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A mathematical model of the rear-trailed top harvester and an evaluation of its motion stability

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Abstract. Improving the quality of sugar beet harvesting to a great extent depends on the first operation in the process, which involves cutting and harvesting sugar beet tops. This technological process is performed with the use of either the haulm harvesting modules of beet harvesters or top harvesting machines as separate agricultural implements, which are aggregated with a tractor. At the same time, front-mounted harvesters are as widely used as trailed asymmetric implements, in which case the aggregating tractor moves on the already harvested area of the field. The purpose of this work is to determine the optimal design and kinematic parameters that would improve the stability in the performance of the technological process of harvesting sugar beet tops by means of developing the basic theory of the plane-parallel motion performed by the rear-trailed asymmetric top harvester. As a result of the analytical study, an equivalent scheme has been composed, on the basis of which a new computational mathematical model has been developed for the plane-parallel motion of the asymmetric top harvester in the horizontal plane on the assumption that the connection between the wheeled tractor and the rear-trailed top harvester is made in the form of a cylindrical hinge joint. Using the results of mathematical modelling, the system of linear second-order differential equations that determines the transverse movement of the centre of mass of the aggregating wheeled tractor and the rotation of its longitudinal symmetry axis by a certain angle about the said centre of mass as well as the angle of deviation of the rear-trailed asymmetric top harvester from the longitudinal symmetry axis of the tractor at an arbitrary instant of time has been obtained. The solving of the obtained system of differential equations provides for determining the stability and controllability of the motion performed by the asymmetric machine-tractor unit, when it performs the technological process of harvesting sugar beet tops.

Key words: harvesting machine, plane-parallel motion, stability of movement, sugar beet tops.

INTRODUCTION

One of the current problems in the beet industry is the high-quality harvesting of the tops (Gruber, 2005; Sarec et al., 2009; Boson et al., 2019), which implies cutting and harvesting the green mass, which is used later, as completely as possible and cutting the root crowns without causing damage to or loss of root bodies (loss of sugar-bearing mass in case of sugar beets). This applies almost equally to the harvesting of sugar beets and fodder beets, carrots and other root crops, the parts of which are harvested separately - first the tops are cut from the roots, then the roots are lifted from the soil (Bulgakov et al., 2017a). The technological process of haulm harvesting is carried out by either the harvesting modules of beet harvesters or top harvesting machines as separate agricultural implements aggregated with tractors, mostly row-crop tractors. Throughout the world, both front-mounted topping implements and trailed asymmetric top gathering machines are widely used, in the latter case the aggregating tractor moving on the already harvested section of the field. Such asymmetric units are especially widely used, when harvesting fodder beet tops Pogorely & Tatyanko, 2004).

It should be noted that today trailed asymmetric agricultural machines are produced in the agricultural industries of many countries in the world and such asymmetric units are successfully used. This primarily applies to the units with trailed asymmetrically positioned harvesting machines, such as the mentioned machines for harvesting sugar and fodder beet tops, rotary windrowers, rotary mowers, trailed forage harvesters and the like (Boson et al., 2019).

While there is an acute need to perform the above-mentioned processes using the described design and it is necessary to ensure their high quality, asymmetric trailed harvesters have one significant disadvantage associated with the resulting unstable movement in the transverse-horizontal plane, which must be overcome in some way. The conditions of unstable movement of the aggregating tractor and the asymmetric harvester installed behind that arise in this case are due to the action of the turning moment produced by the external forces acting on the wheels and implements, when performing the given process (Bulgakov et al., 2020a).

The numerous scientific studies and subsequent engineering developments (Wang & Zhang, 2013; Wu et al., 2013; Zhang et al., 2013; Gu et al., 2014) have made it possible to overcome this disadvantage by using various technical methods. Among the known ways to solve this problem there are two most widely used variants of aggregating asymmetric agricultural machines with aggregating tractors, which are able to provide the relatively stable movement of the unit. The first of them involves a rigid connection between the asymmetric machine and the aggregating tractor during their operation in the technological process. While this alternative provides, in general, the conditions for the stable movement of the asymmetric machine, the existing turning moment creates such conditions, under which the steer wheels of the aggregating tractor must constantly turn to the side opposite to the direction of the side slip. In this case, the controllability of such an asymmetric machine-tractor unit becomes much worse, which not only complicates the conditions of its operation, but can reduce the quality of the specified process overall (Vasilenko, 1996). The first option is most suitable for the operation of simple and light asymmetric machines, such as mowers with small working widths, with tractors of considerable weights.

In the second case, the connection between the rear-trailed asymmetric agricultural machine and the aggregating tractor, when performing the technological process, is provided by a cylindrical hinge, i.e. the conditions for the movement of the tractor and the machine relative to each other are created. Under these conditions, the controllability of the machine-tractor unit under consideration is significantly improved, but additional provisions are required to ensure that the aggregated asymmetric machine does not deviate in the horizontal plane in the process of its operational travelling under the action of the arising turning moment. That is, this method of aggregation is more effective, but it sets a problem of somehow reducing the generated turning-around moment or creating the conditions for its compensation by the 'moment of counteraction'.

Thus, there is an urgent need to find conditions for increasing the stability of motion of any asymmetric machine-tractor unit, and one of the ways is the theoretical research into its movement in the transverse-horizontal plane, i.e. its plane-parallel motion.

When conducting such theoretical research, it is necessary to remember that the stable movement and high-quality functioning of one or another asymmetric agricultural machine-tractor unit should be considered as its response to the input controlling and disturbing actions. In this case, the response of the machine-tractor unit to the controlling actions characterizes its controllability and the response to the disturbing actions - the stability of movement.

The authors have developed a new rotor-type top harvester, which can be used in the form of a front-mounted or rear-trailed aggregated machine. The design and process scheme of using the top harvesting machine hitched behind the aggregating wheeled tractor is presented in Fig. 1. Thus, the presented machine-tractor unit, in which harvesting machine (II) with a working width of 3 rows is asymmetrically attached to the aggregating wheeled tractor (I), is asymmetric.

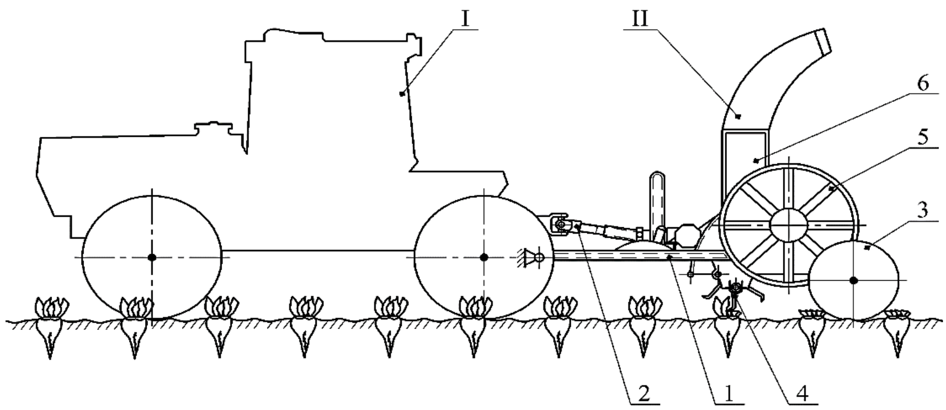

Figure 1. Design and process scheme of asymmetric top harvesting unit in longitudinal-vertical plane: I – wheeled tractor; II – trailed asymmetric harvesting machine; 1 – frame; 2 – universal-joint drive; 3 – pneumatic support-and-finder wheel; 4 – rotary haulm cutting device; 5 – conveying device; 6 – loading device.

In this case, the technological process of cutting the tops is carried out with the use of a rotary topping device (4), which performs the continuous cutting of the entire mass within the working width. In the process of work, the cutting device (4) is positioned relative to the crowns of root crops by means of two pneumatic support-and-finder wheels (3), which move in the inter-row spacing. The cut-off green mass of the haulm is loaded with the use of an arc-shaped loading device (6) into the body of the vehicle moving on the harvested area of the beet field next to the harvester.

Thus, the search for the conditions that ensure the stable movement of this asymmetric top harvesting unit is an urgent and difficult task, the solution of which will significantly improve its technical and operational properties.

Many published works (Hac et al., 2008; Szakács, 2010; Yildiz, S. 2010; Demšar et al., 2012; Li et al., 2016; Anche & Subramanian, 2018; Bulgakov et al., 2020a, 2020b) have been concerned with the research into the stability of the movement performed by trailed asymmetric agricultural machines as parts of various machine-tractor units, based on the generation of their analytical mathematical models. Most of these studies use the basic provisions from the classical theory of the stability of agricultural machine-tractor units in perturbed and unperturbed motion, which is analysed with the use of generated systems of differential equations of motion (Bulgakov et al., 2017b). In this case, the behavior of different machine-tractor units is considered, as a rule, in their plane-parallel motion together with the rear-trailed agricultural machine. Most of the above-mentioned known works are concerned with the research into the machine-tractor units that comprise a wheeled aggregating tractor and a rear-trailed rotary windrower.

Analytical studies on the asymmetric machine-tractor units that consist of a wheeled tractor and a rear-trailed top harvester have not been published until now. It should be noted that in such research it is necessary to take into account the properties of the elastically damping pneumatic tires on the aggregating tractor wheels and the forces arising at the points of contact between these wheels and the ground, which is deformed, as well as the design features of the trailed machine, its size and the external forces acting on the supporting wheels and tools of the machine, the generated traction resistance, the way the machine is attached to the rear hitch of the aggregating tractor, and so on.

The aim of the study was to determine the optimal kinematic and design parameters that will increase the stability of the technological process of harvesting sugar beet tops, basing on the development of the fundamentals for the theory of the plane-parallel motion performed by the rear-trailed asymmetric top harvester.

MATERIALS AND METHODS

The main provisions of the theory of plane-parallel motion performed by the asymmetric top harvesting machine-tractor unit will be considered by the example of an aggregating wheeled tractor, behind which a top harvesting machine is attached. The aggregating tractor moves along the harvested part of the sugar beet plantation, while the haulm harvesting machine, which is positioned to the right of the tractor, moves along the sugar beet crop area and cuts and collects the green mass of the haulm and loads it, through the loading device, into the body of the vehicle that moves next to the tractor (or scatters it on the field). The connection between the wheeled aggregating tractor and the specified haulm topper during the operation of this machine-tractor unit is provided by the vertical cylindrical hinge joint in the rear hitch device of the tractor.

Thus, this harvesting machine-tractor unit is asymmetric, because the top harvester trailed behind the aggregating tractor is offset to the right from its longitudinal axis.

For the analytical study of this machine-tractor unit, it is first necessary to generate its equivalent schematic model, considering the motion of all its points only in a plane parallel to the plane of the field surface. First of all, it should be noted at once that this asymmetric top harvesting machine-tractor unit is a complex dynamic system and due to its asymmetry its movement in the horizontal plane will certainly be unstable.

In addition, to study this movement, it is necessary to formalize some properties of such a machine-tractor unit and on the basis of that make some assumptions. These assumptions will not misrepresent the real process in any way, but will greatly simplify the analytical research. Namely, the main assumptions are as follows:

1. The surface of the field, on which this top harvesting unit moves, is horizontal, and therefore the heel and trim of this machine-tractor unit are neglected.
2. Wheeled aggregating tractor is presented in the form of a solid body having a longitudinal plane of symmetry, which passes through its centre of mass.
3. The oscillations of the traction resistance of the trailed top harvester do not have a significant effect on the speed of translational movement of this unit, as a result of which it is assumed in the first approximation to have a constant value.
4. Interaction between the pneumatic tires on the running wheels of the aggregating tractor and the support-and-finder wheels of the trailed top harvesting machine, on the one hand, and the soil surface, on the other hand, especially in lateral directions, completely corresponds to the so-called hypothesis of side slip applied to the pneumatic tires on the wheels.
5. As only small values of the gyroscopic and stabilizing moments acting on the pneumatic tires on the running wheels of the aggregating tractor and the support-and-finder wheels of the top harvester as well as the moments of their twisting about the vertical axes are observed during the movement of the given unit, they will be left out of consideration.
6. The slip angles of the pneumatic tires on the running wheels of the aggregating tractor, which are located on the same geometric axis, as well as the lateral forces acting on them, are assumed to be quite small.
7. The turn angles of the left and right steer wheels of the aggregating tractor are assumed to be small and equal to each other, because it is reasonable to consider the main movement of this top harvesting machine-tractor unit, when it performs the technological process of harvesting sugar beet tops, to be straight.

Theory and modelling

Taking into account the above-mentioned assumptions, the aggregating wheeled tractor in the equivalent scheme can be presented in an arbitrary position. In this case, its main points are selected and designated with the corresponding letters: the tractor's centre of mass - point S; the centre of the steer wheel axle - point A; the centre of the driving wheel axle - point B; the hitching point of the aggregated top harvester is designated by the point C. At the same time, the stub axles of the steer wheels of the aggregating tractor are included, because of their small size, in the total length of the front wheel axle.

Further, the systems of coordinate axes have to be chosen for this dynamic system and shown in the equivalent diagram with an aim of analysing its movements relative to these systems. First of all, the fixed Cartesian coordinate system $XOYZ$ is rigidly connected to the surface of the beet field, which acts as the plane (the axis Z is not shown), in which the plane-parallel motion of the machine-tractor unit takes place. The separate spatial system of coordinate axes $X_TSY_TZ_T$ has its origin at the centre of mass of the aggregating tractor (point S). Its axis SY_T coincides with the longitudinal axis of the tractor, the axis SX_T is directed normally to the right with respect to the direction of the tractor's movement, and axis SZ_T is directed vertically upwards. For the direction of motion of the tractor's front steer wheels, the movable coordinate system X_AAY_A with its centre located at the point A is assigned. The axis AY_A always coincides with the direction of motion of the tractor's front steer wheels (parallel to the planes of these wheels), while the axis AX_A is directed perpendicular to the axis AY_A and to the right with respect to the course of motion of the tractor.

It is also assumed, taking into account the previously made assumptions, that the aggregating wheeled tractor on its working run performs translational and uniform motion with the speed V_o relative to the stationary coordinate system XOY. However, in the process of performing the work movement, under the influence of external random factors, the aggregating wheeled tractor deviates from its original position, receives an additional speed and begins its relative motion in the plane XOY. In this case, the plane Y_TSX_T associated with the centre of mass of the tractor rotates in the plane XOY around the vertical axis SZ_T that passes through the point S. The angle φ formed by the longitudinal axis of symmetry of the wheeled tractor and the axis OY is the characteristic of this rotation.

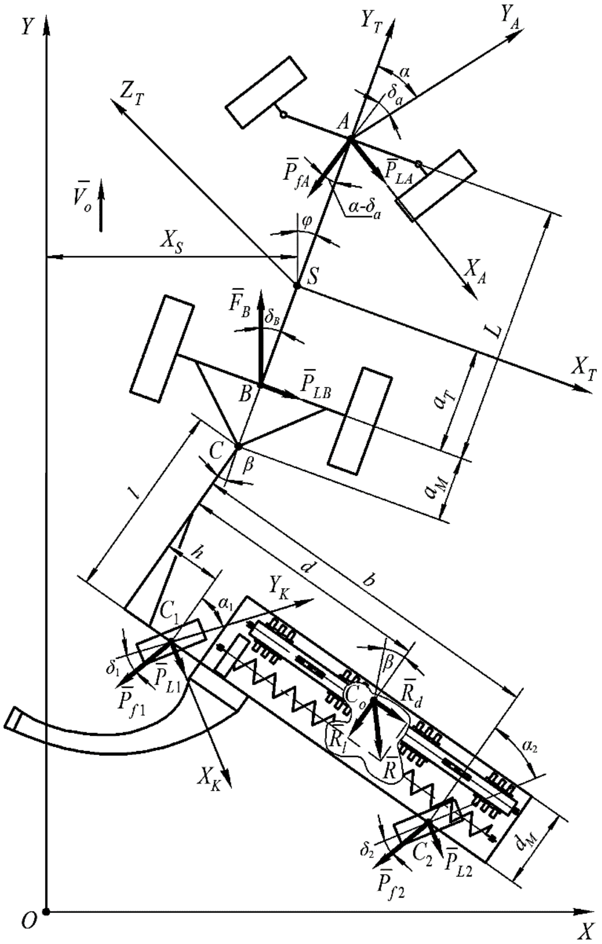

Figure 2. Equivalent scheme of asymmetric top harvesting machine-tractor unit.

During the relative motion of the tractor, its centre of mass moves along the axis OX, which is represented by the change in coordinate X_S (Fig. 2).

Thus, the wheeled aggregating tractor has two degrees of freedom relative to the plane XOY, namely: linear coordinate X_S and angular coordinate φ, which are further used as the generalized coordinates.

The next step is to show on the equivalent diagram the external forces acting on the wheeled aggregating tractor during its plane-parallel motion and determine their physical attributes. These forces are as follows. First of all, it is the driving force $\bar{F}_B$, which is generated by the rear two driving wheels of the tractor and can be applied at the point B.

The driving force $\bar{F}_B$ forms the slip angle δ_B with the longitudinal axis of symmetry of the aggregating tractor. The tractor is also affected by the rolling resistance force $\bar{P}_{fA}$ generated by the two front steer wheels, applied at the point of intersection between their axis and the longitudinal axis SY_T (point A) and deflected from the direction of movement of the tractor's running gear through the slip angle δ_A. There are also lateral forces acting on the tractor from the two axles: $\bar{P}_{LA}$ and $\bar{P}_{LB}$ and applied respectively at points A and B. Finally, the draught resistance force $\bar{P}_{KR}$ is applied to the wheeled aggregating tractor by the top harvester at the point C and deflected from the longitudinal axis of the tractor, that is, from the axis SY_T, through the angle β.

It should be noted at once that the main moment (i. e. the turning moment M_t relative to the point C) of all the external forces acting on the rear-trailed top harvester is not transferred to the aggregating tractor due to the hinge connection of the aggregated asymmetric machine at the point C.

As a result, the influence of the trailed asymmetric machine on the aggregating wheeled tractor is represented only by its traction resistance $\bar{P}_{KR}$ and the angle β of turn (deviation) in the horizontal plane.

Following the above considerations, there is the complete basis for generating the differential equations of the plane-parallel motion performed by the wheeled aggregating tractor. For that purpose, the original Lagrange equations of the second kind of the following form can be used (Dreizler & Lüdde, 2010):

$$\frac{d}{dt}\left(\frac{\partial T_T}{\partial \dot{q}_i}\right)-\frac{\partial T_T}{\partial q_i}=Q_i, \tag{1}$$

where T_T – kinetic energy of a wheeled aggregate tractor; q_i – generalized coordinates; Q_i – generalized forces, according to the corresponding generalized coordinates q_i.

The kinetic energy T_T of the aggregate wheeled tractor relative to the horizontal plane XOY is defined as:

$$T_T=\frac{M_T\cdot V_S^2+J_S\cdot\omega_T^2}{2}, \tag{2}$$

where M_T – mass of the aggregate tractor; V_S – linear velocity of the center of mass of the tractor in the plane XOY; J_s – the moment of inertia of the tractor relative to the vertical axis SZ_T; ω_T – angular speed of rotation of the tractor around the axis SZ_T.

If we express linear V_S and angular ω_T velocity through generalized coordinates X_S and φ, we obtain:

$$V_S=\dot{X}_S, \tag{3}$$

$$\omega_T=\dot{\varphi}. \tag{4}$$

Taking into account expressions (3) and (4), the kinetic energy T_T of the wheeled aggregate tractor can be determined as follows:

$$T_T=\frac{M_T\cdot\dot{X}_S^2+J_S\cdot\dot{\varphi}^2}{2}. \tag{5}$$

Since the kinetic energy T_T of the tractor due to expression (5) depends only on velocities $\dot{X}_S$ and $\dot{\varphi}$ and does not depend on the most generalized coordinates q_i, the partial derivative will be:

$$\frac{\partial T_T}{\partial q_i} = 0 . \tag{6}$$

At the same time, the partial derivatives of the velocities $\dot{X}_S$ and $\dot{\varphi}$ the corresponding generalized coordinates $\dot{X}_S$ and φ will be equal to:

$$\frac{\partial T_T}{\partial \dot{X}_S} = M_T \cdot \dot{X}_S , \tag{7}$$

$$\frac{\partial T_T}{\partial \dot{\varphi}} = J_S \cdot \dot{\varphi} . \tag{8}$$

The time derivatives of expressions (7) and (8) are determined by the following expressions:

$$\frac{d}{dt}\left(\frac{\partial T_T}{\partial \dot{X}_S} \right) = M_T \cdot \ddot{X}_S , \tag{9}$$

$$\frac{d}{dt}\left(\frac{\partial T_T}{\partial \dot{\varphi}} \right) = J_S \cdot \ddot{\varphi} \tag{10}$$

Using the values obtained from expressions (9) and (10), taking into account expression (6) for the two generalized coordinates X_S and φ and substituting them into expression (1), we obtain the following system of differential equations for the plane-parallel motion of the aggregate wheeled tractor in this form:

$$\left. \begin{aligned} M_T \cdot \ddot{X}_S = Q_{X_S} , \\ J_S \cdot \ddot{\varphi} = Q_{\varphi} . \end{aligned} \right\} \tag{11}$$

The next step is to determine the right-hand sides of the differential equations of system (11), i.e., the generalized forces: Q_{X_S} and Q_{φ}.

First, the value of the generalized force Q_{X_S} with reference to the generalized coordinate X_S has to be determined. For that purpose, the dynamical system under consideration is imparted a virtual displacement and the expression is generated for the elementary work of the forces on the virtual displacement δX_S. The result is as follows:

$$\begin{aligned} \delta A_{X_S} &= P_{LA} \cdot \cos(\varphi + \alpha) \cdot \delta X_S - P_{fA} \cdot \sin(\varphi + \alpha - \delta_A) \cdot \delta X_S + \\ &+ P_{LB} \cdot \cos\varphi \cdot \delta X_S - F_B \sin(\delta_B - \varphi) \cdot \delta X_S - P_{KR} \cdot \sin(\varphi + \beta) \cdot \delta X_S , \end{aligned} \tag{12}$$

where α – the angle of rotation of the steered wheels of the tractor.

From expression (12) we obtain that the generalized force Q_{X_S} on the generalized coordinate X_S will be:

$$\begin{aligned} Q_{X_S} = \frac{\delta A_{X_S}}{\delta X_S} &= P_{LA} \cdot \cos(\varphi + \alpha) - P_{fA} \cdot \sin(\varphi + \alpha - \delta_A) + \\ &+ P_{LB} \cdot \cos\varphi - F_B \sin(\delta_B - \varphi) - P_{KR} \cdot \sin(\varphi + \beta). \end{aligned} \tag{13}$$

Thus, this generalized force Q_{X_S} is equal to the sum of the projections of all active external forces applied to the wheeled tractor on the axis OX.

In order to determine the generalized force Q_{φ} with reference to the generalized coordinate φ, again, a virtual displacement of the given dynamical system is assumed, but this time an angular one, and the expression is generated for the elementary work of the forces on the virtual displacement $\delta\varphi$. The following is obtained:

$$\begin{aligned}\delta A_{\varphi} &= P_{LA}\cdot\left(L-a_T\right)\cos\alpha\cdot\delta\varphi - P_{fA}\cdot\left(L-a_T\right)\sin\left(\alpha-\delta_A\right)\cdot\delta\varphi - \\ &- P_{LB}\cdot a_T\cdot\delta\varphi + F_B\cdot a_T\sin\delta_B\cdot\delta\varphi + P_{KR}\cdot\left(a_T+a_M\right)\sin\beta\cdot\delta\varphi.\end{aligned} \quad (14)$$

From expression (14), it is derived that the generalized force Q_{φ} with reference to the generalized angular coordinate φ will be:

$$\begin{aligned}Q_{\varphi} &= \frac{\delta A_{\varphi}}{\delta\varphi} = P_{LA}\cdot\left(L-a_T\right)\cos\alpha - P_{fA}\cdot\left(L-a_T\right)\sin\left(\alpha-\delta_A\right) - \\ &- P_{LB}\cdot a_T + F_B\cdot a_T\sin\delta_B + P_{KR}\cdot\left(a_T+a_M\right)\sin\beta.\end{aligned} \quad (15)$$

That is, the generalized force Q_{φ} on the generalized coordinate φ is equal to the algebraic sum of the moments of all external forces relative to the point S.

Let us further analyze obtained expressions (13) and (15) for generalized forces Q_{X_S} and Q_{φ} and find possibilities for their simplification. These expressions can be simplified because for small angles the value of cosines can be approximately considered as equal to unities, and the value of sines can be considered as equal to the most angles.

Therefore, expression (13) can be represented as follows:

$$Q_{X_S} = P_{LA} - P_{fA}\left(\varphi+\alpha-\delta_A\right) + P_{LB} - F_B\left(\delta_B-\varphi\right) - P_{KR}\left(\varphi+\beta\right), \quad (16)$$

or

$$Q_{X_S} = P_{LA} + P_{LB} - P_{fA}\alpha + P_{fA}\delta_A - F_B\delta_B + \left(F_B - P_{fA} - P_{KR}\right)\varphi - P_{KR}\beta \cdot \quad (17)$$

Given that $F_B - F_{fA} - P_{KR} = 0$, then in the final form we obtain an expression for determining the generalized force Q_{X_S} of the following form:

$$Q_{X_S} = P_{LA} + P_{LB} - P_{fA}\alpha + P_{fA}\delta_A - F_B\delta_B - P_{KR}\beta \cdot \quad (18)$$

Let us perform similar transformations for expression (15), taking into account small values of angles α and β. Finally, we obtain an expression for the generalized force Q_{φ} of the following form:

$$\begin{aligned}Q_{\varphi} &= P_{LA}\left(L-a_T\right) - P_{fA}\cdot\alpha\left(L-a_T\right) - P_{LB}\cdot a_T + P_{fA}\cdot\delta_A\left(L-a_T\right) + \\ &+ F_B\cdot\delta_B\cdot a_T + P_{KR}\cdot\beta\left(a_T+a_M\right).\end{aligned} \quad (19)$$

Forces P_{LA} and P_{LB}, which are included in dependencies (18) and (19), can be replaced by expressions formed on the basis of the so-called hypothesis of 'lateral input' of pneumatic wheel tires (Macmillan, 2002; Kutkov, 2014; Abyzov & Berezin, 2018) of this form:

$$P_{LA} = k_A\cdot\delta_A, \quad (20)$$

$$P_{LB} = k_B\cdot\delta_B, \quad (21)$$

where k_A, k_B – coefficients of lateral input of pneumatic tires of running wheels of the aggregating wheeled tractor.

In expressions (20) and (21) it is necessary to substitute the values of the input angles δ_A and δ_B, which are determined on the basis of the construction of velocity plans of points A and B, that is, the middle of the front (controlled) and rear (traction) axles of the aggregate tractor at their plane and parallel motion in the horizontal plane XOY.

Using the technique outlined in (Hwang et al., 2021), we constructed the above velocity plans, allowed us to graphically find the values of velocities V_A and V_B, then through their projections on the axes X and Y, as well as the corresponding tangents of angles and based on the neglect of small quantities, find the angles themselves δ_A and δ_B. As a result, the final expressions for the lateral forces P_{LA} and P_{LB} of this type:

$$P_{LA} = k_A \cdot \left[\frac{-\dot{X}_S - (L - a_T)\dot{\varphi}}{V_o} + \varphi + \alpha \right], \tag{22}$$

$$P_{LB} = k_B \cdot \left[-\frac{\dot{X}_S - a_T \cdot \dot{\varphi}}{V_o} + \varphi \right]. \tag{23}$$

To determine the position of the supporting copying wheels of the topper at an arbitrary moment of time, consider its diagram separately and show on it a movable coordinate system $Y_K C_1 X_K$, rigidly connected with the left supporting copying wheel of the topper (Fig. 3). The axis $C_1 Y_K$ is always parallel to the plane $C_1 X_K$ of the support wheel, but perpendicular to it and pointing to the right in the direction of the mower harvester's movement. These directions of movement for the right support wheels of the hitch machine in the first approximation can be considered similar. However, the moving coordinate system is not shown here.

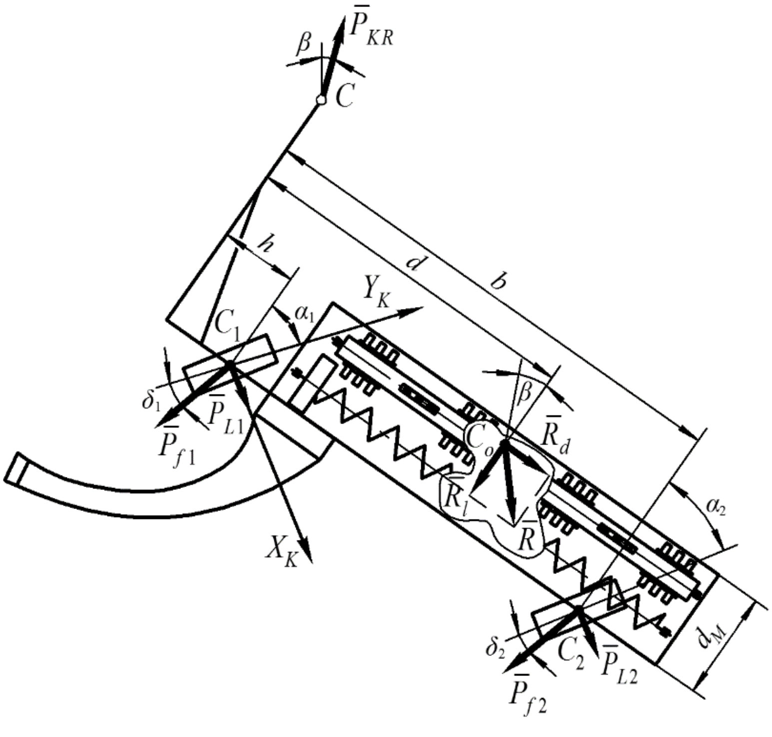

Figure 3. Equivalent diagram of an aggregated harvester.

As presented, the trailed harvester as a dynamic system can be regarded as a physical pendulum with only one degree of freedom - the angle β of rotation in the horizontal plane. This angle β of rotation will be the generalized coordinate in the subsequent differential equations in the form of the original Lagrange equations of dynamics of the form (1).

Let us define in this case the components necessary for expression (1). Thus, the kinetic energy T_H of the aggregated harvesting machine will be:

$$T_H = \frac{J_C \cdot \dot{\beta}^2}{2}, \tag{24}$$

where J_C – the moment of inertia of the hitch harvester relative to the vertical axis passing through the point C; $\dot{\beta}$ – the angular velocity of rotation of the harvesting machine around the point C.

In this case, the necessary partial derivatives included in the original Lagrange equations of the second kind (1) are from the following expressions:

$$\frac{\partial T_H}{\partial \dot{\beta}} = J_C \cdot \dot{\beta}, \tag{25}$$

$$\frac{\partial T_H}{\partial \beta} = 0, \tag{26}$$

$$\frac{d}{dt}\left(\frac{\partial T_H}{\partial \dot{\beta}}\right) = J_C \cdot \ddot{\beta}. \tag{27}$$

Given expressions (22) and (23), the differential equation of motion of a trailed harvester will look like this:

$$J_C \cdot \ddot{\beta} = Q_\beta. \tag{28}$$

To determine the generalized force Q_β, included in expression (28), let us denote the external forces acting on the trailed harvester. External forces acting on the harvester during its technological process include longitudinal $\bar{R}_l$ and transverse $\bar{R}_d$' components of the resistance force $\bar{R}$ of the harvested haulm array, which are applied to the machine at a point C_o; $\bar{P}_{f1}$ rolling resistance force of the left copying wheel of the harvester, applied at a point C_1 and deflected from the wheel plane at the entry angle δ_1; $\bar{P}_{f2}$ - rolling resistance force of the right copying wheel of the tillage machine applied at the point C_2 and deflected from the wheel plane by the entry angle δ_2; lateral forces $\bar{P}_{L1}$ and $\bar{P}_{L2}$, applied according to the left and right copying wheels of the tillage machine at the points C_1 and C_2.

To determine the generalized force Q_β by coordinate β let us use the expression for the elementary work of forces on a possible displacement $\delta\beta$. Then:

$$\begin{aligned} \delta A_\beta &= R_l \cdot d \cdot \delta\beta - R_d\left(l - d_M\right) \cdot \delta\beta + P_{f1}\cos\left(\alpha_1 - \delta_1\right) \cdot h \cdot \delta\beta + \\ &+ P_{f2} \cdot \cos\left(\alpha_2 - \delta_2\right) \cdot b \cdot \delta\beta + P_{f1}\sin\left(\alpha_1 - \delta_1\right) \cdot l \cdot \delta\beta + \\ &+ P_{f2} \cdot \sin\left(\alpha_2 - \delta_2\right) \cdot l \cdot \delta\beta - P_{L1} \cdot \cos\alpha_1 \cdot l \cdot \delta\beta - \\ &- P_{L2} \cdot \cos\alpha_2 \cdot l \cdot \delta\beta + P_{L1} \cdot \sin\alpha_1 \cdot h \cdot \delta\beta + P_{L2} \cdot \sin\alpha_2 \cdot b \cdot \delta\beta, \end{aligned} \tag{29}$$

where d, l, d_M, h and b – construction parameters of the harvesting machine (Fig. 3); a_1 and a_2 – the angles of the left and right gauge wheels of the harvester respectively.

From expression (29) we obtain that the generalized force Q_β on the generalized angular coordinate β will be equal to:

$$Q_\beta = \frac{\delta A_\beta}{\delta\beta} = R_l \cdot d - R_d\left(l - d_M\right) + P_{f1}\cos\left(\alpha_1 - \delta_1\right)\cdot h +$$
$$+ P_{f2}\cdot\cos\left(\alpha_2 - \delta_2\right)\cdot b + P_{f1}\sin\left(\alpha_1 - \delta_1\right)\cdot l +$$
$$+ P_{f2}\cdot\sin\left(\alpha_2 - \delta_2\right)\cdot l - P_{L1}\cdot\cos\alpha_1\cdot l -$$
$$- P_{L2}\cdot\cos\alpha_2\cdot l + P_{L1}\cdot\sin\alpha_1\cdot h + P_{L2}\cdot\sin\alpha_2\cdot b. \quad (30)$$

Analysing the obtained expression (30). As in the previous case, the cosines of small angles can be replaced by units, and the sines by the angles themselves. Moreover, if we take into account the small values of angles a_1 and a_2, then expression (30) can be presented in the following form:

$$Q_\beta = R_l\cdot d - R_d\left(l - d_M\right) + P_{f1}\cdot h + P_{f2}\cdot b + P_{f1}\cdot l\left(\alpha_1 - \delta_1\right) +$$
$$+ P_{f2}\cdot l\left(\alpha_2 - \delta_2\right) - P_{L1}\cdot l - P_{L2}\cdot l + P_{L1}\cdot\alpha_1\cdot h + P_{L2}\cdot\alpha_2\cdot b. \quad (31)$$

Moreover, from Fig. 3 it is clear that $R_d = R_l \cdot \tan\beta$. And since the value of the angle β is small $\tan\beta \approx \beta$, and therefore:

$$R_d = R_l \cdot \beta. \quad (32)$$

As in the case of an aggregate wheeled tractor, the lateral forces P_{L1} and P_{L2}, acting on the supporting copying wheels of the tiller can be replaced by expressions formed on the basis of the hypothesis of 'lateral input' of its pneumatic wheel tires. Namely:

$$P_{L1} = k_1 \cdot \delta_1, \quad (33)$$

$$P_{L2} = k_2 \cdot \delta_2, \quad (34)$$

where k_1, k_2 – coefficients of lateral input of pneumatic tires of basic copying wheels of the harvesting machine; δ_1, δ_2 – angles of input of pneumatic basic copying wheels of the harvesting machine.

As in the previous case, to determine the lateral forces P_{L1} and P_{L2} need to find the input angles δ_1, δ_2 respectively the left and right supporting coping wheels of the harvester. For this purpose, we constructed velocity plans for finding the velocities of points C_1 and C_2, their projections on the axes X and Y taking into account the tangents of the angles and neglecting the small values. As a result, we found the final expressions for these input angles of this form:

$$\delta_1 = \frac{-\dot{X}_S + \dot{\varphi}\left(a_T + a_M\right) + \dot{\beta}\left(l^2 + h^2\right)^{\frac{1}{2}}}{V_o} + \varphi + \beta + \alpha_1, \quad (35)$$

$$\delta_2 = \frac{-\dot{X}_S + \dot{\varphi}\left(a_T + a_M\right) + \dot{\beta}\left(l^2 + b^2\right)^{\frac{1}{2}}}{V_o} + \varphi + \beta + \alpha_2. \quad (36)$$

If we substitute the values of expressions (35) and (36) in expressions (33) and (34), it is possible to obtain the value of lateral forces P_{L1} and P_{L2}, taking into account the input angles δ_1 and δ_2, which can then be used to find the generalized force Q_β.

Now, given expressions (33), (34), which are transformed with expressions (35) and (36), the generalized force Q_β, due to expression (31) can be represented by the following expression:

$$Q_\beta = R_l \cdot d - R_d\left(l - d_M\right)\beta + P_{f1}\cdot h + P_{f2}\cdot b + P_{f1}\cdot l\left(\alpha_1 - \delta_1\right) + \\ + P_{f2}\cdot l\left(\alpha_2 - \delta_2\right) - k_1\cdot\delta_1\cdot l - k_2\cdot\delta_2\cdot l + k_1\cdot\delta_1\cdot\alpha_1\cdot h + k_2\cdot\delta_2\cdot\alpha_2\cdot b. \tag{37}$$

If we consider that the product of two small quantities is even smaller, the sum of the last two terms $(k_1 \cdot \delta_1 \cdot \alpha_1 \cdot h) + (k_2 \cdot \delta_2 \cdot \alpha_2 \cdot b)$ in expression (37) can be neglected. In this case we have:

$$Q_\beta = R_l\left[d - \left(l - d_M\right)\beta\right] + P_{f1}\left[h + \left(\alpha_1 - \delta_1\right)\cdot l\right] + \\ + P_{f2}\left[b + \left(\alpha_2 - \delta_2\right)\cdot l\right] - k_1\cdot\delta_1\cdot l - k_2\cdot\delta_2\cdot l. \tag{38}$$

If we now take into account all of the last expressions, we get the final value for the generalized force Q_β of the trailed harvester in the following form:

$$\begin{aligned} Q_\beta = {} & R_l\left[d - \left(l - d_M\right)\beta\right] + \\ & + P_{f1}\left\{h + \left(\alpha_1 - \left[\frac{-\dot{X}_S + \dot{\varphi}\left(a_T + a_M\right) + \dot{\beta}\left(l^2 + h^2\right)^{\frac{1}{2}}}{V_o} + \varphi + \beta + \alpha_1\right]\right)\cdot l\right\} + \\ & + P_{f2}\left\{b + \left(\alpha_2 - \left[\frac{-\dot{X}_S + \dot{\varphi}\left(a_T + a_M\right) + \dot{\beta}\left(l^2 + b^2\right)^{\frac{1}{2}}}{V_o} + \varphi + \beta + \alpha_2\right]\right)\cdot l\right\} - \\ & - k_1\cdot l\left[\frac{-\dot{X}_S + \dot{\varphi}\left(a_T + a_M\right) + \dot{\beta}\left(l^2 + h^2\right)^{\frac{1}{2}}}{V_o} + \varphi + \beta + \alpha_1\right] - \\ & - k_2\cdot l\left[\frac{-\dot{X}_S + \dot{\varphi}\left(a_T + a_M\right) + \dot{\beta}\left(l^2 + b^2\right)^{\frac{1}{2}}}{V_o} + \varphi + \beta + \alpha_2\right]. \end{aligned} \tag{39}$$

Substituting expressions (18), (19) and (39) for generalized forces Q_{X_S}, Q_φ and Q_β taking into account expressions (22) and (23), which should be substituted into expressions (11) and (28) respectively, and discarding terms of higher order of smallness, after a number of transformations we obtain a system of second order linear differential equations describing the motion of an asymmetric machine-tractor unit in the horizontal plane:

$$\left.\begin{aligned} & A_{11}\cdot\ddot{X}_S + A_{12}\cdot\dot{X}_S + A_{13}\cdot\dot{\varphi} + A_{14}\cdot\varphi + A_{15}\cdot\beta = f_{11}\cdot\alpha, \\ & A_{21}\cdot\ddot{\varphi} + A_{22}\cdot\dot{\varphi} + A_{23}\cdot\varphi + A_{24}\cdot\dot{X}_S + A_{25}\cdot\beta = f_{21}\cdot\alpha, \\ & A_{31}\cdot\ddot{\beta} + A_{32}\cdot\dot{\beta} + A_{33}\cdot\beta + A_{34}\cdot\dot{\varphi} + A_{35}\cdot\varphi + A_{36}\cdot\dot{X}_S = f_{31}, \end{aligned}\right\} \tag{40}$$

where $A_{11} = M_T$;

$A_{12} = \dfrac{k_A + k_B + P_{fA} - F_B}{V_o}$;

$A_{13} = \dfrac{\left(k_A + P_{fA}\right)\cdot(L - \alpha_T) + (F_B - k_B)\cdot\alpha_T}{V_o}$;

$A_{14} = F_B - k_A - k_B - P_{fA}$;

$A_{15} = P_{KR}$;

$A_{21} = J_S$;

$$A_{22} = \frac{(k_A + P_{fA}) \cdot (L - \alpha_T)^2 + (k_B - F_B) \cdot \alpha_T^2}{V_o};$$
$$A_{23} = -A_{13} \cdot V_o;$$
$$A_{24} = A_{13};$$
$$A_{25} = P_{KR}(\alpha_T + \alpha_M);$$
$$A_{31} = J_c;$$
$$A_{32} = \frac{l \cdot \left[(k_1 + P_{f1}) \cdot (l^2 + h^2)^{\frac{1}{2}} + (k_2 + P_{f2}) \cdot (l^2 + b^2)^{\frac{1}{2}}\right]}{V_o};$$
$$A_{33} = R_l(l - d_M) + l(k_1 + k_2 + P_{f1} + P_{f2});$$
$$A_{34} = \frac{l \cdot (\alpha_T + \alpha_M) \cdot (k_1 + k_2 + P_{f1} + P_{f2})}{V_o};$$
$$A_{35} = l(k_1 + k_2 + P_{f1} + P_{f2});$$
$$A_{36} = \frac{A_{35}}{V_o};$$
$$f_{11} = k_A;$$
$$f_{21} = (L - \alpha_T) \cdot k_A;$$
$$f_{31} = R_l \cdot d + P_{f1} \cdot h + P_{f2} \cdot b - \alpha_1 \cdot l \cdot k_1 - \alpha_2 \cdot l \cdot k_2.$$

RESULTS AND DISCUSSION

The obtained mathematical model (40) is the basis for studying the influence of the rotation angle β of the trailed tillage machine on the oscillations of the course angle φ of the tractor, by constructing the amplitude-frequency and phase-frequency characteristics. It is known (David et al., 2009) that the amplitude-frequency response should be as small as possible when the dynamic system works out any perturbation. Ideally, it should be equal to zero. At the same time, the phase-frequency shift (i. e., the delay of the response of the dynamic system to the disturbing influence) should be as large as possible. As a result, the desired amplitude-frequency characteristics should be equal to 0, while the phase-frequency characteristics, on the contrary, should tend to 0 when the dynamic system is working out the perturbation ∞.

Those parameters and modes of operation of the investigated asymmetric machine-tractor unit, which in the working range of oscillations of controlling and disturbing input influences maximally approximate the actual amplitude-frequency and phase-frequency characteristics to the desired ones - will therefore be considered optimal.

To perform numerical calculations of differential equations (40) on the computer, a program was developed. Methodology of practical use of obtained mathematical model (40) is considered on example of analysis of stability of motion in horizontal plane of asymmetric machine-tractor unit developed by us with the following construction parameters: M_T = 4,250 kg, J_S = 4.6 kN m s^2, P_{KR} = 7.3 kN, P_{FA} = 1.7 kN, L = 2.45 m, α_T = 0.98 m; α_M = 1.20 m; k_A = 80 kN rad^{-1}, k_B = 120 kN rad^{-1}.

It should be noted at once that we used for aggregation of the asymmetric mower harvester the power tool of the integral scheme, in which about 60% of the weight is on the front axle, and the rest (40%) - on the rear axle. The driving wheels of both axles in this case, as a rule, are active-primitive and equipped with pneumatic tires of the same size. In this case, the wheels of the front and rear axles of the aggregate tractor are equipped with the same pneumatic tires of size 16.9R30. At air pressure in pneumatic tires equal to 130 kPa coefficient k_A of input resistance of front wheels is 120 kN rad^{-1}. The air pressure in the pneumatic tires of the rear engines is 100 kPa, which corresponds to the value of a similar coefficient k_B, equal to 90 kN rad^{-1}.

As a result of these numerical calculations on the PC of the obtained mathematical model, we plotted the amplitude and phase frequency characteristics, which allow us to estimate the stability of the working motion of the asymmetric machine-tractor unit (Fig. 4 and Fig 5).

On the basis of the obtained graphical dependencies, we will analyze how some construction and technological factors of the given harvester machine affect the amplitude and phase frequency characteristics of oscillations of the course angle φ of the wheel tractor when changing the frequency of oscillations of the angle β of deviation of the trailed harvester in the horizontal plane.

First, we consider the influence of the translational velocity V_o of a given harvesting machine. The analysis of the obtained amplitude-frequency characteristics testifies to the following. First, as the frequency of the disturbing oscillations (i.e., the angle β) increases, the amplification factor of the considered dynamic system of this input effect at each speed mode of motion of the harvesting machine gradually decreases (Fig. 4).

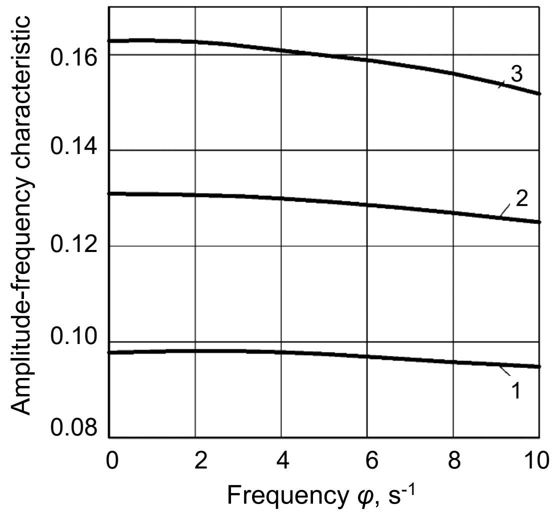

Figure 4. Amplitude-frequency characteristic of the course angle φ of the tractor of the integral configuration when it works out disturbances in the form of oscillations of the turn angle β of the mower harvester at different speeds of the machine: 1) V_o = 1.5 m s^{-1}; 2) V_o = 2.0 m s^{-1}; 3) V_o = 2.5 m s^{-1}.

There is every reason to believe that this result is logical, since the greater the frequency of oscillations of perturbation ω, the greater the stabilizing role played by the inertial properties of the dynamic system, which is considered.

Secondly, with the increase of speed V_o of the harvesting machine movement, it becomes more sensitive to perturbing influences. For example, at frequency of oscillations of the angle β at a level ω = 4 s^{-1} and speed of machine movement 1.5 m s^{-1} its amplitude-frequency characteristic makes 0.09 (curve 1, Fig. 4). In simplified to understand form, this means that for the vibration amplitude of the perturbation (angle β) at 5° (and this is palpable) the vibration amplitude of the course angle φ of the aggregate tractor is only 0.5°, which is almost imperceptible.

At a speed V_o of movement of the harvesting machine at the level of 2.5 m s^{-1} and the same frequency of vibration perturbation ω = 4 s^{-1} the amplitude-frequency characteristic of the dynamic system increases to a mark 0.15 (curve 3, Fig. 4). Compared with the previous velocity mode (when the amplitude-frequency response is equal to 0.09) this is almost 1.5 times greater. However, by the same amplitude of oscillations of the angle $\beta = 5°$ the amplitude of oscillations of the course angle of the tractor φ does not exceed 0.7°.

Thus, at a speed of 2.5 m s^{-1} in a range of vibration frequency of disturbing influences (angle β fluctuations) $\omega = 0–10$ s^{-1} the amplitude-frequency characteristic of the given machine-tractor at application of the integral aggregate tractor changes already in a range 0.163–0.152 (curve 3, Fig. 4).

As we see, in qualitative terms, an increase in the speed V_o of the machine-tractor unit under consideration leads to an undesirable increase in the amplitude-frequency response when it reproduces the external disturbing influence in the form of oscillations of the angle β of rotation of the harvester. In quantitative terms, this influence is such that it cannot worsen the practical stability of motion of the given harvesting machine.

Now, as for the delayed response of the considered dynamic system to the disturbing influence. Under the condition of increasing its frequency, the phase-frequency response at each speed mode of the machine-tractor unit movement increases (Fig. 5).

That is, the greater the value of the perturbation frequency ω, the more influential are the inertial properties of the unit and the greater is its delay (in this case, the phase shift) to the action of perturbing influences. The same inertial properties of the tillage machine are also responsible for the fact that its reaction time to disturbances increases as its speed V_o increases. So, if at $V_o = 1.5$ m s^{-1} and $\omega = 10$ s^{-1} the phase shift of dynamic system (i.e. phase-frequency characteristic) makes - 15° (curve 1, Fig. 5), then already at speed $V_o = 2.5$ m s^{-1} and at the same frequency ω this parameter increases to a mark - 25° (curve 3, Fig. 5). The phase shift difference in this case is 10° or 0.17 rad.

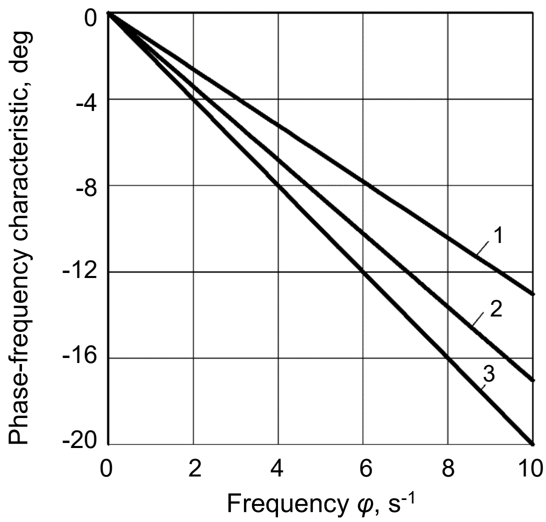

Figure 5. Phase-frequency characteristic of the course angle φ of the tractor of the integral configuration when it works out disturbances in the form of oscillations of the tiller turning angle β at different speeds of the machine: 1) $V_o = 1.5$ m s^{-1}; 2) $V_o = 2.0$ m s^{-1}; 3) $V_o = 2.5$ m s^{-1}.

CONCLUSIONS

1. Using the developed equivalent scheme (Fig. 2) of motion of an asymmetric harvesting machine-tractor unit, a new calculated mathematical model of its plane-parallel motion in the horizontal plane is constructed.

2. A new system of linear differential equations (40) of the second order is obtained, which describes the dynamics of the transverse displacement of the center of mass of the aggregate wheeled tractor, its course angle, and the angle of deviation of the trailed hitch from the longitudinal axis of the tractor at any time.

3. The obtained system of differential equations (40) after its solution on the PC made it possible to establish the stability and controllability of the movement of the asymmetric harvesting machine-tractor unit when performing the technological process of harvesting sugar beet tops.

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