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Analysis of non-stationary flow interaction with simple form objects

I. Tipans, J. Viba, M. Irbe and S.K. Vutukuru*

Riga Technical University, Faculty of Mechanical Engineering, Transport and Aeronautics, Department of Theoretical Mechanics and Strength of Materials, Viskalu Street 36A, LV – 1006, Riga, Latvia
*Correspondence: vshravankoundinya1989@gmail.com

Abstract. The paper is devoted to the analysis of a non-stationary rigid body interaction in a fluid flow. Initially, an approximate method for determining the forces due to fluid interaction with the rigid body is offered. For this purpose, the plane movement of a mechanical system with an infinite DOF (degrees of freedom) is reduced to 5 DOF motion: 3 DOF for the body and 2 DOF for the areas of compression and vacuum in fluid flow. Differential equations of non-stationary motion are formed by the laws of classical mechanics. The use of an approximate method has been quantified by computer modelling. The average difference in results was found to be small (< 5%). The analysis of the fluid (air) interaction is carried out for a rigid body of two simple geometries - flat plate and diamond. The results obtained are used to refine the parameters of the proposed approximate method that is addressed in the present study for fluid interaction with the non-stationary rigid body. Theoretical results obtained in the final section are used in the analysis of the movement of prismatic bodies in order to obtain energy from the fluid flow.

Key words: degree of freedom, energy, fluid interaction, mathematical modelling, flow parameters, rigid body.

INTRODUCTION

Analysis of fluid interaction with a rigid body is a challenging task; unfortunately, this interaction phenomenon is not extensively investigated. The subject of the study is the determination of fluid interactions with a rigid body in non-stationary motion by using a mathematical model according to classical mechanics. Existing computational fluid dynamic methods are based on deforming and moving mesh techniques that fail when the mesh deformation is severe, which requires frequent re-meshing, time-consuming and is computationally expensive (Crank, 1984; Finlayson, 1992). The present article is the first of its kind, as no research article in this field in the past was based on a mathematical model that is simple, easy to understand for the study of non–stationary rigid body flow interactions. The only disadvantage of the proposed model is that the model does not consider the viscous effect of the fluid.

In order to understand the importance of mathematical modelling technique, we refer to scientific works that involve mathematical modelling for the case of stationary rigid body fluid interaction. For the case when relative velocity vectors are constant, the motion is well researched and described (Sears, 2011). It follows that only the velocity of relative motion, the density of the fluid, the area perpendicular to the velocity and the experimental drag coefficient are necessary to determine the reduced interaction of fluid force (Hoerner, 1965). (1)

$$\overline{F}^{(F)} = \frac{D \cdot v^2 \cdot \rho \cdot A}{2} \left(\frac{-\bar{v}}{|\bar{v}|} \right) \quad (1)$$

where $\overline{F}^{(F)}$ – interaction force; D – drag coefficient; $\bar{v}$ – flow velocity; $\rho -$ air density; $A -$ object frontal area.

Non-stationary interaction tasks between the continuous medium (fluid flow) and the rigid body motion has complex interaction (Hossenfelder, 2018). In that case, non-stationary interaction can only be solved numerically with space-time programming methods (Beal & Viroli, 2015). A practical model of space-time function applications was used and theoretical and empirical results were presented (Beal et al., 2013).

In the present article, a new method for describing the interaction of non – stationary rigid body, fluid flow in engineering tasks is presented. The numerical validation of the new method was carried out with the help of a two-dimensional continuous flow model for two body shapes, i.e., diamond and rectangular prisms. As a result, it will be shown that engineering calculations do not require a step by step space–time calculation to find coefficient D in formula (1). Instead of the formula (1), a new non-stationary flow interaction formula will be obtained, which will include the object shape, state, and flow rate direction parameters.

MATERIALS AND METHODS

New mathematical model

To simplify calculations for analysis, optimization and synthesis tasks, our work offers a mathematical model without considering the viscous effects of a fluid. For that, the interaction fluid is divided into two spaces: one space on the pressure side and the other on the vacuum side.

The essence of the model is described for flat plate body (Fig. 1) and for the diamond-shaped rigid body (Fig. 2) respectively. On the pressure side, the theorem of change in linear momentum in the differential form is applied (Goldstein et al., 2015) and (Meriam et al., 2015).

Accordingly, it can be expressed as:

$$dm \cdot v \cdot cos(\beta) = d \quad \cdot dt; \quad (2)$$

$$dm = v \cdot cos(\beta) \cdot dt \cdot dL \cdot B \cdot \rho, \quad (3)$$

where dm – mass of the elementary volume of the flow, having velocity v against the inclined surface; $\beta -$ theangle between the flow and the surface at the normal point of impact; dN – the impact force in the direction of the normal surface of the elementary area; t – time; dL – the elementary length of the surface; B – the width of the object, which is constant in the case of a two dimensional task, ρ – density.

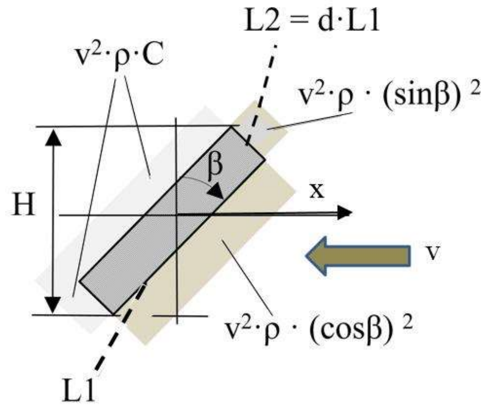


Figure 1. Pressure distribution for rectangle cross section flat plate body.

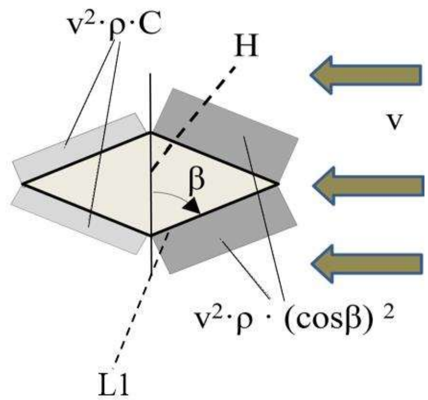


Figure 2. Pressure distribution for the diamond body.

By integration of equation (2) additional pressure in the flow direction for the edges *L1, L2* for flat plate and diamond can be calculated. Pressure distribution for each of edges *L1, L2* for the here given bodies shown in Fig. 1 and Fig. 2. as (4), (5):

$$v^2 \cdot \rho \cdot (cos\beta)^2 \tag{4}$$

or

$$v^2 \cdot \rho \cdot (sin\,\beta)^2. \tag{5}$$

In the proposed new model, at the vacuum side along the edge just behind the body, it is recommended to be taken as constant pressure $\Delta p2$, proportional to the density ρ multiplied by the flow velocity v square in the following form (6):

$$\Delta p2 = v^2 \cdot \rho \cdot C, \tag{6}$$

where: C– constant, to be found out experimentally or by computer modelling.
Thus it is possible to find the total force in the direction of the flow for the rigid body.

As an approximation, the rectangle body will get force *Fxr* as given in Eq. (7), and for the diamond-shaped rigid body *Fxd*, along the direction of flow as given by the Eq. (8).

$$Fxr = \text{-}\,\mathrm{H} \cdot B \cdot v^2 \cdot \rho \cdot \left[C + \frac{cos(\beta)^3 + d \cdot sin(\beta)^3}{cos(\beta) + d \cdot sin(\beta)}\right]; \tag{7}$$

$$Fxd = \text{-}\,\mathrm{H} \cdot B \cdot v^2 \cdot \rho \cdot [C + (cos\beta)^2], \tag{8}$$

where d – the ratio of edges *L2/L1*; B – body width; H - section height, perpendicular to the flow (Fig. 1., Fig. 2.). β – the angle between the flow and the surface at the normal point of impact; ρ – density of the fluid.

From the obtained correlations (7) and (8) it can be shown that an approximate analytical method can be applied in the scope of identification of fluid and body interactions problems. The validation is required for the application of the proposed method, which was performed by using computer simulations discussed in the following section.

2D diamond shaped body transient analysis

From the analysis and modelling graphs obtained, it can be concluded that on the pressure side, the interaction forces quickly reach a stable state. In contrast, on the suction, vacuum side, the flow settled after a certain period of time (Fig. 3). Therefore, the parameters of this body – stationary flow interaction could be used for approximate calculations. It was found that the interaction forces changed with time so an average value is taken (Table 1) at a velocity of 10 m.sec^{-1}.

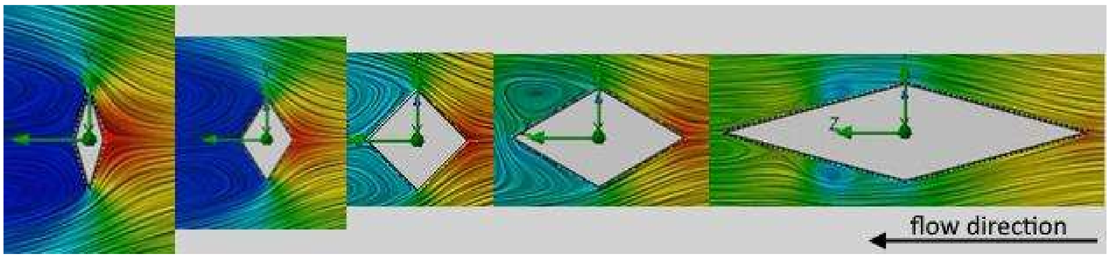


Figure 3. Stream lines and pressure distribution for the diamond body.

Table 1. Average interaction force and average interaction coefficient

β(degrees)	Name	Unit	Averaged value
15	Force (Fxde)	[N]	0.19587
	Interaction coefficient (Dex)		1.440817
30	Force(Fxde)	[N]	0.176314
	Interaction coefficient(Dex)		1.296965
45	Force (Fxde)	[N]	0.164733
	Interaction coefficient(Dex)		1.21177
60	Force(Fxde)	[N]	0.157237
	Interaction coefficient(Dex)		1.156635
75	Force(Fxde)	[N]	0.073616
	Interaction coefficient(Dex)		0.541519

Accordingly, from Table 1. interaction coefficient *C* was identified: *D* – theoretical interaction coefficient (9), Dex – calculated interaction coefficient((10), (Table 1) and approximated D_p – interaction coefficient as fifth degree polynomial function (11):

$$D = C + (\cos\beta)^2; \quad (9)$$

$$Dex = \frac{Fxde}{Av^2\rho}; \quad (10)$$

$$D_p = 1.5 + 3.7266\beta^3 - 1.5249\beta^4 - 0.10135\beta^5 - 2.8129\beta^2 + 0.2823\beta, \quad (11)$$

where *Fxde* – interaction force for diamond plate along the flow direction (Table 1). From Eqs (9) and (10) it follows that $C = 0.5$ when $\beta = 0$.

Therefore, the following approximation formula is recommended for the interaction coefficient in case of the diamond shaped body (12):

$$D = 0.5 + (\cos\beta)^2. \quad (12)$$

Similarly, in this work modelling results for a rectangle flat plate were approximated and compared to the approximate formula (7). In a similar procedure like in diamond plate, it was found that the coefficient C is about 0.5, similar to (12). The estimation of the accuracy of the approximate formula (7) and (8) for diamond-shaped and rectangular objects at $C = 0.5$ is depicted in Fig. 4. and Fig. 5. The difference is not large and the percentage average value is less than 5%.

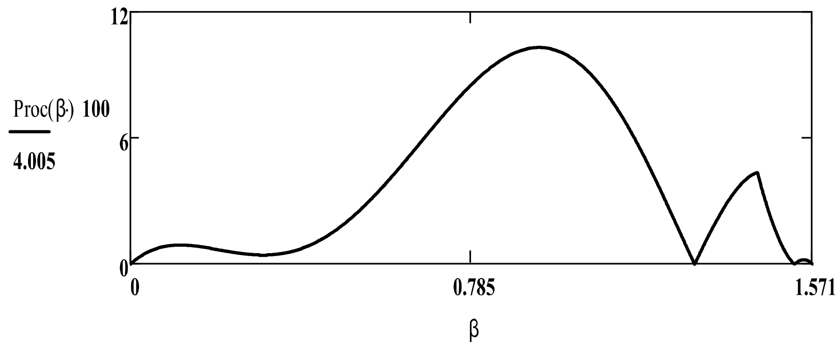


Figure 4. β radians against percentage difference. The accuracy of the approximate formula (8) for diamond plate when C = 0.5, expressed as a percentage. The mean value is about 4%.

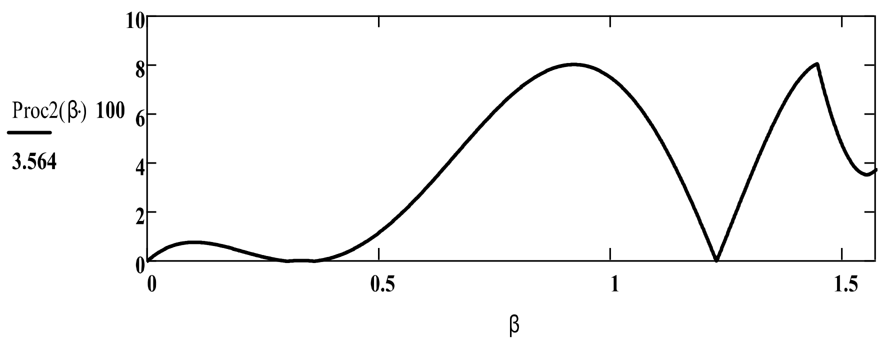


Figure 5. β radians against percentage difference. The accuracy of the approximate formula (7) for rectangle plate when C = 0.5, expressed as a percentage. The mean value is about 3,6%.

Based on analysis from Fig. 4. and Fig. 5. the proposed new method can be applied to engineering calculations with sufficient accuracy (less than 5%), excluding work-based calculations with space-time programs. In the subsequent section as an example, the calculation for a thin flat plate showing the effectiveness of applying the new method to generate energy from airflow is discussed.

RESULTS AND DISCUSSION

Theory and modelling for moving plate

The thin plate (d ~ 0, Fig. 1.) two dimensional (2D) analysis model in the translation motion is shown in Fig. 6. Model includes linear spring with stiffness c and linear damping with proportionality coefficient b. According to the approximation theory, taken into account relative interaction velocity Vr:

$$Vr = (V + v), \tag{13}$$

where V – fluid velocity; v – plate velocity along x axis.
Differential equation of plate motion along x axis will be (14):

$$m\ddot{x} = -cx - b\dot{x} - A\rho[0.5 + (\cos\beta)^2](V + \dot{x})^2 \frac{V+\dot{x}}{|V+\dot{x}|}, \tag{14}$$

where A – surface area of the plate; ρ – density; β – plate angle against flow.
The renewable energy is represented as the damping force $-b\dot{x}$ (). Accordingly, power P will be (15):

$$P = b(\dot{x})^2. \tag{15}$$

Accordingly the average power Pa will be (16):

$$Pa \quad \frac{\int_0^t b(\dot{x})^2 \cdot dt}{t}. \tag{16}$$

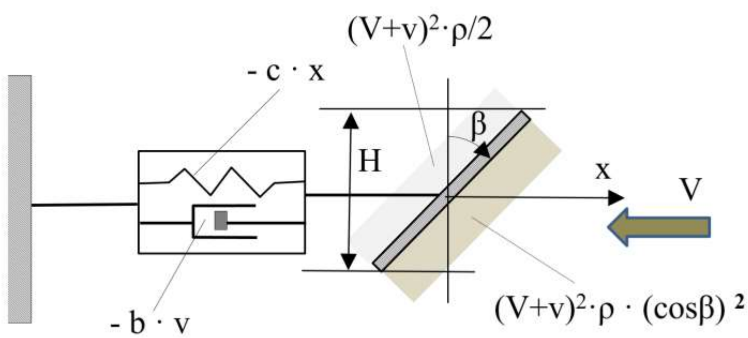


Figure 6. Model for obtaining renewable energy from fluid.

Analysis of equation (14) allows to conclude that there are five possibilities to analyse the efficiency of the given system. They are: two parameters (c, b) of a system and three time or phase coordinates control actions (β, V and A).

Some equation modelling results are shown in (Figs 7–10). Comments are given under all graphs. For all results presented in graphs, systems parameters are:

$A = 0.004$ m^2; $V = 10$ m s^{-1}; $\rho = 1.25$ kg m^{-3}. Average power, Pa is calculated as a percentage of maximal power $Pmax$, when $Pmax$ is taken as a function of flow velocity (17):

$$P\,max = \quad {}^2 \cdot \frac{V}{3}. \tag{17}$$

Angle β time control modelling results are shown in (Figs 7–10), for $\beta = \frac{\pi}{2.5} sin(7t)$

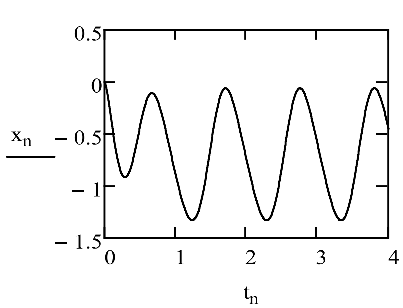


Figure 7. Displacement x as time t function.

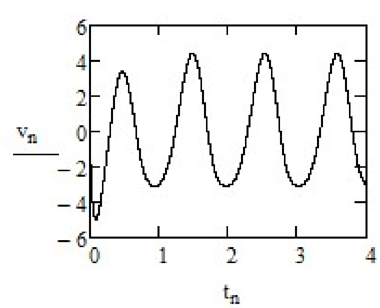
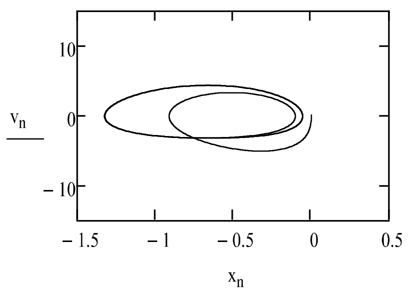


Figure 8. Plate centre velocity v as time t function.



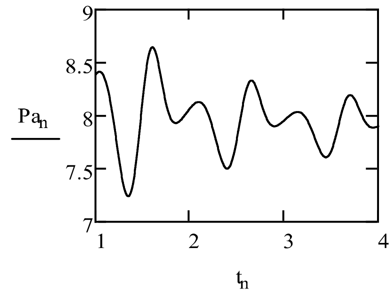


Figure 9. Motion in phase plane
v = v(x).

Figure 10. Average power *Pa* of generator force *bv*.

Motion modelling (Figs 7–10) in a case of varying the angle of the plate to the flow (angle β as a mono harmonic function of time $\beta = \frac{\pi}{2.5} sin(7t)$) show the following:

- motion occurs very quickly, practically within two, three back and forth oscillating cycles;
- it is possible to synthesize the optimal parameters of the system (i.e. stiffness, area, frequency, amplitude), which would provide the maximum power for the given limits;
- additional optimum angle controls can be obtained such as bi – harmonics $\beta = \frac{\pi}{2.5}[sin\ \ 7t) + 2\,sin(14t)]$, poly-harmonics.

Fluid flow velocity V control modelling results are shown in (Figs 11–14). By function: $V = V0 \cdot (2 - 0.5\,sin(10t))$, were $V0 = 10\ m\ s^{-1}$.

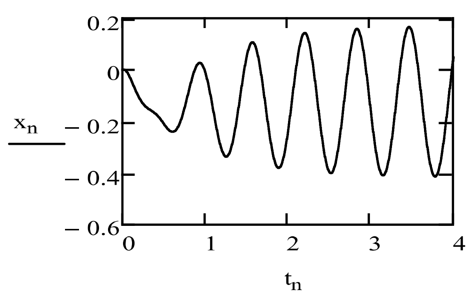


Figure 11. Displacement *x* as time *t* function.

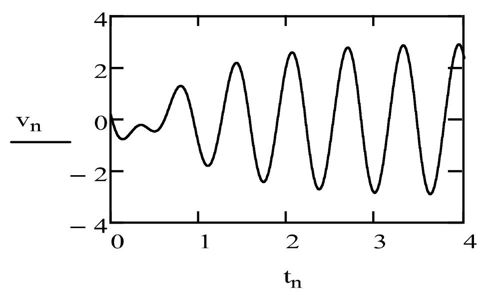


Figure 12. Plate centre velocity *v* as time *t* function.

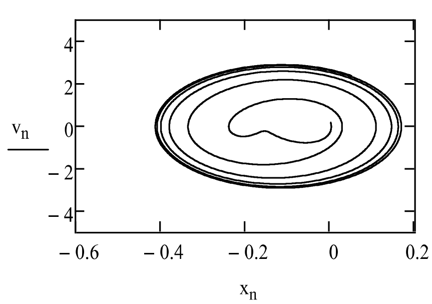


Figure 13. Motion in phase plane
v = v(x).

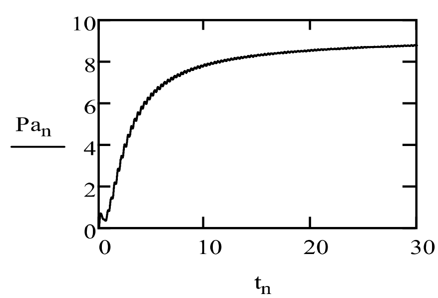


Figure 14. Average power *Pa* of generator force *bv*.

Results of the mathematical modelling in the case of changing the flow velocity as harmonic time function (Figs 11–14) shows the following:

- a new opportunity to generate energy, changing the flow rate harmonically is discovered.

- opportunity to use new forms of control for flow such as harmonic, bi-harmonic, pulse is open. The summary of results of the present work is as follows:

1. In the present paper a new approximated method was developed for calculating flow and rigid body interaction according to classical mechanics.
2. The essence of the method is based on the evidence of numerical and analytical modelling that the interaction can be split separately on the sides of the additional pressure and then on the suction, vacuum.
3. The most interesting is the interaction of the suction side which was almost constant, depending only on the fluid flow rate.
4. The average difference in the results as obtained is found to be less than 5%.

CONCLUSIONS

The article provides new knowledge through mathematical modelling for the analysis of fluid and rigid body interactions. The proposed analytical method being mathematical is highly advantageous as can be easily understood. Some important conclusions that can be drawn from the resent work are as follows:

1. The developed method allows performing the tasks of analysis, optimization and synthesis in a simplified way in the interaction of objects with fluids.

1. For specific tasks, there is no need to use large space - time computing programs for solving engineering problems.

2. Further, the proposed mathematical model can be extended for calculations of flying or diving robot systems as well as in the extraction of energy from the fluid flow.

3. In future studies, it would be advisable to observe the effect of flow viscosity on the accuracy of the developed method.

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