2).

As the potato tuber, while residing in the mentioned trough between the spirals 1 and 2, at the same time stays between two adjacent coils of the spiral 1 that continues to rotate, the said coils slipping on the tuber's surface, but restraining the tuber on both sides and retaining it in the groove between them, move it along the MN axis.

It makes no difference for the potato tuber's translation along the MN axis, whether the tuber moves along the helical groove formed by the two adjacent coils or the tuber is quiescent and the groove moves along the tuber. Since the spiral 2 rotates with the same sense as the spiral 1 and the coils of both the spirals wind identically, the coil of the spiral 2 thrusting against the potato tuber slips on its surface, acting as a thrust surface.

Thus, the contact between the potato tuber and the surfaces of the spirals 1 and 2 during the tuber's translation along the MN axis occurs at the three points K_1, K_2 and K_3, as shown in Fig. 2.

But, the translational motion of the potato tuber along the MN axis is only theoretically close to straight-line motion (basing on the purely geometrical properties of the helical line). In effect, due to a number of random factors, in particular, due to the variable mass of the separated potato heap passing on the surface of all the three spirals, the said spirals perform linear, at a first approximation, oscillations on a vertical line, i.e. perpendicular to the MN axis, which contribute to the sifting of the soil fed together with the potato tubers and other plant residues as well as the cleaning of the tubers' side surfaces from the stuck soil. In this process, the potato tubers perform three-dimensional oscillations about their centres of mass, and in some cases also angular displacements about some of their axes. Evidently, the said oscillations and angular displacements are of random nature. But, for the most part, the potato tubers situated on the surfaces of the separator's spiral springs perform together with the spirals linear translational oscillations oriented vertically, i.e. perpendicular to the MN axis.

Nevertheless, the principal motion of the potato tuber is its translational displacement along the MN axis (i.e. along the trough between the spirals 1 and 2) under the action of the reaction forces applied by the helical groove or, to be more accurate, by the coil that pushes against the potato tuber, propelling it along the MN axis. The said helical groove acts as the constraint that shapes, for its part, the line of the normal reaction force applied to the potato tuber during its translation along the MN axis, and

the equation of that line should be taken as the equation of constraint. Apparently, the main cleanup of the potato tubers from the stuck soil and the sifting of soil and other foreign materials from the potato heap fed for separation take place during the translation of potato tubers along the MN axis alongside with the oscillations of the spirals.

Therefore, it is reasonable first to investigate the process of the translation of a potato tuber situated in the trough between the two spirals under the action of the spirals' helical coils.

RESULTS AND DISCUSSION

In order to generate the analytical mathematical model of the mentioned process, the equivalent schematic model of the interaction between the potato tuber and the surfaces of the above-mentioned separator spiral coils (Fig. 3) has to be set up. In the case under consideration, the potato tuber resides in the trough between the spirals 1 and 2, and on the spiral 1 it is situated in the space between two adjacent coils, its surface supported by both the spiral coils. It is easily evident that, enumerating the two above-mentioned coils in the direction of the tuber's progression along the MN axis, the first coil acts as the pushing element, the second coil acts as the bearing part. The coil of the spiral 2, as it was already pointed out earlier, acts as the thrust part. Hence, as can be seen in Fig. 2 and Fig. 3, at the points of the spirals' contacts with the tuber's surface (its shape being close to spherical) K_1, K_2 and K_3 the normal reaction forces N_1, N_2 and N_3, respectively, are applied. The force of gravity acting on the tuber $\bar{G}$ is applied to the potato tuber's centre of mass (point C) and is vectored vertically down.

Figure 3. Equivalent schematic model of interaction between potato tuber's body and spiral separator's surface.

The potato tuber's motion under the action of the system of forces shown in the equivalent schematic model (Fig. 3) will be investigated with reference to the fixed Cartesian coordinate system xOyz with its origin (point O) situated on the MN longitudinal axis of the spiral 1, its axis Oz coinciding with the MN longitudinal axis of the spiral 1, the axis Oy being directed vertically up and the axis Ox being directed to the right and contained in the spiral's cross-section plane.

After selecting the coordinate system, the forces applied to the potato tuber, as shown in Fig. 3, have to be described.

They are, first of all: N_1, N_2 and N_3 – the normal reaction forces exerted by the surfaces of the coils of the spirals 1 and 2, respectively, vectored along the common normal lines on the surfaces of the coils and the tuber at the points of contact K_1, K_2 and K_3, respectively, their vectors crossing the tuber's centre (point C). Thus, their lines of
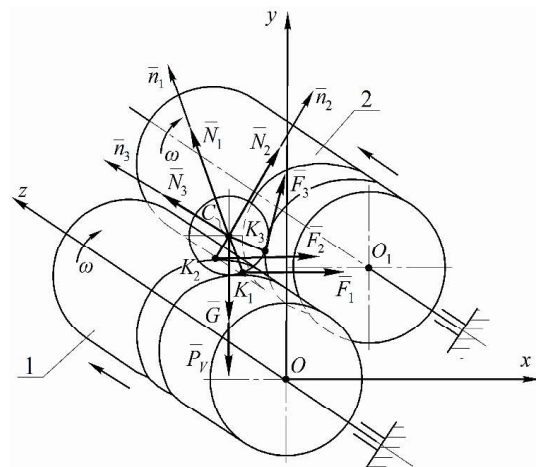
1594

action intersect at the point C.

F_1, F_2 and F_3 – friction forces, which are generated, when the coils of the spirals 1 and 2 as well as the spiral 3, respectively, slip on the surface of the potato tuber at the points of contact K_1, K_2 and K_3, respectively. They are vectored in the same directions, as the spirals' senses of rotation, along the common tangents of the coils and the surface of the potato tuber.

As is known, the F_i forces of sliding friction are expressed as follows:

$$F_i = f \cdot N_i, \ i = 1,2,3, \tag{1}$$

where f – coefficient of sliding friction for the potato tuber sliding on the spiral's material (most often spring steel). In case of potato tubers, it can be assumed that $f = 0.2 \ldots 0.3$ (Guo & Campanella, 2017).

G – potato tuber's force of gravity (N), which can be found, as is known, from the following expression:

$$G = mg \tag{2}$$

where m – mass of the potato tuber (kg); g – gravity acceleration (m s^{-2}).

$\bar{P}_V$ – force of the fed potato heap's dynamic action on the spiral separator that causes the spiral's bending (bending force), vectored vertically down.

The first step is to set up the potato tuber's equation of motion in vector notation on the basis of the constructed equivalent schematic model (Fig. 2):

$$m\bar{a} = \bar{G} + \bar{N}_1 + \bar{N}_2 + \bar{N}_3 + \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{P}_V, \tag{3}$$

where $\bar{a}$ – potato tuber's translation acceleration under the action of the described system of forces (m s^{-2}).

In terms of the projections on the axes of the fixed Cartesian coordinate system xOyz, the vector Eq. (3) takes the following form:

$$
\left.
\begin{aligned}
m\ddot{x} &= N_1 \cos\left(x, \hat{\bar{n}}_1\right) + N_2 \cos\left(x, \hat{\bar{n}}_2\right) + N_3 \cos\left(x, \hat{\bar{n}}_3\right) - F_1 \cos\left(x, \hat{\bar{V}}_1\right) - \\
&\quad - F_2 \cos\left(x, \hat{\bar{V}}_2\right) - F_3 \cos\left(x, \hat{\bar{V}}_3\right), \\
m\ddot{y} &= N_1 \cos\left(y, \hat{\bar{n}}_1\right) + N_2 \cos\left(y, \hat{\bar{n}}_2\right) + N_3 \cos\left(y, \hat{\bar{n}}_3\right) - F_1 \cos\left(y, \hat{\bar{V}}_1\right) - \\
&\quad - F_2 \cos\left(y, \hat{\bar{V}}_2\right) - F_3 \cos\left(y, \hat{\bar{V}}_3\right) - G - P_V, \\
m\ddot{z} &= N_1 \cos\left(z, \hat{\bar{n}}_1\right) + N_2 \cos\left(z, \hat{\bar{n}}_2\right) + N_3 \cos\left(z, \hat{\bar{n}}_3\right) - F_1 \cos\left(z, \hat{\bar{V}}_1\right) - \\
&\quad - F_2 \cos\left(z, \hat{\bar{V}}_2\right) - F_3 \cos\left(z, \hat{\bar{V}}_3\right).
\end{aligned}
\right\} \tag{4}
$$

In order to simplify the obtained system of differential Eq. (4), it is assumed that the potato tuber sits symmetrically in the trough between the spirals 1 and 2, i.e. its centre of mass (point C) is located in the middle of the said trough. In that event, as may be inferred from Fig. 2, the following is obtained:

$$m\ddot{x} = \left(N_1 + N_2\right)\cos\left(x,\overset{\wedge}{\overline{n}_1}\right) - N_3\cos\left(x,\overset{\wedge}{\overline{n}_3}\right) - $$

$$- \left(F_1 + F_2\right)\cos\left(x,\overset{\wedge}{\overline{V}_1}\right) - F_3\cos\left(x,\overset{\wedge}{\overline{V}_3}\right),$$

$$m\ddot{y} = \left(N_1 + N_2\right)\cos\left(y,\overset{\wedge}{\overline{n}_1}\right) + N_3\cos\left(y,\overset{\wedge}{\overline{n}_3}\right) - $$

$$- \left(F_1 + F_2\right)\cos\left(y,\overset{\wedge}{\overline{V}_1}\right) - F_3\cos\left(y,\overset{\wedge}{\overline{V}_3}\right) - G - P_V,$$

$$m\ddot{z} = \left(N_1 - N_2\right)\cos\left(z,\overset{\wedge}{\overline{n}_1}\right) + N_3\cos\left(z,\overset{\wedge}{\overline{n}_3}\right) - $$

$$- \left(F_1 + F_2\right)\cos\left(z,\overset{\wedge}{\overline{V}_1}\right) - F_3\cos\left(z,\overset{\wedge}{\overline{V}_3}\right).$$

$$(5)$$

where $\overline{n}_1, \overline{n}_2, \overline{n}_3$ – common normal lines on the surfaces of the coils and the tuber at the points of contact K_1, K_2 and K_3, respectively; $\overline{V}_1, \overline{V}_2, \overline{V}_3$ – velocity vectors of the potato tuber's relative motion along the spirals' coils at the points of contact K_1, K_2 and K_3, respectively, vectored along the common tangents of the surfaces of the coils and the potato tuber in the directions opposite to the coils' circumferential velocities at the points of contact.

The next step is to determine the direction cosines of the angles between the axes of the coordinate system xOyz and the normal lines on the coils' surfaces at the K_1, K_2 and K_3 points of contact between the potato tuber and the coils of the spirals 1 and 2 included in the system of differential Eq. (5).

The direction cosines $\cos\left(x,\overset{\wedge}{\overline{n}_i}\right)$, $\cos\left(y,\overset{\wedge}{\overline{n}_i}\right)$, $\cos\left(z,\overset{\wedge}{\overline{n}_i}\right)$, $i = 1, 2, 3$, are defined by the following dependencies:

$$\cos\left(x,\overset{\wedge}{\overline{n}_i}\right) = \frac{\partial f_k}{\partial x} \cdot \left(\Delta f_k\right)^{-1},$$

$$\cos\left(y,\overset{\wedge}{\overline{n}_i}\right) = \frac{\partial f_k}{\partial y} \cdot \left(\Delta f_k\right)^{-1},$$

$$\cos\left(z,\overset{\wedge}{\overline{n}_i}\right) = \frac{\partial f_k}{\partial z} \cdot \left(\Delta f_k\right)^{-1},$$

$$i = 1, 2, 3; k = 1, 2,$$

$$(6)$$

where Δf_k – modulus of gradient of the function $f_k = f_k(x, y, z)$ determined from the following expression:

$$\Delta f_k = \sqrt{\left(\frac{\partial f_k}{\partial x}\right)^2 + \left(\frac{\partial f_k}{\partial y}\right)^2 + \left(\frac{\partial f_k}{\partial z}\right)^2}, \quad k = 1, 2 \qquad (7)$$

$f_k = f_k(x, y, z)$, $k = 1, 2$ – equation of constraint that corresponds to the equation of surface of the spiral winding.

For the cylindrical spiral 1 with the specified dimensions and the longitudinal axis

1596

coinciding with the coordinate axis Oz, the equation of constraint in the Cartesian coordinate system xOyz appears as:

$$f_1 = \frac{S^2}{4\pi^2} \left[\frac{x \cdot \sin\dfrac{2\pi z}{S} - y \cdot \cos\dfrac{2\pi z}{S}}{\sqrt{x^2 + y^2}} \right] \cdot \cos\left(\frac{S}{2\pi\sqrt{x^2 + y^2}} \right) +$$

$$+ \left(\sqrt{x^2 + y^2} - R \right)^2 - r^2 = 0. \tag{8}$$

Since the longitudinal axis of symmetry of the spiral 2 is offset to the right along the Ox axis by the centre-to-centre distance equal to a, its equation of constraint in the Cartesian coordinate system xOyz appears as follows:

$$f_2 = \frac{S^2}{4\pi^2} \left[\frac{(x-a)\sin\dfrac{2\pi z}{S} - y \cdot \cos\dfrac{2\pi z}{S}}{\sqrt{(x-a)^2 + y^2}} \right] \cos\left(\frac{S}{2\pi\sqrt{(x-a)^2 + y^2}} \right) +$$

$$+ \left[\sqrt{(x-a)^2 + y^2} - R \right]^2 - r^2 = 0, \tag{9}$$

where S – spiral winding pitch.

At this stage, it is necessary first to determine the relevant partial derivatives and the gradient of the function that represents the equation of constraint. In order to achieve that, the equations of constraints (8) and (9) have to be differentiated with respect to the x, y, z variables, then, in the obtained expressions, the x, y and z variables have to be substituted by their parametric expressions, which will be determined as follows. According to Krause & Minkin (2005), the parametric equations of the helical lines (coils) of the spirals 1 and 2 have to be written down. They appear as follows:

– for the spiral 1:

$$\left. \begin{aligned} x &= R\cos\left(\psi_{01} + \psi\right), \\ y &= R\sin\left(\psi_{01} + \psi\right), \\ z &= -S\psi \cdot (2\pi)^{-1}, \end{aligned} \right\} \tag{10}$$

– for the spiral 2:

$$\left. \begin{aligned} x &= a + R\cos\left(\psi_{02} + \psi\right), \\ y &= R\sin\left(\psi_{02} - \psi\right), \\ z &= -S\psi \cdot (2\pi)^{-1}, \end{aligned} \right\} \tag{11}$$

where R – radius of the spiral; $\psi = \omega t$ – independent angular parameter of the spiral, which defines the position of the cross-section within the spiral's length; ψ_{01}, ψ_{02} – initial values of the independent angular parameters of the spirals 1 and 2, respectively, at the initial instant $t = 0$; a – centre-to-centre distance between the spirals 1 and 2.

Considering the fact that the corresponding cross-section of the spiral 2 (in the place of contact of the mentioned spirals with the potato tuber at the points K_1, K_2 and K_3 is displaced by an angle of π with respect to the angular parameter of the spiral 1, the

1597

following initial values of the angular parameters are assumed:

$$\psi_{01} = 0, \ \psi_{02} = \pi. \tag{12}$$

Thereupon, the parametric Eq. (10) and (11), subject to (12), take the following form:

– for the spiral 1:

$$\left.\begin{aligned} x &= R\cos(\omega t), \\ y &= R\sin(\omega t), \\ z &= -S\omega t \cdot (2\pi)^{-1}, \end{aligned}\right\} \tag{13}$$

– for the spiral 2:

$$\left.\begin{aligned} x &= a + R\cos(\omega t + \pi), \\ y &= R\sin(\omega t + \pi), \\ z &= -S\omega t \cdot (2\pi)^{-1}, \end{aligned}\right\} \tag{14}$$

or

$$\left.\begin{aligned} x &= a - R\cos(\omega t), \\ y &= -R\sin(\omega t), \\ z &= -S\omega t \cdot (2\pi)^{-1}. \end{aligned}\right\} \tag{15}$$

As a result of the above-mentioned operations of differentiating the constraint equations and substituting the variables x, y and z in the obtained values of partial derivatives by their parametric expressions (13) and (15), the following expressions are obtained for the partial derivatives in the parametric form:

$$\frac{\partial f_1}{\partial x} = \frac{\partial f_2}{\partial x} = A\sin(\omega t) + B\cos(\omega t) \cdot \sin(2\omega t), \tag{16}$$

$$\frac{\partial f_1}{\partial y} = \frac{\partial f_2}{\partial y} = A\cos(\omega t) + B\sin(\omega t) \cdot \sin(2\omega t), \tag{17}$$

$$\frac{\partial f_1}{\partial z} = C \cdot \cos(2\omega t), \tag{18}$$

$$\frac{\partial f_2}{\partial z} = -C \cdot \cos(2\omega t), \tag{19}$$

where

$$A = -\frac{S^2}{4\pi^2 R} \cdot \cos\left(\frac{S}{2\pi R}\right), \tag{20}$$

$$B = \frac{S^2}{4\pi^2 R} \cdot \cos\left(\frac{S}{2\pi R}\right) - \frac{S^3}{8\pi^3 R^2} \cdot \sin\left(\frac{S}{2\pi R}\right), \tag{21}$$

$$C = \frac{S}{2\pi} \cdot \cos\left(\frac{S}{2\pi R}\right). \tag{22}$$

1598

Thereat, the gradients of the functions $f_1(x, y, z)$ and $f_2(x, y, z)$ are equal and, according to (7), are determined by the following formula:

$$\Delta f_1 = \Delta f_2 = \sqrt{\begin{aligned}&\left[A\sin(\omega t) + B\cos(\omega t)\cdot\sin(2\omega t) \right]^2 + \\ &+ \left[A\cos(\omega t) + B\sin(\omega t)\cdot\sin(2\omega t) \right]^2 + C^2\cos^2(2\omega t)\,.\end{aligned}} \tag{23}$$

Taking into account the expressions (6) and (16)–(23), finally, the target values of the direction cosines of the angles between the coordinate axes and the spiral coils' normal reaction forces at the K_1, K_2 and K_3 points of contact between the potato tuber and the surfaces of coils of the spirals 1 and 2 are obtained:

$$\cos\left(x, \hat{\overline{n}}_1 \right) = \cos\left(x, \hat{\overline{n}}_2 \right) = -\cos\left(x, \hat{\overline{n}}_3 \right) =$$

$$= \frac{A\sin(\omega t) + B\cos(\omega t)\cdot\sin(2\omega t)}{\sqrt{\begin{aligned}&\left[A\sin(\omega t) + B\cos(\omega t)\cdot\sin(2\omega t) \right]^2 + \\ &+ \left[A\cos(\omega t) + B\sin(\omega t)\cdot\sin(2\omega t) \right]^2 + C^2\cos^2(2\omega t)\end{aligned}}}, \tag{24}$$

$$\cos\left(y, \hat{\overline{n}}_1 \right) = \cos\left(y, \hat{\overline{n}}_2 \right) = \cos\left(y, \hat{\overline{n}}_3 \right) =$$

$$= \frac{A\cos(\omega t) + B\sin(\omega t)\cdot\sin(2\omega t)}{\sqrt{\begin{aligned}&\left[A\sin(\omega t) + B\cos(\omega t)\cdot\sin(2\omega t) \right]^2 + \\ &+ \left[A\cos(\omega t) + B\sin(\omega t)\cdot\sin(2\omega t) \right]^2 + C^2\cos^2(2\omega t)\end{aligned}}}, \tag{25}$$

$$\cos\left(z, \hat{\overline{n}}_1 \right) = -\cos\left(z, \hat{\overline{n}}_2 \right) = \cos\left(z, \hat{\overline{n}}_3 \right) =$$

$$= \frac{C\cdot\cos(2\omega t)}{\sqrt{\begin{aligned}&\left[A\sin(\omega t) + B\cos(\omega t)\cdot\sin(2\omega t) \right]^2 + \\ &+ \left[A\cos(\omega t) + B\sin(\omega t)\cdot\sin(2\omega t) \right]^2 + C^2\cos^2(2\omega t)\end{aligned}}}. \tag{26}$$

Further, the cosines of the angles between the vectors of the relative velocity of the potato tuber's motion along the spiral windings at the points of contact K_1, K_2 and K_3 and the coordinate axes Ox, Oy and Oz, i.e. the values $\cos\left(x, \overset{\wedge}{\overline{V_i}}\right)$, $\cos\left(y, \overset{\wedge}{\overline{V_i}}\right)$ and $\cos\left(z, \overset{\wedge}{\overline{V_i}}\right)$, $i = 1, 2, 3$, have to be determined.

Since the velocity vectors $\overline{V_1}$ and $\overline{V_2}$ are collinear, the following holds true:
$$\cos\left(x, \overset{\wedge}{\overline{V_2}}\right) = \cos\left(x, \overset{\wedge}{\overline{V_1}}\right), \; \cos\left(y, \overset{\wedge}{\overline{V_2}}\right) = \cos\left(y, \overset{\wedge}{\overline{V_1}}\right), \; \cos\left(z, \overset{\wedge}{\overline{V_2}}\right) = \cos\left(z, \overset{\wedge}{\overline{V_1}}\right).$$

The direction cosines $\cos\left(x, \overset{\wedge}{\overline{V_i}}\right)$, $\cos\left(y, \overset{\wedge}{\overline{V_i}}\right)$ and $\cos\left(z, \overset{\wedge}{\overline{V_i}}\right)$ can be found with the use of the following expressions (Vasilenko, 1996):

$$\cos\left(x, \overset{\wedge}{\overline{V_i}}\right) = \frac{\dot{x}}{V_i} = \frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}},$$

$$\cos\left(y, \overset{\wedge}{\overline{V_i}}\right) = \frac{\dot{y}}{V_i} = \frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}},$$

$$\cos\left(z, \overset{\wedge}{\overline{V_i}}\right) = \frac{\dot{z}}{V_i} = \frac{\dot{z}}{\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}},$$

$$i = 1, 2, 3.$$

(27)

For that purpose, the velocities $\overline{V_i} = 1, 2, 3$, of the potato tuber's relative motion along the coils of the spirals 1 and 2 , respectively, have to be determined. That requires differentiating the systems of Eq. (13) and (15) on time t. The following is obtained for the spiral 1:

$$\left.\begin{aligned} \dot{x} &= -R\omega \cdot \sin(\omega t), \\ \dot{y} &= R\omega \cdot \cos(\omega t), \\ \dot{z} &= -S\omega \cdot (2\pi)^{-1}, \end{aligned}\right\}$$

(28)

and for the spiral 2:

$$\left.\begin{aligned} \dot{x} &= R\omega \cdot \sin(\omega t), \\ \dot{y} &= -R\omega \cdot \cos(\omega t), \\ \dot{z} &= -S\omega \cdot (2\pi)^{-1}. \end{aligned}\right\}$$

(29)

The systems of Eq. (28) and (29) represent the formulae for finding the projections of the circumferential velocities of the points on coils during the rotation of the spirals 1 and 2, respectively, on the coordinate axes Ox, Oy and Oz.

Considering the fact that the velocities $\bar{V}_i$, $i = 1, 2, 3$, of the potato tuber's relative motion along the coils are vectored in opposition to the vectors of the circumferential velocities of the points on coils, the projections of their vectors on the coordinate axes Ox, Oy and Oz are opposite in sign, therefore, the systems of Eq. (28) and (29) take the following form:

$$\left.\begin{aligned} \dot{x} &= R\omega \cdot \sin(\omega t), \\ \dot{y} &= -R\omega \cdot \cos(\omega t), \\ \dot{z} &= S\omega \cdot (2\pi)^{-1}, \end{aligned}\right\} \tag{30}$$

and

$$\left.\begin{aligned} \dot{x} &= -R\omega \cdot \sin(\omega t), \\ \dot{y} &= R\omega \cdot \cos(\omega t), \\ \dot{z} &= S\omega \cdot (2\pi)^{-1}, \end{aligned}\right\} \tag{31}$$

Hence, the velocity modulus of the potato tuber's relative motion along the spirals' coils is equal to:

$$V_i = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}, \quad i = 1, 2, 3, \tag{32}$$

or, after substituting the values (30) and (31) and carrying out the corresponding transformations, the following is obtained:

$$V_i = \frac{\omega}{2\pi}\sqrt{4\pi^2 R^2 + S^2}, \quad i = 1, 2, 3 \cdot \tag{33}$$

Thereafter, in accordance with the expressions (27), the required cosines of the angles between the vectors $\bar{V}_i$, $i = 1, 2, 3$, and coordinate axes Ox, Oy and Oz are finally obtained. They appear as follows:

$$\cos\left(x, \overset{\wedge}{\bar{V}_1}\right) = \cos\left(x, \overset{\wedge}{\bar{V}_2}\right) = \frac{2\pi R \cdot \sin(\omega t)}{\sqrt{4\pi^2 R^2 + S^2}}, \tag{34}$$

$$\cos\left(y, \overset{\wedge}{\bar{V}_1}\right) = \cos\left(y, \overset{\wedge}{\bar{V}_2}\right) = -\frac{2\pi R \cdot \cos(\omega t)}{\sqrt{4\pi^2 R^2 + S^2}}, \tag{35}$$

$$\cos\left(z, \overset{\wedge}{\bar{V}_1}\right) = \cos\left(z, \overset{\wedge}{\bar{V}_2}\right) = \frac{S}{\sqrt{4\pi^2 R^2 + S^2}}, \tag{36}$$

$$\cos\left(x, \overset{\wedge}{\bar{V}_3}\right) = -\frac{2\pi R \cdot \sin(\omega t)}{\sqrt{4\pi^2 R^2 + S^2}}, \tag{37}$$

$$\cos\left(y, \overset{\wedge}{\bar{V}_3}\right) = \frac{2\pi R \cdot \cos(\omega t)}{\sqrt{4\pi^2 R^2 + S^2}}, \tag{38}$$

$$\cos\left(z, \overset{\wedge}{\bar{V}_3}\right) = \frac{S}{\sqrt{4\pi^2 R^2 + S^2}} \cdot \tag{39}$$

By substituting the expressions (24)–(26) and (34)–(39) in the system of differential Eq. (5), the following system of differential equations is obtained:

$$m\ddot{x} = \left(N_1 + N_2 - N_3\right) \times$$

$$\times \frac{A\sin(\omega t) + B\cos(\omega t) \cdot \sin(2\omega t)}{\sqrt{\begin{array}{l}\left[A\sin(\omega t) + B\cos(\omega t) \cdot \sin(2\omega t)\right]^2 + \\ + \left[A\cos(\omega t) + B\sin(\omega t) \cdot \sin(2\omega t)\right]^2 + C^2\cos^2(2\omega t)\end{array}}} -$$

$$-\left(F_1 + F_2 + F_3\right)\frac{2\pi R\sin(\omega t)}{\sqrt{4\pi^2 R^2 + S^2}},$$

$$m\ddot{y} = \left(N_1 + N_2 + N_3\right) \times$$

$$\times \frac{A\cos(\omega t) + B\sin(\omega t) \cdot \sin(2\omega t)}{\sqrt{\begin{array}{l}\left[A\sin(\omega t) + B\cos(\omega t) \cdot \sin(2\omega t)\right]^2 + \\ + \left[A\cos(\omega t) + B\sin(\omega t) \cdot \sin(2\omega t)\right]^2 + C^2\cos^2(2\omega t)\end{array}}} -$$

$$-\left(-F_1 - F_2 + F_3\right)\frac{2\pi R\cos(\omega t)}{\sqrt{4\pi^2 R^2 + S^2}} - G - P_V,$$

$$(40)$$

$$m\ddot{z} = \left(N_1 - N_2 + N_3\right) \times$$

$$\times \frac{C\cos(2\omega t)}{\sqrt{\begin{array}{l}\left[A\sin(\omega t) + B\cos(\omega t) \cdot \sin(2\omega t)\right]^2 + \\ + \left[A\cos(\omega t) + B\sin(\omega t) \cdot \sin(2\omega t)\right]^2 + C^2\cos^2(2\omega t)\end{array}}} -$$

$$-\left(F_1 + F_2 + F_3\right)\frac{S}{\sqrt{4\pi^2 R^2 + S^2}},$$

where the coefficients A, B and C are determined in accordance with the expressions (20), (21) and (22), respectively.

Thus, the system of differential Eq. (40) has been obtained, which describes the potato tuber's motion under the action of the rotating spirals' coils in the absolute coordinate system xOyz, when the tuber resides in the trough between the adjacent spirals.

Meanwhile, as the spirals' rotation is characterised by constant angular velocities of, $\omega = $ const, the potato tuber under the conditions of such steady-state motion moves at a constant velocity of $\bar{V}_i$, $i = 1, 2, 3,$ relative to the surfaces of coils of the spirals 1 and 2, which velocity can be found from the expression (33). But, in the absolute coordinate system the projections of the potato tuber's velocity will be as follows: $V_x = V_y = 0$ and $V_z = S\omega \cdot (2\pi) = $ const, because the potato tuber in the form of a material point moves only along the Oz axis.

Thereby, the $\ddot{x}$, $\ddot{y}$ and $\ddot{z}$ accelerations of the tuber along the three coordinate axes Ox, Oy and Oz can be considered as equal to zero. Hence, putting the left-hand sides of the differential equations in the system (40) to zero, in view of the formula (1), the following system of linear algebraic equations in the unknown quantities N_1, N_2 and N_3 with variable coefficients is obtained:

$$\left(N_1 + N_2 - N_3\right)\frac{A\sin\left(\omega t\right) + B\cos\left(\omega t\right)\cdot\sin\left(2\omega t\right)}{\sqrt{\left[A\sin\left(\omega t\right)+B\cos\left(\omega t\right)\cdot\sin\left(2\omega t\right)\right]^2 + \atop +\left[A\cos\left(\omega t\right)+B\sin\left(\omega t\right)\cdot\sin\left(2\omega t\right)\right]^2 + C^2\cos^2\left(2\omega t\right)}} -$$

$$-\left(fN_1 + fN_2 + fN_3\right)\frac{2\pi R\sin\left(\omega t\right)}{\sqrt{4\pi^2 R^2 + S^2}} = 0,$$

$$\left(N_1 + N_2 + N_3\right)\frac{A\cos\left(\omega t\right) + B\sin\left(\omega t\right)\cdot\sin\left(2\omega t\right)}{\sqrt{\left[A\sin\left(\omega t\right)+B\cos\left(\omega t\right)\cdot\sin\left(2\omega t\right)\right]^2 + \atop +\left[A\cos\left(\omega t\right)+B\sin\left(\omega t\right)\cdot\sin\left(2\omega t\right)\right]^2 + C^2\cos^2\left(2\omega t\right)}} -$$

$$-\left(-fN_1 - fN_2 + fN_3\right)\frac{2\pi R\cos\left(\omega t\right)}{\sqrt{4\pi^2 R^2 + S^2}} - mg - P_V = 0, \qquad (41)$$

$$\left(N_1 - N_2 + N_3\right)\frac{C\cos\left(2\omega t\right)}{\sqrt{\left[A\sin\left(\omega t\right)+B\cos\left(\omega t\right)\cdot\sin\left(2\omega t\right)\right]^2 + \atop +\left[A\cos\left(\omega t\right)+B\sin\left(\omega t\right)\cdot\sin\left(2\omega t\right)\right]^2 + C^2\cos^2\left(2\omega t\right)}} -$$

$$-\left(fN_1 + fN_2 + fN_3\right)\frac{S}{\sqrt{4\pi^2 R^2 + S^2}} = 0.$$

In essence, the system of Eq. (41) comprises the equations of the potato tuber's equilibrium, when it resides in the trough between the adjacent separator spirals 1 and 2 in contact with the coils of these spirals at the points K_1, K_2 and K_3 (Fig. 1) at the random instant of time t. The fulfilment of the conditions set by (41) ensures the theoretically stable position of the potato tuber in the trough with the constant points of contact K_1, K_2 and K_3 over the period of its translation along the spiral's longitudinal axis, i.e. the axis Oz, up to the moment of its final departure from the spirals.

In this case, the potato tuber, as already mentioned, moves at a constant velocity of $V_z = S\omega \cdot \left(2\pi\right)^{-1}$ along the Oz axis up to the point of its departure from the spirals onto the discharge conveyor.

The system of Eq. (41) can be solved analytically with the use of Cramer's rule. In order to do that, the system is to be transformed into the form that allows applying the said method. Hence, the following designations are introduced:

$$\cfrac{A\sin(\omega t)+B\cos(\omega t)\cdot\sin(2\omega t)}{\sqrt{\begin{array}{l}[A\sin(\omega t)+B\cos(\omega t)\cdot\sin(2\omega t)]^2+\\ +[A\cos(\omega t)+B\sin(\omega t)\cdot\sin(2\omega t)]^2+C^2\cos^2(2\omega t)\end{array}}}=A_1 \tag{42}$$

$$\cfrac{A\cos(\omega t)+B\sin(\omega t)\cdot\sin(2\omega t)}{\sqrt{\begin{array}{l}[A\sin(\omega t)+B\cos(\omega t)\cdot\sin(2\omega t)]^2+\\ +[A\cos(\omega t)+B\sin(\omega t)\cdot\sin(2\omega t)]^2+C^2\cos^2(2\omega t)\end{array}}}=A_2 \tag{43}$$

$$\cfrac{C\cos(2\omega t)}{\sqrt{\begin{array}{l}[A\sin(\omega t)+B\cos(\omega t)\cdot\sin(2\omega t)]^2+\\ +[A\cos(\omega t)+B\sin(\omega t)\cdot\sin(2\omega t)]^2+C^2\cos^2(2\omega t)\end{array}}}=A_3 \tag{44}$$

$$\frac{2\pi R\sin(\omega t)}{\sqrt{4\pi^2R^2+S^2}}=B_1 \tag{45}$$

$$\frac{2\pi R\cos(\omega t)}{\sqrt{4\pi^2R^2+S^2}}=B_2 \tag{46}$$

$$\frac{S}{\sqrt{4\pi^2R^2+S^2}}=B_3. \tag{47}$$

Substituting the expressions (42–47) in the system of Eq. (41), the following system of equations is obtained:

$$\left.\begin{array}{l}(N_1+N_2-N_3)A_1-(fN_1+fN_2+fN_3)B_1=0,\\ (N_1+N_2+N_3)A_2-(-fN_1-fN_2+fN_3)B_2=mg+P_V,\\ (N_1-N_2+N_3)A_3-(fN_1+fN_2+fN_3)B_3=0.\end{array}\right\} \tag{48}$$

After certain transformations, the system of equations suitable for solving with the use of Cramer's rule is obtained:

$$\left.\begin{array}{l}(A_1-fB_1)N_1+(A_1-fB_1)N_2+(-A_1-fB_1)N_3=0,\\ (A_2+fB_2)N_1+(A_2+fB_2)N_2+(A_2-fB_2)N_3=mg+P_V,\\ (A_3-fB_3)N_1+(-A_3-fB_3)N_2+(A_3-fB_3)N_3=0.\end{array}\right\} \tag{49}$$

The principal determinant of the system (49) has to be written down. It is as follows:

1604

$$\Delta = \begin{vmatrix} A_1 - fB_1 & A_1 - fB_1 & -A_1 - fB_1 \\ A_2 + fB_2 & A_2 + fB_2 & A_2 - fB_2 \\ A_3 - fB_3 & -A_3 - fB_3 & A_3 - fB_3 \end{vmatrix}. \tag{50}$$

The next step is to write down the determinants Δ_i required for finding the unknown quantities N_i, $i = 1, 2, 3$:

$$\Delta_1 = \begin{vmatrix} 0 & A_1 - fB_1 & -A_1 - fB_1 \\ mg + P_V & A_2 + fB_2 & A_2 - fB_2 \\ 0 & -A_3 - fB_3 & A_3 - fB_3 \end{vmatrix}, \tag{51}$$

$$\Delta_2 = \begin{vmatrix} A_1 - fB_1 & 0 & -A_1 - fB_1 \\ A_2 + fB_2 & mg + P_V & A_2 - fB_2 \\ A_3 - fB_3 & 0 & A_3 - fB_3 \end{vmatrix}, \tag{52}$$

$$\Delta_3 = \begin{vmatrix} A_1 - fB_1 & A_1 - fB_1 & 0 \\ A_2 + fB_2 & A_2 + fB_2 & mg + P_V \\ A_3 - fB_3 & -A_3 - fB_3 & 0 \end{vmatrix}. \tag{53}$$

Then, pursuant to Cramer's rule, the following solution of the system of equations is obtained:

$$N_1 = \frac{\Delta_1}{\Delta}, \quad N_2 = \frac{\Delta_2}{\Delta} \text{ and } N_3 = \frac{\Delta_3}{\Delta}. \tag{54}$$

The obtained values of the normal reaction forces N_1, N_2 and N_3 provide for the stable position of the potato tuber during its translation along the longitudinal axes of the separator spirals.

The necessary and sufficient condition of the existence and uniqueness of a solution for the system of Eq. (49) and, hence, also (41), is the system's principal determinant not being equal to zero, i.e. the condition that $\Delta \neq 0$.

The next stage in the development of the mathematical model for the process of cleaning potatoes on the surface of the spiral separator is the compilation of the computer programme and the numerical solution of the obtained system of Eq. (49), which describes the motion of the potato tuber on the surface of the spiral separator, with the use of the PC. That would enable finding the optimum parameters of the spiral separator. Further, it is also necessary to investigate analytically the possible angular displacements of the potato tuber about the axes that pass through its centre of mass during its stay on the surface of the spiral separator under the action of the moments produced by the friction forces.

CONCLUSIONS

1. An analytical mathematical model has been developed for the process of cleaning potatoes on cantilevered cleaning spirals. The model allows determining the efficient design and kinematic parameters of the spiral separator with the use of analytical methods.

2. It has been established that the main work process of the transportation and cleaning of potato tubers takes place in the troughs between two adjacent spirals of the separator.

3. As a result of the analytical investigation, the system of linear equations of the potato tuber's motion, which, in effect, are the equations of the relative equilibrium of the tuber's position in the trough between the coils of two adjacent spirals, has been generated.

4. Solving the obtained system of equations on the basis of Cramer's rule and with the use of the PC will make it possible to plot the graphical dependencies that show the effect of the spiral separator's design and kinematic parameters on the behaviour of the process of separating the potato heap and cleaning the tubers from stuck soil.

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