Atmospheric attenuation of the Ku band along the space-earth path due to clouds and rain

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Abstract. The weather conditions formed in troposphere causes the greatest signal attenuation in satellite communication systems especially at frequencies above 10 GHz. This paper describes possible signal attenuations on the satellite-earth path due to rain and clouds. It was measured whether it is advantageous to use the Ku band for data transmission over other bands. The measurement was carried out in the Czech Republic using a beacon signal from Eutelsat 12W satellite at 12.5 GHz. Clouds and the rainfall rate at the measured location were obtained from the CHMI portal. The measurements show that the clouds cause negligible attenuation. Significant attenuation was caused by rainfall. The measured values of slant path rain show a significant decrease in signal strength even in light rain. The measured cumulative rainfall rate was found to be close to the ITU-R model. The measurements show that the Ku band is advantageous for data transmission in rain poorer regions including Central Europe.

Key words: signal, attenuation, weather, satellite, rain, clouds.

INTRODUCTION

Telecommunications satellites are designed for data transmission. Signal transmission is ensured by a ground station, which sends a signal to the satellite, transposes it to another frequency, amplifies and sends it back to Earth (Elbert, 2004).

Radio signal transmission is very important for modern technology. Accurate transmission of information is very important in many specializations, including agriculture. Communication of ground units with satellites can gather information that can be used in the future to improve and refine agricultural practices. The radio spectrum is not unlimited, so it is necessary to allocate bands to certain applications with caution. It is also important to develop new principles for increasingly better and more secure information transfer. Without the possibility of transmitting information using satellites, it would not be possible to use the conventional satellite applications. The measured transmission of information in the Central European region mainly uses the Ku-band and

the signal is transmitted through satellites located in space. These satellites are located in a geostationary orbit, which is about 37,000 kilometres from Earth, and therefore, for various reasons, attenuation occurs. On the Earth-satellite path, electromagnetic waves pass through different environments and each environment has a different effect on signal attenuation (Lee et al., 2011).

The literature shows that the greatest attenuations occur in the troposphere, where the weather is forming, other attenuations on the Earth-satellite route are negligible in practical use (Yeo et al., 2014).

Several models for rain attenuation have been developed based on empirical measurements. Best accuracy models provide in regions with mild to moderate rainfall rate. These models as SC EXCELL, BRYANT or ITU-R method don't work very well in tropical areas with heavy rainfall rate (Crane, 1982); Bryant et al., 2001; Capsoni et al., 2009).

The attenuation in the Ku band can be calculated with great accuracy, knowing certain parameters, so in practice it should not happen that newly installed terrestrial antennas will not be able to receive a signal with sufficient C/N reserve. In this paper, the use of the Ku band for the Central European region, namely in the Czech Republic, was measured and calculated.

The model ITU-R618 was chosen for the measurement. It was developed using geophysical observations of point rain speed statistics, horizontal structure and vertical temperature structure of the atmosphere, based on measured data from countries with mild climate. This model seems to be ideal for the Central European region (Crane, 1980).

On the contrary, it would not be very suitable for tropical regions as it could underestimate the attenuation in these regions. Other models have been designed specifically for regions with higher precipitation (Boonchuk et al., 2005; Lakanchanh et al., 2006).

MATERIALS AND METHODS

The measurements took place from 10.7.2018 to 6.3.2019. For the measurements was used fixed antenna Prodelin 1.8m directed to the satellite Eutelsat 12W B. The measurement was performed at the beacon frequency of this satellite. The received signal was converted by the LNB convertor to a frequency that can be recorded by a spectrum analyser and sent via coaxial and fiber optic cables to the Vista link LNB matrix from which the signal was connected to the Rhode & Schwarz FSC3 spectrum analyser. The correctness of the received signal is verified on the Ericsson RX 8200 SN 28097 receiver. For each measurement, data on the intensity of precipitation or cloudiness was assigned from the radar maps.

Attenuation due to clouds and fog calculation

Clouds or fog consist of tiny particles, which are usually smaller than 0.1 mm. The ITU Recommendation provides methods to predict signal attenuation due to cloud and fog on the Earth's satellite path. The methods described apply to frequencies less than 200 GHz. However, for correct prediction it is necessary to follow several principles. Terrestrial telecommunications systems shall use frequencies higher than 10 GHz. It is

also necessary to know the total column water content contained in clouds for the attenuation calculation. (ITU-R, 2019)

The temperature T=0 °C is used to calculate the probability of attenuation due to cloud cover. Furthermore, it is necessary to know the elevation and specific coefficient K_i and the value of the column water content, which can be read from the maps. The values read from the attached documents are approximately the following (ITU-R, 2019):

- 20% per year -0.1 (kg m⁻²)
- 10% per year -0.2 (kg m⁻²)
- 5% per year -0.3 (kg m⁻²)
- 1% per year -0.5 (kg m⁻²)

Specific attenuation by clouds and fog is expressed by:

$$\gamma_c(f, T) = Kl(f, T)M \quad [dB \text{ km}^{-1}] \tag{1}$$

 γ_c – specific attenuation [dB km⁻¹]; K_l – specific attenuation coefficient [(dB km⁻¹)/(g m³)]; [dB km⁻¹]; M – liquid concentration in cloud or fog [g m⁻³]; f – frequency [GHz]; T – cloud liquid water temperature [K]. Liquid water density in fog:

- 0.05 g m⁻³ fog visibility about 300 m
- 0.5 g m⁻³ fog visibility about 50 m

Specific attenuation coefficient K_1 :

$$K_1(f,T) = \frac{0.819f}{\varepsilon''(1+\eta^2)} \left[\left(\frac{dB}{km}\right) / \left(\frac{g}{m^3}\right) \right]$$
 (2)

$$\eta = \frac{2 + \varepsilon'}{\varepsilon''} \tag{3}$$

The complex permittivity of water:

$$\varepsilon'(f) = \frac{f(\varepsilon_0 - \varepsilon_1)}{f_p[1 + (\frac{f}{f_p})^2]} + \frac{f(\varepsilon_1 - \varepsilon_2)}{f_p[1 + (\frac{f}{f_s})^2]}$$
(4)

$$\epsilon'(f) = \frac{\epsilon_0 - \epsilon_1}{[1 + (\frac{f}{f_p})^2]} + \frac{\epsilon_1 - \epsilon_2}{[1 + (\frac{f}{f_s})^2]} + \epsilon_2$$
 (5)

where:

$$\varepsilon = 77.66 + 103.3 (\theta - 1)$$
 (6)

$$\varepsilon 1 = 0.0671\varepsilon_0 \tag{7}$$

$$\varepsilon 2 = 3.52 \tag{8}$$

$$\theta = 300 / T \tag{9}$$

The principal relaxation frequency and secondary relaxation frequency:

$$fp = 20.2 - 146 (\theta - 1) + 316 (\theta - 1)2 \text{ [GHz]}$$
 (10)

$$fs = 39.8 fp [GHz]$$
 (11)

Slant path cloud attenuation is given by:
$$A = \frac{L_{red} K_l(f, 273.15)}{\sin \varphi} \ [db] \ pro \ 90^{\circ} \ge \varphi \ge 5^{\circ}$$
 (12)

 φ – elevation angle; L_{red} – total columnar content of liquid water reduced to a temperature of 273.15K at kg m⁻².

Total columnar content of liquid water at measured region $L_{red} = 0.5$ [kg m⁻²].

Calculated attenuation due clouds and fog with probability of 1% per year for Eutelsat 12W B satellite for variable elevations at Table 1.

Table 1. Attenuation due to clouds and fog [dB] at frequencies [GHz]

Elev. [°]	10.8	11.0	11.2	11.4	11.6	11.8	12.0	12.2	12.4	12.6	12.8	13.0
20.00	0.16	0.16	0.17	0.18	0.18	0.19	0.19	0.20	0.21	0.21	0.22	0.23
22.00	0.14	0.15	0.15	0.16	0.17	0.17	0.18	0.18	0.19	0.19	0.20	0.21
24.00	0.13	0.14	0.14	0.15	0.15	0.16	0.16	0.17	0.17	0.18	0.19	0.19
26.00	0.12	0.13	0.13	0.14	0.14	0.15	0.15	0.16	0.16	0.17	0.17	0.18
27.20	0.12	0.12	0.13	0.13	0.14	0.14	0.15	0.15	0.15	0.16	0.16	0.17
28.00	0.11	0.12	0.12	0.13	0.13	0.14	0.14	0.15	0.15	0.16	0.16	0.17
30.00	0.11	0.11	0.12	0.12	0.12	0.13	0.13	0.14	0.14	0.15	0.15	0.16
32.00	0.10	0.11	0.11	0.11	0.12	0.12	0.13	0.13	0.13	0.14	0.14	0.15

Calculation of long-term rain attenuation statistics

An important factor affecting the Earth-satellite link is rain. According to ITU-R P.618, rain attenuation can be calculated based on precipitation totals. The procedure provides estimates of long-term rainfall statistics with slope attenuation at frequencies below 55 GHz. (ITU-R, 2015)

To calculate the signal attenuation due to rain, it is necessary to know the model of the attenuation of rain for the measured location. Data for the final value are obtained from monthly precipitation totals and monthly measured temperatures. In tropical regions where rain attenuation is a major problem, many rainfall patterns are developed for specific regions, and models are tested using beacon frequencies on both uplink and downlink (Yeo et al., 2014).

The following procedure is described in ITU-R Recommendation P.618 and further refers to other recommendations. The resulting expression was calculated for attenuation in 0.01% of the average year. Subsequently, the attenuation was calculated in other percentages of the average year in the range of 0.001-5% based on the attenuation for 0.01%.

Rain height h_r is a key parameter for both direct and statistical models. The actual height and width of the melting layer can be estimated using weather radar data. According to ITU-R P.839, the annual average rainfall, higher than the mean sea level hmR, is obtained from an average annual isotherm height of 0 °C.

$$h_r = h_0 + 0.36 [km] (13)$$

 h_0 was found from the document ITU-R 639 (ITU-R, 2013b)

In the second step the length of the signal path through the rain was calculated. The following formula was used for this particular case since the elevation angle was greater than 5°. If the elevation angle $\theta < 5$ °, it is necessary to use a different formula.

$$L_{s} = \frac{(h_{r} - h_{s})}{\sin \theta} [km] \tag{14}$$

 h_s – ground station altitude; θ – elevation angle.

In next step is calculated Horizontal projection L_s , of the slant-path length:

$$L_G = L_s \cos \theta \text{ [km]} \tag{15}$$

In the next step it is necessary to obtain a precipitation rate $R_{0.01}$ of the average year with an integration time of 1 minute. If the aggregate rate cannot be obtained from long-term statistics of the area, it can be estimated from the precipitation maps described in ITU-R Recommendation P.837. In next step was obtained the specific attenuation:

$$\gamma_R = k(R_{0.01})^{\alpha} [dB \text{ km}^{-1}]$$
 (16)

 $R_{0.01}$ – point rainfall rate for the location for 0.01% of an average year (mm h⁻¹).

Now it is necessary to calculate the coefficients k and α , which depend on the angle of elevation of the antenna and the angle of polarization. The coefficients are also frequency dependent. The ITU-R P.838 recommendation for selected frequencies can be used to calculate the coefficients kH, kV, α H and α V.

$$k = \frac{[k_H + k_V + (k_H + k_v)cos^2\theta cos2\tau]}{2}$$
 (17)

The next step is to calculate the horizontal decrease factor $r_{0,01}$

$$\alpha = \frac{[k_H + \alpha_H + k_V \alpha_V + (k_H \alpha_H + k_V \alpha_V) \cos^2 \theta \cos 2\tau]}{2k}$$
(18)

for 0.01% of the total time:

$$r_{0.01} = \frac{1}{1 + 0.78\sqrt{\frac{L_G \gamma_R}{f}} - 0.38(1 - e^{-2L_G})}$$
(19)

 L_R and χ is calculated as:

$$\zeta = \tan^{-1} \left(\frac{h_R - h_S}{L_G r_{0.01}} \right) \quad [\circ]$$
 (20)

If $\zeta > \theta$:

$$L_R = \frac{L_G r_{0.01}}{\cos \theta} \text{ [km]} \tag{21}$$

Otherwise:

$$L_R = \frac{(h_R - h_S)}{\sin \theta} [km]$$
 (22)

$$\chi = 36 - |\varphi|[^{\circ}] \tag{23}$$

For latitude of the earth station $\phi > 36^{\circ}$ applies $\chi = 0$, which corresponds to the latitude of the Central European region.

The next step is to calculate the vertical adjustment factor $v_{0.01}$. The L_R must be obtained according to ITU.R P.618(ITU-R, 2015)

$$v_{0.01} = \frac{1}{1 + \sqrt{\sin\theta} \left(31\left(1 - e^{-\left(\frac{\theta}{1+\chi}\right)}\right) \frac{\sqrt{L_R \gamma_R}}{f^2} - 0.45\right)}$$
(24)

In the next step, the effective path length is calculated:

$$L_E = L_R \, v_{0.01} \, [\text{km}] \tag{25}$$

The next step is to calculate the predicted attenuation for 0.01% of the current year:

$$A_{0.01} = \gamma_R L_E \text{ [dB]} \tag{26}$$

From the $A_{0.01}$ attenuation it is possible to determine attenuation for other percentages of the year in the range of 0.001-5%

$$A_p = A_{0.01} \left(\frac{p}{0.01}\right)^{-(0.655 + 0.33 \ln(p) - 0.045 \ln(A_{0.01}) - \beta(1-p) \sin \theta} [dB]$$
 (27)

Where p is the desired percentage and β is determined as follows:

If
$$p \ge \text{or } |\varphi| \ge 36^\circ$$
: $\beta = 0$ (28)

$$p < 1\% |\varphi| < 36^{\circ} \text{ a } \theta \ge 25^{\circ} : \beta = -0.005(|\varphi| - 36)$$
 (29)

Otherwise:

$$\beta = -0.005(|\phi| - 36) + 1.8 - 4.25 \sin \theta \tag{30}$$

RESULTS AND DISCUSSION

Table 2 shows attenuations with a probability of 0.01% to 5% of time per year. The values are primarily calculated for a probability of 0.01% for the Earth-satellite link, where the location of the ground station is at 50.0310° N and 14.3913° E at an altitude of 310 m above sea level, for Eutelsat 12W B at 12.5°W. The calculations were carried out for the frequency of 12.5005 GHz, on which the empirical measurement was also performed.

Table 2. Attenuation due to rain [dB] for the satellite Eutelsat 12W in frequencies [GHz]

Probability [%]	11.2	11.4	11.6	11.8	12	12.2	12.4	12.5	12.6	12.8
0.01	8.42	8.59	8.75	8.91	9.07	9.22	9.38	9.45	9.53	9.68
0.1	2.26	2.31	2.36	2.41	2.46	2.5	2.55	2.57	2.59	2.64
0.5	0.85	0.87	0.89	0.91	0.93	0.95	0.97	0.98	0.99	1.01
1	0.64	0.66	0.67	0.69	0.7	0.72	0.73	0.74	0.74	0.76
5	0.19	0.19	0.2	0.2	0.21	0.21	0.22	0.22	0.22	0.22

From the calculated Table 3 for a certain frequency it is also possible to calculate the table of the frequency dependence on the attenuation due to precipitation.

Table 3. Long-term rain attenuation [dB] at the intensity of precipitation [mm h⁻¹]

Frequency [GHz]	1	1.5	2	4	10	20	30	40
11	0.44	0.65	0.86	1.67	3.43	5.69	7.48	8.98
11.5	0.45	0.67	0.88	1.71	3.55	5.94	7.84	9.44
12	0.45	0.68	0.89	1.74	3.60	6.18	8.19	9.89
12.5	0.46	0.68	0.91	1.77	3.77	6.41	8.52	10.32
13	0.46	0.69	0.92	1.79	3.88	6.63	8.84	10.74

Measured values

The difference between the measured average clear and cloudy attenuation is 0.235 dB. Graph 1 compares clear weather with cloud cover. Signal attenuation caused the signal to decrease by 2.73%. Rain attenuation was measured at certain rainfall rates. For precipitation up to 1 mm h⁻¹, the measured attenuation is 0.425 dB. Rain up to 2 mm h⁻¹ caused a loss of 1.014 dB. Rain up to 4 mm h⁻¹ caused a loss of 1.709 dB. All measured attenuation values due to rain are recorded in Table 2. Values with a total rainfall above 4 mm h⁻¹ are not compared, as insufficient number of values was measured for statistical analysis.

Diagram 1. Measured data clear sky vs cloudy

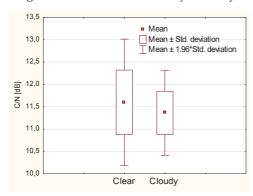


Diagram 2. Measured data clear sky vs rain

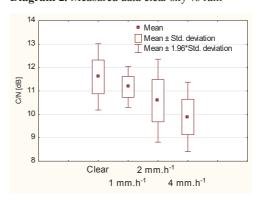


Table 4 shows statistical calculations. A two-sample t-test was used for evaluation and a significance level of $\alpha=0.05$ was used. For clear vs clouds and clear vs 1 mm h⁻¹, this significance level cannot be rejected. For values of clear vs 2 mm h⁻¹ and 4 mm h⁻¹, this significance level is rejected. All statistical calculations were performed in StatSoft, Inc. software. (2013). STATISTICA, version 12.0.1133.15.

Table 4. Statistical data processing

	Mean clear	Mean variable	t	Е	р
clear vs cloudy	11.6	11.36471	1.015006	25	0.319826
clear vs 1 mm h ⁻¹	11.6	11.175	1.692038	20	0.106161
clear vs 2 mm h ⁻¹	11.6	10.58571	2.933984	22	0.007681
clear vs 4 mm h ⁻¹	11.6	9.890909	5.274598	19	0.000043

In general, signal attenuation due to rain occurs at 10 GHz or more as the wavelength of the signal decreases and approaches the size of a raindrop. It is also true that the higher the rate of rainfall in a given area, the larger the droplets and the closer they are to the size of the carrier wave in the Ku-band. A prerequisite for the correct calculation of the total annual signal attenuation is preceded by the correct selection of the rain model for the given region. In the Central European region, annual precipitation is much lower than in the tropics. For this reason, the ITU-R model is used. In tropical areas, the choice of rain pattern may vary. G.N. Ezeh et al. compared the three rain models, the ITU-R model, the Ajayi method and the Alnutt model. The simulated results in the Matlab program are based on the Ajai method as the most accurate method in the tropical region. Different models are developed in each region. There are more of these

models in the tropical regions, as there is an effort for the most accurate result that can be used in antenna design (Ezeh et al., 2014).

If the reception of the antenna signal is not continuous, it is economically advantageous to choose the smallest available antenna in the Central European region. Khairi Abdul Rahim et al. compares 0.6m and 13.2m antennas. The article compares an antenna for home use and a professional antenna. The measured distance from C / N noise was measured in clear weather. C / N values differ between antennas by 9.37dB. This difference is really striking and indicates how important the size of the antenna is in receiving the signal. As the measurement took place in Malaysia, which is located in a tropical area and the intensity of precipitation is several times higher than in the Central European region, the usability of the antenna with a diameter of 0.6 m is very limited. An antenna with a diameter of 13.2 m is more suitable for tropical areas (Rahim et al., 2009).

CONCLUSION

The attenuations were measured at the beacon frequency of the satellite, which serves, among other things, to set the satellite, so it is necessary that this frequency is permanently accessible. Therefore, its value reached 11.6 dB carrier to noise ratio in clear weather. This is a very high value for Earth-satellite data transmission. The measured values do not fully correspond to the calculated values in the cloud attenuation, which may be caused by the use of other measuring instruments than used in the processing of the prediction method or attenuation in optical cables on the LNB -spectrum analyzer route. Nevertheless, the trend of slight signal attenuation due to cloudiness is perceptible. Clouds in the Czech Republic vary between 20–40%, but the measurements show that the signal attenuation on the link has a very small contribution, in this particular case the attenuation due to cloudiness was 2.73% compared to clear weather.

It can be seen that light rain will cause more attenuation than clouds. Values for precipitation up to 1 mm h⁻¹ and 4 mm h⁻¹ correlate with calculated values. The measured attenuation at precipitation up to 2 mm h⁻¹ was slightly higher than predicted by the ITU model. This inaccuracy may be caused by the same problem that occurred when measuring attenuation due to cloudiness.

With increasing frequency, the signal attenuation increases as well. Also, the higher the rainfall rate in a given area, the larger are the drops and the closer they are to the size of the carrier wave. The attenuation from the average year was calculated. In the measured case, it would be 81.47% attenuation for 52 minutes of the average year, without total signal loss. According to the measurement results, the Ku-band band is suitable for signal transmission on the Earth - satellite route in the Czech Republic. The measurements show that the ITU model for rain attenuation was chosen correctly and is suitable for the less wealthy region of Central Europe. Because the signal loss in the Czech Republic is at a low level, the Ku band should also be used for data transmission in agriculture.

ACKNOWLEDGEMENTS. It is a project supported by the CULS IGA TF (2019:31170/1312/3113 and 2020:31180/1312/3103).

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