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The theory of cleaning the crowns of standing beet roots with the use of elastic blades

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Abstract. A standing beet root crown cleaner has been designed. The design comprises the vertical drive shaft that carries two flat elastic cleaning blades installed on axes and connected through the articulated connection. The aim of the study was to develop the new theory of cleaning the crowns of standing roots with the use of an elastic blade installed on the vertical drive shaft in order to determine its optimal design and kinematic parameters. The first step was to design an equivalent schematic model of the interaction between the elastic cleaning blade installed on the vertical drive shaft and the spherical surface of the beet root fixed in the soil. The interaction between the blade and the root's crown took place at the point, where all the forces that can arise during such interaction are applied. A three-dimensional coordinate system was set and the design and kinematic parameters of the considered interaction were designated. Using the original differential equations projected on the set coordinate axes, the system of four nonlinear differential equations of the three-dimensional motion of the elastic cleaning blade on the spherical surface of the root crown was set up, then it was transformed into the system of two differential equations in the normal form. Further, to determine the force that strips off the remaining haulm, which is part of the obtained system of differential equations, the problem of its analytical determination was solved separately. Also, the additional equivalent schematic model of the interaction between the elastic blade as a cantilever beam and the root's crown was designed, the differential equation of the beam's deflection curve (taking into account the beam's simultaneous bending and twisting) was set up and, on the basis of it, the projections of the stripping force on the coordinate axes were found. The values of the force were substituted in the earlier obtained system of differential equations.

Key words: cleaner, elastic blade, harvesting, haulm, sugar beet root.

INTRODUCTION

The state of the art of the implements for mechanized sugar beet harvesting suggests cutting the bulk of herbage from the roots' crowns immediately prior to extracting the roots from the soil, normally with the use of rotary haulm cutting tools set at a higher cutting height, which is followed by final cleaning (after-cleaning) of the crowns from the remaining tops, while the roots remain standing in the ground throughout the procedure (Pogorely & Tatyanko, 2004; Gruber, 2005; Sarec et al., 2009; Schulze Lammers, 2011). There is quite a number of engineering developments, theoretical and experimental studies (Pogorely et al., 1983; Zhang et al., 2013, Gu et al., 2014; Bulgakov et al., 2016a; Bulgakov et al., 2016b; Bulgakov et al., 2017) aimed at solving the problem of cleaning the crowns of standing roots from the remaining haulm. A considerable number of various standing root crown cleaners exists in practice, they can be bladed, drum-type, annular, sectoral, paraboloidal etc (Eichorn, 1999; Wang & Zhang et al., 2013; Wu et al., 2013). But, looking at their design features, virtually all standing root crown cleaners can be divided into the two groups: the ones with horizontal drive shafts and those with vertical drive shafts rotating their cleaning parts. The widely used types of cleaning parts are elastic blades (from rubber or other elastic materials), brushes from wire, loops (flexible and rigid), metal rings, chains, specially shaped drums, discs with serrated or otherwise profiled surface etc. Also, the cleaners with horizontal drive shafts rotating their cleaning parts can move progressively along the row of roots, in case their rotating shafts are positioned along the row direction, or perpendicularly to the row of beet roots. During this movement, the complete cleaning is performed over the implement's working width and the sweeping-out of all plant residues to the harvested part of the beet field is done.

The primary requirement to the design and kinematic parameters of the implements cleaning root crowns from haulm residues is that they must ensure, during the cleaner's translational movement along the planted rows of sugar beet, its secure contact with a greater area of the spherical (or near-spherical) surface of the root crown (Bentini et al., 2005; Bulgakov et al., 2016a). At the same time, the cleaning parts, which perform by some means the removal of haulm residues from the said surface, must provide for the discharge of the residues, mainly into the plantation inter-row spaces. Also, the beet root crown surface after its final cleaning has to be free from haulm residues with equal degree of quality on all sides of the crown. However, this is not always achieved: for example, the rear side of the beet root, looking along the cleaner's translational movement, is often left totally uncleaned.

It is to be pointed out that, depending on the type of cleaning parts utilised in various standing root crown cleaners and the kinematic parameters of the movements they perform, the haulm residues can be removed by hitting, stripping, scraping, direct cutting (scalping) or by some combination of these methods. Moreover, the cleaners of root crowns from haulm residues shall meet a certain number of established agrotechnical requirements: they must not dislodge the roots from the soil, have to damage possibly little the root crown surface itself, provide for the transfer of the separated mass of residues out of the row zone etc., as well as the requirements of the engineering state of the art: they must have a simple design, the minimum energy and metal intensity, the considerable durability of the cleaning parts etc. However, under the condition of ensuring high cleaning process productivity and relatively simple movements of the cleaners' cleaning parts coupled with the varied properties and states of the beet plantations, it proves not always possible to meet the said requirements in full, as we already mentioned earlier.

It is to be noted that the majority of the beet harvesters produced throughout the world do not provide for the cleaning of standing root crowns from haulm residues at all, – on the contrary, they perform the direct cutting (and, undoubtedly, loss) of the whole crown together with the haulm residues. Meanwhile, the root crown (which includes the crown itself and the zone of dormant eyes) can make 10 to 20% of the root's total mass and contain up to 10% of its sugar. Thus, it is hardly reasonable to lose knowingly this part of the root.

The aim of our research was to develop the new theory of the cleaning of standing root crowns with the use of an elastic cleaning blade installed on the vertical drive shaft in order to determine the blade's optimal design and kinematic parameters.

MATERIALS AND METHODS

The methods of the mathematics, the theoretical mechanics and the theory of strength of materials were used in the development of the mathematical model of the interaction between an elastic cleaning blade and the crown of a beet root fixed in the soil.

The design and process schematic model of such a cleaner is presented in Fig. 1. The cleaner comprises vertical drive shaft 1 with disc 2 fixed on its end face, arms 3 are pivotally connected to the periphery of the disc, the lower ends of the arms feature axes 4, on which elastic cleaning blades are installed radially and in the cantilever mode. The upper ends of arms 3 are connected to drive shaft 1 through screw mechanisms 6 and articulated joints 7 with mechanisms 8 of their shifting and fixing on shaft 1, which facilitates the pre-setting of blades 5 at various angles in the vertical plane. Cleaning blades 5 are able to rotate on axes 4 and tilt in the radial direction (with regard to the centre line of drive shaft 1).

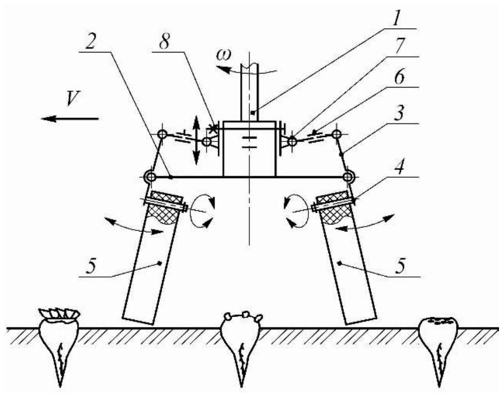


Figure 1. Design and process schematic model of new standing root crown cleaner with vertical drive shaft: 1 – vertical drive shaft; 2 – disc; 3 – double-arm lever; 4 – axis; 5 – cantilevered elastic cleaning blade; 6 – adjustment tensioner; 7 – adjustment slide; 8 – adjustment screw.

The work process of cleaning the standing root crowns from haulm residues with the use of the described type of cleaner proceeds as follows. Cantilevered vertical drive shaft 1 moves progressively along the row of planted beet roots, from which the bulk of haulm was cut earlier. Because of the rotation of shaft 1 with a pre-set angular velocity ω, cleaning blades 5 that are under the action of centrifugal forces tilt from the vertical position by some angle, forming as a result the 'cleaning cone' as it is called (the vertex of which is on drive shaft 1, the base facing down and the generatrices represented by the end faces of cleaning blades 5), which provides for the formation of a sufficiently wide cleaning zone. The cleaner set at a certain height above the soil surface level (in case of a significant quantity of residues on the root crowns and the presence of other vegetable remains in the row the said height must be possibly lower) moves progressively along the row of roots and, as a result of the rotation of drive shaft 1, its cleaning blades 5 hit the front parts of the root crowns, then, bending, move their flat surfaces over the very surfaces of the root crowns and in this phase the work process of cleaning, i.e. the stripping of haulm residues from the root crown surfaces takes place. The use of tilt control screw mechanisms 6, articulated joints 7 and shifting and fixing mechanisms 8 allows to pre-set elastic cleaning blades 5 at different angles to the centre line of cantilevered drive shaft 1. We have carried out the experimental research and field testing of the described cleaner of root crowns from haulm residues and they have produced positive results.

The general view of the cleaner of sugar beet root crowns from haulm residues is presented in Fig. 2.

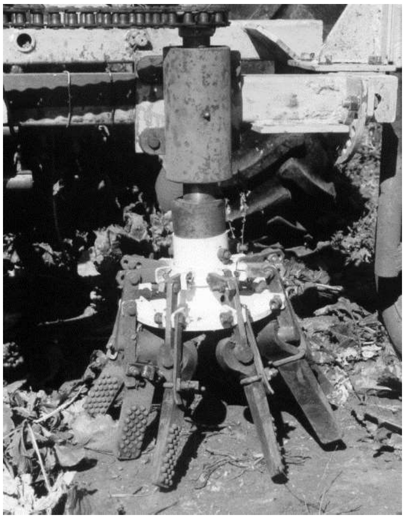


Figure 2. Working head for cleaning root crowns from haulm residues mounted on tractor during experimental investigations.

Fig. 3 shows the view of the cleaning head (a) and one of its eight cleaning blades (b).

a) 

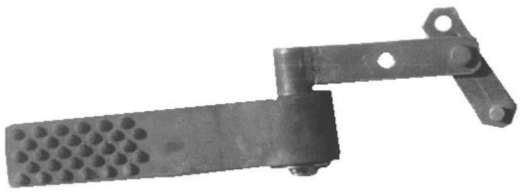
b) 

Figure 3. Beet root crown cleaner (a) and cleaning blade (b).

During the cleaner's operation, the vertical drive shaft rotates at a pre-set revolution rate and at the same time moves progressively along the row of beet roots. The cleaning blades hit the beet root crowns, stripping off them haulm residues, and take the latter out of the row zone.

The intensity of cleaning is adjusted by changing the vertex angle of the cone formed by the cleaning blades as well as changing the length and width of the cleaning blades themselves.

In order to determine the optimal design and kinematic parameters of the new standing root crown cleaner designs we proposed the theory of the interaction between the elastic cleaning blade and the root crown in the process of stripping haulm residues from the root's surface, i.e. we developed a model of the movement of the elastic blade, which rotates about the vertical axis and at the same time moves progressively, on the spherical surface of the beet root crown.

In this case, the beet root is modelled as a solid rigidly fixed in the soil, only its crown protruding above the soil surface to a height of h, and that crown is approximated by a spherical surface with a radius of r (Fig. 4). Above it, the cleaner with a vertical axis of rotation, which rotates its cleaning blades with an angular velocity of ω, moves progressively along the row of roots (i.e. in the vertical and longitudinal plane that contains the root's centre line). As it was mentioned above, the cleaning blade is tilted from its vertical position through some angle, therefore, each time when the cleaner meets a root crown, two impacts take place in effect: the first is from the blade oriented at an obtuse angle to the direction of movement (i.e. from the outer part of the 'cleaning cone', when the cleaner only starts interacting with the root crown) and the second is from the blade oriented already at an acute angle to the direction of movement, i.e. from the inner part of the 'cleaning cone'. During the first impact, the cleaning blade bends toward the inside of the cone, its outer part interacts with the beet root crown and it 'rolls' over the crown's surface (mainly on the small front part of the root crown), stripping the haulm residues, until the end of the contact with the surface of the crown, which is followed by the straightening of the blade. During the second impact, the cleaning blade hits the root crown with its inner part (when the drive shaft axis has already passed the root's centre line, i.e. the root is already inside the cleaning cone), it bends back even more along the direction that is opposite to the cleaner's translational movement and also its inner part rolls over the root crown's spherical surface towards the following end of the contact with it.

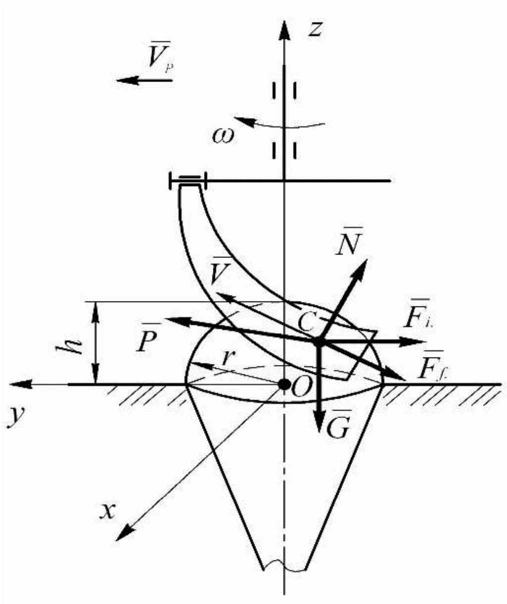


Figure 4. Equivalent schematic model of interaction between elastic cleaning blade and beet root crown.

In order to develop the analytical model of the interaction between the elastic cleaning blade and the beet root's crown we have to make an equivalent schematic model, in which the movement of the elastic cleaning blade on the spherical surface of the root crown will be examined in the absolute (fixed) three-dimensional Cartesian coordinate system *xOyz* (Fig. 4).

Under the action of the force generated during the contact between the elastic cleaning blade and the beet root that takes place at point *C* during the cleaner's translational movement (with a velocity of $\bar{V}_p$ as well as the action of the drive's angular momentum the blade will undergo simultaneously the bending and twisting deformations.

Force $\bar{P}$ generated by the mentioned deformations is in effect the working force that strips the haulm residues from the surface of the beet root crown. In addition to the said force $\bar{P}$, frictional force $\bar{F}_{f.}$, centrifugal force of inertia $\bar{F}_{i.}$, normal constraint force $\bar{N}$ and weight $\bar{G}$ of the blade itself, directed as shown in Fig. 4, are also applied to the root crown at contact point *C*. Hereafter, the movement of the point *C* of the contact between the blade and the root crown will be considered as the movement of nominal material point *C* specifically under the action of the mentioned forces: $\bar{P}$, $\bar{N}$, $\bar{F}_{f.}$, $\bar{F}_{i.}$, $\bar{G}$. That said, point *C* itself moves over the surface of the root crown with translational movement velocity $\bar{V}$.

RESULTS AND DISCUSSION

First of all, the differential equation of the movement of point *C* on the surface of the beet root crown in the vector form must be generated. It will have the following representation (Dreizler & Lüdde, 2010):

$$m\bar{a} = \bar{P} + \bar{N} + \bar{F}_{f.} + \bar{F}_{i.} + \bar{G}, \tag{1}$$

where *m* – mass of cleaning blade (kg); α – acceleration in the movement of the cleaning blade on the root crown (m s^{-2}).

The equation (1) in the projections on the axes of the assumed Cartesian coordinate system *xOyz* will be represented by a system of differential equations and appear as follows:

$$\left.\begin{aligned} m\ddot{x} &= P_x + N_x + F_{f.x} + F_{i.x}, \\ m\ddot{y} &= P_y + N_y + F_{f.y} + F_{i.y}, \\ m\ddot{z} &= P_z + N_z + F_{f.z} + G. \end{aligned}\right\} \tag{2}$$

The right members of the equations in the system (2) represent the sums of the projections of the forces applied at the point *C* of the contact between the elastic cleaning blade and the beet root crown on the respective coordinate axes.

Further, the analytical expressions of the force of friction $F_{f.}$, as well as the centrifugal force of inertia $F_{i.}$ and their projections on the axes x and y have to be written down. They will appear as follows, respectively:

– for the force of friction:

$$F_{f.} = f_{f.}N \tag{3}$$

where $\bar{f}_{f.}$ – coefficient of friction of the elastic blade on the beet root crown surface;

– for the centrifugal force of inertia:

$$F_{i.} = m\omega^2 R \tag{4}$$

– for the projections of the centrifugal force $\bar{F}_{i.}$ on the mentioned axes *x* and *y*:

$$F_{i.x} = m\omega^2 R\cos\omega t,$$
$$F_{i.y} = -m\omega^2 R\sin\omega t, \tag{5}$$

where m – mass of cleaning blade, (kg), ω – angular speed of rotation of the cleaner's drive shaft (s^{-1}); R – radius of rotation of point C about axis z (m).

Considering the fact that frictional force vector $\overline{F}_{f.}$ has a direction that is opposite to the direction of velocity vector $\overline{V}$ of the movement of contact point C as well as basing on (3) and (5), the system of differential equations (2) can be written as follows:

$$\left.\begin{aligned} m\ddot{x} &= P_x + N\cos\left(x,\hat{\overline{N}}\right) - f_{f.}N\cos\left(\dot{x},\hat{\overline{V}}\right) + m\omega^2 R\cos\omega t,\\ m\ddot{y} &= P_y + N\cos\left(y,\hat{\overline{N}}\right) - f_{f.}N\cos\left(\dot{y},\hat{\overline{V}}\right) - m\omega^2 R\sin\omega t,\\ m\ddot{z} &= P_z + N\cos\left(z,\hat{\overline{N}}\right) - f_{f.}N\cos\left(\dot{z},\hat{\overline{V}}\right) - mg. \end{aligned}\right\} \tag{6}$$

Further, the direction cosines that appear in the right members of the equations (6) have to be determined. According to Vasilenko (1996), they are as follows:

$$\cos\left(x,\hat{\overline{N}}\right) = \frac{\partial f}{\partial x}\cdot\frac{1}{\Delta f};\ \cos\left(y,\hat{\overline{N}}\right) = \frac{\partial f}{\partial y}\cdot\frac{1}{\Delta f};\ \cos\left(z,\hat{\overline{N}}\right) = \frac{\partial f}{\partial z}\cdot\frac{1}{\Delta f};\ \cos\left(\dot{x},\hat{\overline{V}}\right) = \frac{\dot{x}}{V};$$
$$\cos\left(\dot{y},\hat{\overline{V}}\right) = \frac{\dot{y}}{V};\ \cos\left(\dot{z},\hat{\overline{V}}\right) = \frac{\dot{z}}{V}. \tag{7}$$

where $\Delta f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2}$ – modulus of the gradient of function $f(x,y,z)$; $f(x,y,z)=0$ – constraint equation; $V = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$ – modulus of velocity vector.

In this case, the constraint is the beet root crown itself, which, as already mentioned earlier, can be approximated by a spherical solid and that can be represented by the following constraint equation:

$$f(x,y,z) = x^2 + y^2 + z^2 - r^2 = 0, \tag{8}$$

where r – radius of sphere of the beet root crown (m).

Thus, taking into account the expressions for the direction cosines of force vectors $\overline{N}$ and $\overline{F}_{t.}$ and adding the equation of sphere (8) to the system of equations (6), the following system of differential equations of the three-dimensional motion of the elastic cleaning blade on the beet root crown surface is obtained:

$$\left.\begin{aligned} m\ddot{x} &= P_x + \frac{N}{\Delta f}\frac{\partial f}{\partial x} - f_{f.}N\frac{\dot{x}}{V} + m\omega^2 R\cos\omega t,\\ m\ddot{y} &= P_y + \frac{N}{\Delta f}\frac{\partial f}{\partial y} - f_{f.}N\frac{\dot{y}}{V} - m\omega^2 R\sin\omega t,\\ m\ddot{z} &= P_z + \frac{N}{\Delta f}\frac{\partial f}{\partial z} - f_{f.}N\frac{\dot{z}}{V} - mg,\\ x^2 &+ y^2 + z^2 - r^2 = 0. \end{aligned}\right\} \tag{9}$$

Hence, this system (9) of differential equations describes the movement of the blade contact point C on the spherical surface of the crown under the action of the forces that are applied by the cleaning blade to the beet root crown.

Further, the partial derivatives of the function $f(x,y,z)$ with respect to the variables x, y and z and the gradient of function Δf, which are parts of the system of equations (9) have to be calculated. The result will be as follows:

$$\frac{\partial f}{\partial x} = 2x, \quad \frac{\partial f}{\partial y} = 2y, \quad \frac{\partial f}{\partial z} = 2z, \tag{10}$$

$$\Delta f = \sqrt{(2x)^2 + (2y)^2 + (2z)^2} = 2\sqrt{x^2 + y^2 + z^2} = 2r.$$

After substituting the obtained expressions (10) in the system of differential equation (9), the following system of equations with respect to the variables x, y, z, N is obtained:

$$\left.\begin{aligned} m\ddot{x} &= P_x + \frac{x}{r}N - f_{f.}N\frac{\dot{x}}{V} + m\omega^2 R\cos\omega t, \\ m\ddot{y} &= P_y + \frac{y}{r}N - f_{f.}N\frac{\dot{y}}{V} - m\omega^2 R\sin\omega t, \\ m\ddot{z} &= P_z + \frac{z}{r}N - f_{f.}N\frac{\dot{z}}{V} - mg, \\ x^2 &+ y^2 + z^2 - r^2 = 0. \end{aligned}\right\} \tag{11}$$

The next step is to reduce the obtained system of four equations with the four variables x, y, z, N to a system of two equations with the two variables x, y and a separate formula for determining the normal constraint force N.

For this purpose, the following transformations will be carried out. After differentiating the equation of sphere (8) with respect to time t, we obtain:

$$2x\dot{x} + 2y\dot{y} + 2z\dot{z} = 0,$$

or

$$x\dot{x} + y\dot{y} + z\dot{z} = 0. \tag{12}$$

On the second differentiation of the equation of sphere (8) with respect to time t, we obtain:

$$x\ddot{x} + \dot{x}^2 + y\ddot{y} + \dot{y}^2 + z\ddot{z} + \dot{z}^2 = 0$$

and from that we have

$$(x\ddot{x} + y\ddot{y} + z\ddot{z}) + (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = 0$$

Since $\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = V^2$, the following formula for the squared velocity V of the movement of the point C on the beet root crown is concluded from the last expression:

$$V^2 = -(x\ddot{x} + y\ddot{y} + z\ddot{z}). \tag{13}$$

The obtained formulae (12) and (13) will be applied for the further transformation of the system of differential equations (11). After multiplying the first equation of the system (11) by x, the second equation by y, and the third equation by z, the following is obtained:

$$\left.\begin{aligned} m\ddot{x}\cdot x &= P_x\cdot x+\frac{x^2}{r}N-f_{f.}N\frac{\dot{x}}{V}\cdot x+m\omega^2R\cdot x\cdot\cos\omega t,\\ m\ddot{y}\cdot y &= P_y\cdot y+\frac{y^2}{r}N-f_{f.}N\frac{\dot{y}}{V}\cdot y-m\omega^2R\cdot y\cdot\sin\omega t,\\ m\ddot{z}\cdot z &= P_z\cdot z+\frac{z^2}{r}N-f_{f.}N\frac{\dot{z}}{V}\cdot z-mg\cdot z,\end{aligned}\right\} \tag{14}$$

Adding term by term all three equations of the system (14), we come to:

$$\begin{aligned} m\left(x\ddot{x}+y\ddot{y}+z\ddot{z}\right) &= \left(P_x x+P_y y+P_z z\right)+\frac{N}{r}\left(x^2+y^2+z^2\right)-\\ &-f_{f.}\frac{N}{V}\left(x\dot{x}+y\dot{y}+z\dot{z}\right)+m\omega^2Rx\cos\omega t-m\omega^2Ry\sin\omega t-mgz.\end{aligned} \tag{15}$$

Since $x^2+y^2+z^2=r^2$, the expression (15) can be represented as follows:

$$\begin{aligned} m\left(x\ddot{x}+y\ddot{y}+z\ddot{z}\right) &= \left(P_x x+P_y y+P_z z\right)+rN-\\ &-f_{f.}\frac{N}{V}\left(x\dot{x}+y\dot{y}+z\dot{z}\right)+m\omega^2R\left(x\cos\omega t-y\sin\omega t\right)-mgz.\end{aligned} \tag{16}$$

Taking into account the formulae (12) and (13), we obtain:

$$\begin{aligned} -mV^2 &= \left(P_x x+P_y y+P_z z\right)+rN+m\omega^2R\left(x\cos\omega t-\right.\\ &\left.-y\sin\omega t\right)-mgz.\end{aligned} \tag{17}$$

From the equation (17) the value of the normal pressure N at the point C of the contact between the cleaning blade and the beet root crown is found. It is equal to:

$$N=-\frac{1}{r}\left[P_x x+P_y y+P_z z+mV^2+m\omega^2R\left(x\cos\omega t-y\sin\omega t\right)-mgz\right]. \tag{18}$$

Further, from the equation (12) the following can be obtained:

$$\dot{z}=-\frac{x\dot{x}+y\dot{y}}{z},$$

then

$$\dot{z}^2=\frac{\left(x\dot{x}+y\dot{y}\right)^2}{z^2},$$

or

$$\dot{z}^2=\frac{\left(x\dot{x}+y\dot{y}\right)^2}{r^2-\left(x^2+y^2\right)}. \tag{19}$$

Taking into consideration (19), we obtain the expression for the squared velocity V of the movement of the cleaning blade on the beet root crown surface, which does not contain the variable z, but only contains the variables x, y and their first-order derivatives, that is:

$$V^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2 = \dot{x}^2 + \dot{y}^2 + \frac{(x\dot{x} + y\dot{y})^2}{r^2 - (x^2 + y^2)} \tag{20}$$

Substituting the formula (18) in the first two equations of the system (11), we obtain:

$$\left.\begin{aligned}
&m\ddot{x} = P_x - \left(\frac{x}{r} - f_{f.}\frac{\dot{x}}{V}\right)\Big[P_x x + P_y y + mV^2 + m\omega^2 R\big(x\cos\omega t - \\
&\quad - y\sin\omega t\big) - mgz\Big]\frac{1}{r} + m\omega^2 R\cos\omega t,\\
&m\ddot{y} = P_y - \left(\frac{y}{r} - f_{f.}\frac{\dot{y}}{V}\right)\Big[P_x x + P_y y + mV^2 + m\omega^2 R\big(x\cos\omega t - \\
&\quad - y\sin\omega t\big) - mgz\Big]\frac{1}{r} - m\omega^2 R\sin\omega t,\\
&x^2 + y^2 + z^2 - r^2 = 0.
\end{aligned}\right\} \tag{21}$$

Finally, taking into account the formula (20) and the last equation of the system (21), a system of two differential equations of motion with the two variables x and y is obtained:

$$\left.\begin{aligned}
&m\ddot{x} = P_x - \left[\frac{x}{r} - f_{f.}\frac{\sqrt{r^2 - x^2 - y^2}\cdot\dot{x}}{\sqrt{(\dot{x}^2 + \dot{y}^2)(r^2 - x^2 - y^2) + (x\dot{x} + y\dot{y})^2}}\right]\times\\
&\times\left[P_x x + P_y y + m\dot{x}^2 + m\dot{y}^2 + \frac{m(x\dot{x} + y\dot{y})^2}{(r^2 - x^2 - y^2)} + \right.\\
&\left. + m\omega^2 R(x\cos\omega t - y\sin\omega t) - mg\sqrt{r^2 - x^2 - y^2}\right]\frac{1}{r} + m\omega^2 R\cos\omega t,\\
&m\ddot{y} = P_y - \left[\frac{y}{r} - f_{f.}\frac{\sqrt{r^2 - x^2 - y^2}\cdot\dot{y}}{\sqrt{(\dot{x}^2 + \dot{y}^2)(r^2 - x^2 - y^2) + (x\dot{x} + y\dot{y})^2}}\right]\times\\
&\times\left[P_x x + P_y y + m\dot{x}^2 + m\dot{y}^2 + \frac{m(x\dot{x} + y\dot{y})^2}{(r^2 - x^2 - y^2)} + \right.\\
&\left. + m\omega^2 R(x\cos\omega t - y\sin\omega t) - mg\sqrt{r^2 - x^2 - y^2}\right]\frac{1}{r} - m\omega^2 R\sin\omega t.
\end{aligned}\right\} \tag{22}$$

The obtained system of equations (22) is a system of nonlinear differential equations with respect to the unknown functions $x(t)$ and $y(t)$ in what is known as the normal form, when the derivatives of higher order are expressed in terms of unknown functions and lower-order derivatives. Since the system of differential equations under consideration is nonlinear, it can be solved only with the use of approximate numerical techniques on a PC under the specified initial conditions.

After determining the unknown functions $x(t)$ and $y(t)$, it is possible to find the unknown function $z(t)$ from the constraint equation (8):

$$z = \sqrt{r^2 - x^2 - y^2}\,. \tag{23}$$

Then it becomes possible to calculate the normal constraint force N with the use of the expression (18) and perform the comparative assessment of the obtained value with respect to the force acceptable for beet roots under the condition of their remaining undamaged. The said condition appears as follows:

$$N \le [N], \tag{24}$$

where $[N]$ – acceptable normal force.

Thus, the system of differential equations (22) of the three-dimensional motion of the elastic cleaning blade on the surface of the beet root crown has been obtained. Nevertheless, this system of differential equations contains the force $\overline{P}$, which causes the separation of haulm residues from the beet root crown during its cleaning in the standing condition. Without determining the said force it is impossible to solve the obtained system of differential equations (22).

The force in question $\overline{P}$ can be determined on the basis of the conditions of bending and twisting of the elastic cleaning blade during its movement on the root crown surface. According to the principle of superposition, it is possible to analyse the deformations of bending and twisting of the elastic cleaning blade separately. The bending deformation arises in consequence of the contact between the cleaning blade and the beet root crown during the cleaner's translational movement along the planted row and also the action of the centrifugal force $\overline{F}_{i.}$ of the cleaning tool rotation.

In order to solve the set problem analytically, the cleaning blade will be analysed separately as cantilever beam AB with a fixed end (Fig. 5), to which the force $\overline{P}_{b.}$ is applied at a distance of d, which is the distance from the contact point C to the free end of the blade (point B). The Cartesian coordinate system xAy, the axis Ax of which coincides with the centre line of the blade, is assumed.

The force P_b can be determined from the differential equation of the beam deflection curve (Grote, K.-H. & Antonsson, E.K., 2008). The moment of deflection in a random beam section at a distance of x from the origin of coordinates (point A) is:

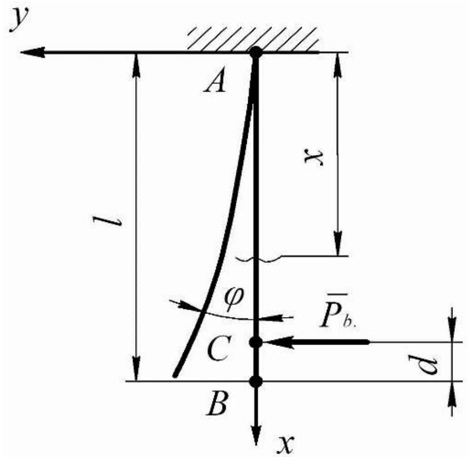


Figure 5. Deformation of elastic cleaning blade under the action of force $\overline{P}_{b}$.

$$M_x = -P_{b.}\left(l - d - x\right). \tag{25}$$

The differential equation of the beam deflection curve will appear as follows:

$$EJ\frac{d^2y}{dx^2}=M(x), \tag{26}$$

where EJ – beam stiffness; y – beam deflection.
Then, taking into account the formula (25), the following is obtained:

$$EJ\frac{d^2y}{dx^2}=-P_{b.}(l-d-x)\cdot \tag{27}$$

In order to determine the amount of deflection $y(x)$, it is necessary to integrate the obtained differential equation (27) two times. The first integration will produce:

$$EJ\frac{dy}{dx}=-\int P_{b.}(l-d-x)\,dx,$$

or:

$$EJ\frac{dy}{dx}=-P_{b.}(l-d)x+P_{b.}\frac{x^2}{2}+C_1\cdot \tag{28}$$

The second integration will give:

$$EJ\,y(x)=-\int P_{b.}(l-d)x\,dx+\int P_{b.}\frac{x^2}{2}dx+C_1x+C_2,$$

or:

$$EJ\,y(x)=-P_{b.}(l-d)\frac{x^2}{2}+P_{b.}\frac{x^3}{6}+C_1x+C_2\cdot \tag{29}$$

where C_1, C_2 – arbitrary constants of integration.

In order to determine the arbitrary constants of integration C_1 and C_2, the following initial conditions will be set: at $x = 0$:

$$\frac{dy}{dx}=0,$$
$$y(x)=y(0)=0\,.$$

It can be found from the equation (28), after substituting in it the initial conditions, that $C_1 = 0$, and it is found from the equation (29) that $C_2 = 0$. Then the expressions (28) and (29) will finally appear as follows:

$$EJ\,\varphi(x)=EJ\frac{dy}{dx}=-P_{b.}\left[(l-d)x-\frac{x^2}{2}\right], \tag{30}$$

$$EJ\,y(x)=-P_{b.}\left[(l-d)\frac{x^2}{2}-\frac{x^3}{6}\right], \tag{31}$$

or

$$\varphi(x)=-\frac{P_{b.}}{EJ}\left[(l-d)x-\frac{x^2}{2}\right], \tag{32}$$

$$y(x) = -\frac{P_{b.}}{EJ}\left[(l-d)\frac{x^2}{2} - \frac{x^3}{6}\right]. \tag{33}$$

In the obtained equations (30) – (33): $\varphi(x) = \frac{dy}{dx}$ – angular displacement of the cleaning blade section at the arbitrary point x; $y(x)$ – amount of deflection of the cleaning blade at the arbitrary point x. Further, the angular displacement of section φ and amount of deflection y at the point C of contact between the cleaning blade and the beet root crown, i.e. at the point $x = l - d$, will be determined:

– angular displacement:

$$\varphi(l-d) = -\frac{P_{b.}}{EJ}\left[(l-d)^2 - \frac{(l-d)^2}{2}\right],$$

after transformation:

$$\varphi(l-d) = -\frac{P_{b.}}{2EJ}(l-d)^2. \tag{34}$$

– deflection:

$$y(l-d) = -\frac{P_{b.}}{EJ}\left[\frac{(l-d)^3}{2} - \frac{(l-d)^3}{6}\right],$$

after transformation:

$$y(l-d) = -\frac{P_{b.}}{3EJ}(l-d)^3. \tag{35}$$

The expressions (34) and (35) can be regarded as the functions of the angular displacement of section and deflection of the elastic cleaning blade with respect to the value d, because the value d is variable during the movement of the cleaning blade on the root crown surface.

For the case, when $d = 0$, i.e. when the contact point C is at the free end of the cleaning blade, the following is obtained:

$$\varphi(l) = -\frac{P_{b.}l^2}{2EJ}, \tag{36}$$

$$y(l) = -\frac{P_{b.}l^3}{3EJ}. \tag{37}$$

If $\varphi(l)$ or $y(l)$ are known values, then it can be determined from the equations (36) and (37) that:

$$P_{b.} = -\frac{2EJ\varphi(l)}{l^2}, \tag{38}$$

$$P_{b.} = -\frac{3EJ\,y(l)}{l^3}. \tag{39}$$

The force $\overline{P}_{pr.}$ that has the same modulus as the force $\overline{P}_{b.}$, but has the opposite direction, is the force pressing the elastic cleaning blade to the beet root crown. This force is a useful force, which performs the stripping of haulm residues as a consequence of the bending deformation. It can be assumed approximately that the said force is in line with the course of the cleaner's movement over the row of beet roots.
Hence, the pressing force $\overline{P}_{pr.}$ is equal to $P_{pr.} = -P_{b.}$, i.e.:

$$P_{pr.} = \frac{2EJ\varphi(l)}{l^2}, \tag{40}$$

or

$$P_{pr.} = \frac{3EJy(l)}{l^3}. \tag{41}$$

Further, the twisting deformation of the cleaner's cleaning blade due to the rotary motion of the tool about its axis is to be analysed. The twisting deformation of the blade occurs because of the action of the moment of rotation imparted by the drive of the cleaning tool. It can be conventionally assumed that the blade itself is a shaft with a rectangular cross-section of $h \times b$, where h – blade thickness and b – blade width.

It is known that the angle of torsion Θ for rectangular cross-section shafts is determined by the formula (Grote & Antonsson, 2008):

$$\Theta = \frac{M_{tw.}l}{GJ_{tw.}}, \tag{42}$$

where $M_{tw.}$ – moment of rotation causing the twisting; l – length of shaft; G – shear modulus of the material; $J_{tw.}$ – moment of inertia under torsion.
The moment of inertia $J_{tw.}$ is determined by the following formula:

$$J_{tw.} = \alpha h^4. \tag{43}$$

The coefficient α depends on the ratio $b : h$ and is selected from the tables (Grote & Antonsson, 2008). Further, knowing the blade's torsion angle Θ, it is possible to determine from the formula (42) the moment of rotation that causes the blade's twisting. It will be:

$$M_{tw.} = \frac{GJ_{tw.}\Theta}{l}. \tag{44}$$

Then the force $P_{tw.}$ that twists the blade can be expressed as follows:

$$P_{tw.} = \frac{M_{tw.}}{b},$$

where b – blade width,
or, taking into account the expression (44):

$$P_{tw.} = \frac{GJ_{tw}\Theta}{lb}. \tag{45}$$

The cleaning force $\overline{P}_{p.}$, which has a modulus equal to that of the force $\overline{P}_{tw.}$ and the opposite direction, is a useful force, which performs the stripping of haulm residues due to the blades elasticity in its twisting, because $P_{p.} = P_{tw.}$

It can be assumed that this force has a direction along the tangent to the trajectory of the rotary motion of the cleaning blade on the beet root crown. Then the resulting haulm residue stripping force *P* will be as follows:

$$\overline{P} = \overline{P}_{pr.} + \overline{P}_{p.} \cdot \qquad (46)$$

The vector equation (46) in terms of its projections on the axes of the system of coordinates *xOyz* (46) can be written as follows:

$$\begin{aligned} P_x &= P_{pr.x} + P_{p.x}, \\ P_y &= P_{pr.y} + P_{p.y}, \\ P_z &= P_{pr.z} + P_{p.z}. \end{aligned} \qquad (47)$$

Since:

$$P_{pr.x} = 0,\ P_{pr.y} = P_{pr.},\ P_{pr.z} = 0,$$

$$P_{p.x} = P_{p.} \sin \omega t,\ P_{p.y} = P_{p.} \cos \omega t,\ P_{p.z} = 0,$$

then the equations (47) will appear as follows:

$$\begin{aligned} P_x &= P_{p.x}, \\ P_y &= P_{pr.} + P_{p.y}, \\ P_z &= 0, \end{aligned}$$

or, taking into account the expressions of the projections:

$$\begin{aligned} P_x &= P_{p.} \sin \omega t, \\ P_y &= P_{pr.} + P_{p.} \cos \omega t, \\ P_z &= 0 \cdot \end{aligned}$$

After substituting in the latter equations the expressions of the forces $P_{pr.}$ and $P_{p.}$ in accordance with (41) and (45), we obtain, respectively:

$$P_x = \frac{GJ\Theta}{lb} \sin \omega t, \qquad (48)$$

$$P_y = \frac{3EJ\, y(l)}{l^3} + \frac{GJ_{tw}\Theta}{l\, b} \cos \omega t \cdot \qquad (49)$$

The obtained expressions (48) and (49) for the stripping force projections are substituted in the system of differential equations (22). The final result will be as follows:

$$\left.\begin{aligned}
&m\ddot{x}=\frac{GJ_{tw.}\Theta}{lb}\sin\omega t-\left[\frac{x}{r}-f_{f.}\frac{\sqrt{r^2-x^2-y^2}\cdot\dot{x}}{\sqrt{\left(\dot{x}^2+\dot{y}^2\right)\left(r^2-x^2-y^2\right)+\left(x\dot{x}+y\dot{y}\right)^2}}\right]\times\\
&\times\left[\frac{GJ_{tw.}\Theta}{lb}\sin\omega t\cdot x+\left(\frac{3EJ\,y(l)}{l^3}+\frac{GJ_{tw.}\Theta}{lb}\cos\omega t\right)y+m\dot{x}^2+m\dot{y}^2+\right.\\
&\left.+\frac{m\left(x\dot{x}+y\dot{y}\right)^2}{r^2-x^2-y^2}+m\omega^2R\left(x\cos\omega t-y\sin\omega t\right)-mg\sqrt{r^2-x^2-y^2}\right]\times\\
&\times\frac{1}{r}+m\omega^2R\cos\omega t,\\
&m\ddot{y}=\frac{3EJ\,y(l)}{l^3}+\frac{GJ_{tw.}\Theta}{lb}\cos\omega t-\\
&-\left[\frac{y}{r}-f_{f.}\frac{\sqrt{r^2-x^2-y^2}\cdot\dot{y}}{\sqrt{\left(\dot{x}^2+\dot{y}^2\right)\left(r^2-x^2-y^2\right)+\left(x\dot{x}+y\dot{y}\right)^2}}\right]\times\\
&\times\left[\frac{GJ_{tw.}\Theta}{lb}\sin\omega t\cdot x+\left(\frac{3EJ\,y(l)}{l^3}+\frac{GJ_{tw.}\Theta}{lb}\cos\omega t\right)y+m\dot{x}^2+m\dot{y}^2+\right.\\
&\left.+\frac{m\left(x\dot{x}+y\dot{y}\right)^2}{r^2-x^2-y^2}+m\omega^2R\left(x\cos\omega t-y\sin\omega t\right)-mg\sqrt{r^2-x^2-y^2}\right]\times\\
&\times\frac{1}{r}-m\omega^2R\sin\omega t.
\end{aligned}\right\}\quad(50)$$

Thus, we have obtained the system of two differential equations (50) of the motion of the elastic cleaning blade on the spherical root crown surface, which models the process of haulm residue stripping and contains two unknown variables (*x* and *y*) as well as kinematic and design parameters of the cleaner with a vertical axis of rotation.

The obtained system of differential equations (50) was solved numerically with the use of the Runge-Kutta method on the PC, resulting in the plotting of the diagrams shown in Figs 6–11. It becomes obvious from the graphs in Fig. 6 that the use of reinforced rubber, which has the highest elasticity modulus *E*, as the material of the cleaning blade ensures the generation of the sufficiently strong stripping force *P* (graph 2). Any smaller values of the elasticity modulus *E* will not always facilitate the high quality cleaning of sugar beet root crowns from short, green and sufficiently strong haulm residues, while its higher values will result in the considerable damage to the root crowns.

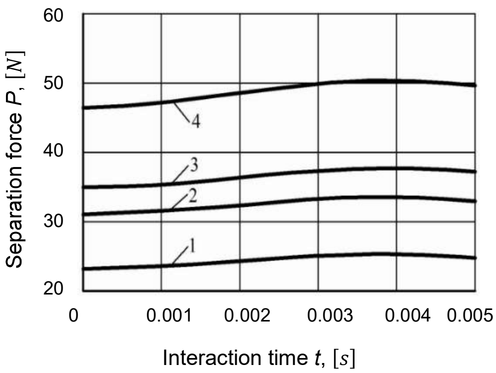


Figure 6. Relation between cleaning blade stripping force P and time t at different values of elasticity modulus E: 1) 1.80; MPa; 2) 2.42 MPa; 3) 3.17 MPa; 4) 4.3 MPa.

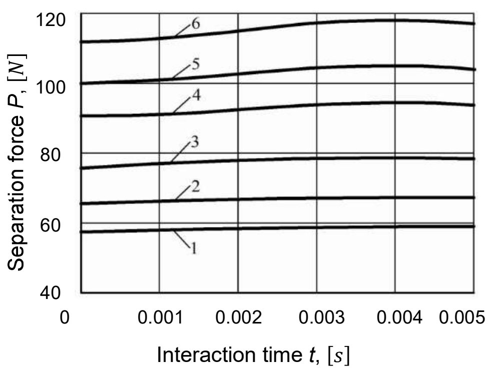


Figure 7. Relation between cleaning blade stripping force P and time t at different values of blade length: 1) 32 cm; 2) 28 cm; 3) 24 cm; 4) 20 cm; 5) 18 cm; 6) 16 cm (blade width 2 cm).

It can be concluded from the graphs in Figs 7–11 that the magnitude of the stripping force P to a great extent depends on the length of the blade and its width. On the basis of the results obtained from the numerical calculations on a PC a conclusion can be drawn that the optimal dimensions of the blade are its length equal to 20–25 cm and its width equal to 4–5 cm, in which case the total stripping force P will not exceed its maximum permitted value of 30.0 N (Pogorely et al., 1983).

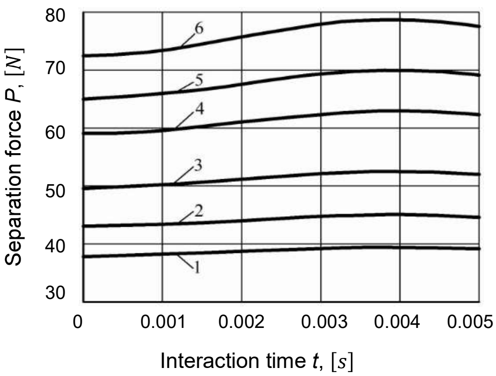


Figure 8. Relation between cleaning blade stripping force P and time t at different values of blade length: 1) 32 cm, 2) 28 cm, 3) 24 cm, 4) 20 cm, 5) 18 cm, 6) 16 cm (blade width 3 cm).

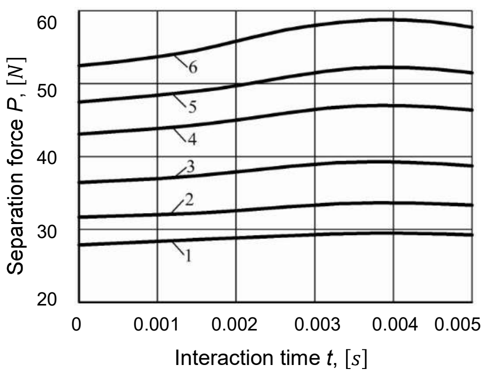


Figure 9. Relation between cleaning blade stripping force P and time t at different values of blade length: 1) 32 cm, 2) 28 cm, 3) 24 cm, 4) 20 cm, 5) 18 cm, 6) 16 cm (blade width 4 cm).

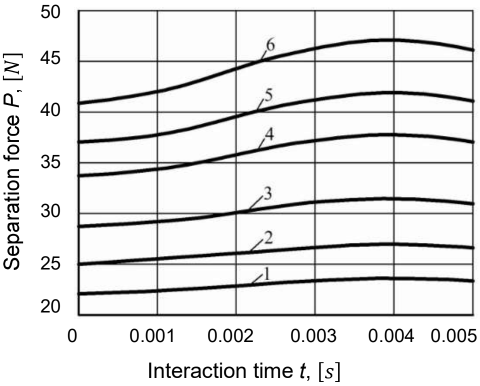


Figure 10. Relation between cleaning blade stripping force P and time t at different values of blade length: 1) 32 cm, 2) 28 cm, 3) 24 cm, 4) 20 cm, 5) 18 cm, 6) 16 cm (blade width 5 cm).

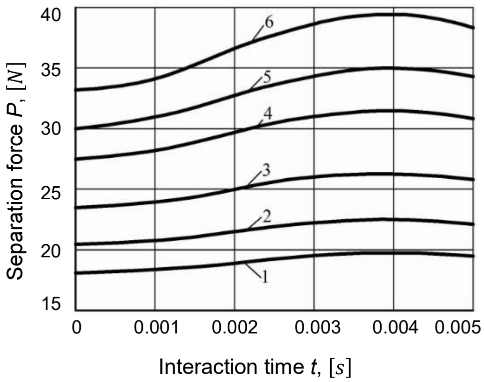


Figure 11. Relation between cleaning blade stripping force P and time t at different values of blade length: 1) 32 cm, 2) 28 cm, 3) 24 cm, 4) 20 cm, 5) 18 cm, 6) 16 cm (blade width 6 cm).

CONCLUSIONS

1. A new design of the standing root crown cleaner has been developed. The cleaner consists of the vertical drive shaft, on which two elastic cleaning blades installed as cantilevers with the use of articulated joints and axes. The experimental studies and field tests of the cleaner have produced positive results.

2. The theory of the interaction between an elastic cleaning blade that is installed as a cantilever on the vertical drive shaft and the crown of a beet root fixed in the soil has been developed. For that purpose initially the equivalent schematic model of the interaction between the elastic cleaning blade and the spherical surface of the beet root crown was formulated. It was assumed that the interaction between the blade and the root crown took place at the point, where all the forces acting in such interaction were applied. The three-dimensional coordinate system was set and the design and kinematic parameters of the interaction were designated.

3. Basing on the use of the original differential equations in the form of their projections on the assumed coordinate axes, the system of four nonlinear differential equations of the three-dimensional motion of the elastic cleaning blade on the spherical surface of the root crown was set up. Thereafter, the system of differential equations was transformed into a system of two differential equations in the normal form.

4. In order to determine the haulm residue stripping force, the additional equivalent schematic model of the interaction between the cleaning blade as a cantilever-fixed beam and the root crown was composed, then the differential equation of the beam deflection curve was set up (taking into account the simultaneous bending and twisting of the beam) and on its basis the projections of the stripping force on the assumed coordinate axes were obtained.

5. The finally obtained system of differential equations of the movement of the elastic cleaning blade on the spherical surface of the beet root crown with the specified haulm residue stripping force taken into account can be solved with the use of numerical methods on a PC after compiling the respective software. Substituting then various

design and kinematic parameters of the cleaner, it is possible to select their optimal values.

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