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Mathematical model of vibration digging up of root crops from soil

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Abstract. A new theory of vibrational digging up of root crops from the soil has been developed. The Hamilton-Ostrogradski variational principle is used, on the basis of which we have received the differential equation of longitudinal oscillations of the root in the soil with an infinite number of degrees of freedom. Solution of the given equation provided the possibility to determine the main parameters of the tools that are used in modern beet harvesters.

Key words: root crop, digging tool, vibrational digging up, variational principle, differential equation, constructive parameters.

INTRODUCTION

The reason for large-scale use of vibrational digging tools in root harvesters of the modern technical level is their significantly lower draught level, actual ability to dig up beetroots from the soil without damage and losses. Oscillations of the digging tool create conditions, under which the soil that adheres to roots is intensely beaten down when they are dug up, which facilitates high level of qualitative indicators. That's why development of new constructions of vibrational digging tools, as well as research of their operation for the purpose of determination of the optimal constructive and kinematic parameters is a current task of the branch of mechanisation of sugar beet growing (Sarec et al., 2009; Lammers, 2011).

Statement of the problem. Analytical research of the process of interaction between working elements of the vibration digging tool with the root, that allows to obtain kinematic, constructive and technological parameters, and gives the opportunity to determine their optimal value.

MATERIAL AND METHODS

Fundamental theoretical and experimental research of the vibrational digging up of the root crops of sugar beet was published in the paper (Babakov, 1968), in which the root is modelled as a body having elastic properties, and it is presented as a rod with variable cross section with one attached end. Transverse oscillations of the root analysed in the given paper are described using the differential equation, in particular derivatives of fourth order. The technological process of digging up of the root from the soil with vibrational application of forces is not actually analysed here; instead it is stated that using the additionally prepared equations of kinetostatics the conditions are found for its digging up from the soil under the action of the disturbing force applied in the cross vertical plane. It is stated in the given paper that this particular direction of oscillations will be the best way to foster high quality digging up of the root crops from the soil.

Paper (Vasilenko et al., 1970) presents the theory of the digging tool of a regular digging share type, and states the conditions for digging up of the root from the soil with translational motion of the digger, taking the condition of avoiding damage to the root into account. The given paper demonstrates how the expression is obtained for determination of the allowed velocity of translational motion of the digging tool with its pre-set constructive parameters. In the given case the process of digging up of the root from the soil is performed under the action of forces that emerge on the working surfaces of the digging shares as a result of the transitional motion of the digging tool along the rows of the roots.

The developed theory of own and forced oscillations of the body of the root (Pogorely et al., 1983) is necessary for assessment of the action of the given oscillations on the process of destruction of connections of the root with the soil.

However, the given methods are not sufficient for performance of full analysis of the actual process of digging-up of the root from the soil.

Goal of the research. To develop a calculation and mathematical model and to analytically analyse the root – tool system in order to study the process of oscillations of the root during its vibrational digging-up from the soil.

RESULTS OF THE RESEARCH

The case where oscillatory motions of the vibration digging-up tool are applied to the beetroot in longitudinal vertical area will now be analytically analysed. It will be assumed that the root that is located in the soil is a complex solid elastic system with an infinite number of degrees of freedom, also modelled as the rod with variable cross section with the attached low end.

At the same time, since the Lagrange equation of the second kind in generalised coordinates serves as theoretical basis for most research of oscillations of holonomic systems with a finite number of degrees of freedom, for the purpose of performance of oscillations of holonomic systems with an infinite number of degrees of freedom the so called Hamilton-Ostrogradski principle of stationary action is used (Babakov, 1968).

In the theory of longitudinal, torsional and transverse oscillations of straight rods the Hamilton-Ostrogradski functionals are applied, which in the most generalised form look as follows:

$$S = \int_{t_1}^{t_2} \int_{0}^{h} L\left(t, x, y, \frac{\partial y}{\partial t}, \frac{\partial y}{\partial x}, \frac{\partial^2 y}{\partial t^2}, \frac{\partial^2 y}{\partial t \partial x}, \frac{\partial^2 y}{\partial x^2}\right) dx\, dt \qquad (1)$$

where: $L = T - \Pi$ is the Lagrange function; T is kinetic energy of the system; Π is the potential energy of the system.

It will be assumed that the root that is located in the soil to be the rod with variable cross section along its length with one end attached (Fig. 1). The Hamilton-Ostrogradski principle will now be applied for research of longitudinal oscillations of the root that occur under the action of the vertical disturbing force that changes according to the harmonic law of the following type (Bulgakov et al., 2005):

$$Q_{зб.} = H \sin \omega t, \qquad (2)$$

where: H is the amplitude of forced oscillations; ω is the frequency of forced oscillations.

As we can see from the scheme (Fig. 1), the root having a cone-like body (the top angle of which equals 2γ, and the top part of which is located above the level of the surface of the soil), is modelled as the rod with variable cross section with the attached low end (point O). In the centre of gravity, designated as point C, the force $\overline{G}$ is applied – the weight force of the root. h is its total length. Through the axis of symmetry of the root the vertical axis x is drawn, the beginning of which matches the point O. Connection of the root with the soil is determined by the general reaction of the soil $\overline{R}_x$, which is located along the axis x.

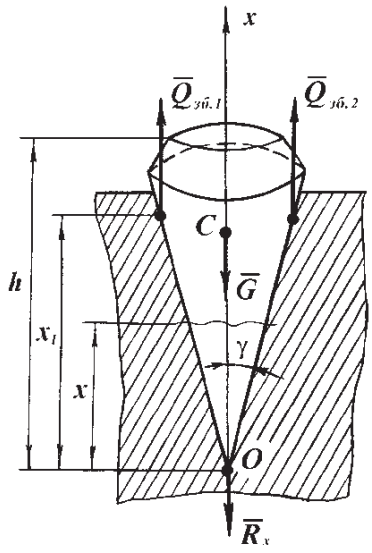


Figure 1. Scheme of the forces having an action on the root at the time of gripping by the vibration digging tool.

The disturbing force $\overline{Q}_{зб.}$ stated above is simultaneously applied to the root from two digging-up plough shares from its two sides, and that's why it is presented in the scheme by two components $\overline{Q}_{зб.1}$ and $\overline{Q}_{зб.2}$. The given forces are applied on the distance x_1 from the origin of coordinates (point O), and they are the source of oscillations of the root in longitudinal vertical area, that destroy connection of the root with the soil and form conditions for digging up of the latter from the soil. The functional S of Hamilton-Ostrogradski for the analysed vibrational process will now be made. For this purpose the necessary symbols will be applied:

$F(x)$ is the area of cross section of the root at some point located at the distance x from the low end m^2; E is the Young's modulus for material of the root N m^{-2}; $y(x,t)$ is the longitudinal dislocation of some cross section of the root at the time point t, m; $Q(x,t)$ is the intensity of longitudinal external load directed along the axis of the root N m^{-1}; $\mu(x)$ is the mass per length of the root kg m^{-1}.

Then kinetic energy of the oscillatory motion of the root will be:

$$T = \frac{1}{2}\int_o^h \mu(x)\left(\frac{\partial y}{\partial t}\right)^2 d\,x \tag{3}$$

Potential energy of the elastic deformation is designated as follows:

$$\Pi_1 = \frac{1}{2}\int_o^h E\cdot F(x)\left(\frac{\partial y}{\partial x}\right)^2 d\,x \tag{4}$$

Potential stretching energy of the longitudinal load $Q(x,t)$ will look as follows:

$$\Pi_2 = \frac{1}{2}\int_o^h Q(x,t)y\,d\,x \tag{5}$$

The Lagrange function L will be made.

Since

$$L = T - \Pi_1 + \Pi_2, \tag{6}$$

then, taking the expressions (3), (4) and (5) into consideration, we get:

$$L = \frac{1}{2}\int_o^h \left[\mu(x)\left(\frac{\partial y}{\partial t}\right)^2 - E\cdot F(x)\left(\frac{\partial y}{\partial x}\right)^2 + Q(x,t)y\right]d\,x. \tag{7}$$

By inserting the expression (7) into the expression (1), we will have:

$$S = \frac{1}{2}\int_{t_1}^{t_2}\int_{o}^{h}\left[\mu(x)\left(\frac{\partial y}{\partial t}\right)^2 - E\cdot F(x)\left(\frac{\partial y}{\partial x}\right)^2 + Q(x,t)y\right]dx\,dt. \qquad (8)$$

Further, expressions of all values that are included in the functional (8) will be found. Since the root has the shape of a cone, we find that its area of cross section $F(x)$ at the point that is located at an arbitrary distance x from the point O, will be:

$$F(x) = \pi x^2 \tan^2\gamma\,. \qquad (9)$$

It is obvious that the mass per length of the root can be determined using the following expression:

$$\mu(x) = \rho\cdot F(x),$$

or, given the (9),

$$\mu(x) = \rho\cdot\pi x^2\tan^2\gamma\,, \qquad (10)$$

where ρ is the density of the root in kg m^{-3}.

Since the value $Q(x,t)$, included in the functional (8), is the intensity of distributed load, that is measured in N m^{-1}, then in each specific case the disturbing force must correspond to dimension of the intensity of the load. By using the so called impulse function of the first order $\sigma_1(x)$ (Babakov, 1968) it is possible to determine the intensity of the distributed load, and to include into the composition of the load divided along the length the concentrated forces and moments of forces.

Respectively, if $Q_{зб.}(t)$ is the concentrated disturbing force applied to point x_1 and measured in Newtons, then the function:

$$Q_{зб.}(x,t) = Q_{зб.}(t)\cdot\sigma_1(x-x_1) \qquad (11)$$

has the dimension in N m^{-1} and expresses intensity of the concentrated load in the point x_1.

The function $\sigma_1(x-x_1)$ equals zero for all x, except for $x = x_1$, where it is transformed into infinity.

Let the disturbing force acting according to the law

$$Q_{зб.}(t) = H\sin\omega t\,, \qquad (12)$$

be applied to the root at the distance x_1 from the starting point (point O in Fig. 1). Then, according to (11) we can write:

$$Q_{зб.}(x,t) = H\sin\omega t\cdot\sigma_1(x-x_1). \qquad (13)$$

Since the root is connected with the soil, which is an elastic environment, application of the disturbing force of type (12) to the root leads to emergence of the force of resistance of the soil to movement of the root due to its oscillations. This force also has an action on the process of own oscillations of the root in the soil, especially at the beginning of the oscillations process, until connections of the root with the soil are destroyed.

It is obvious that the force of resistance of the soil (for the entire body of the root) is the distributed load along the area of contact of the root with the soil, and that's why we must determine its intensity as the force of resistance of the soil to movement of a length unit of the root.

Let c be the coefficient of the elastic deformation of the soil applied to the area of the contact measured in N m^{-2}. It will now be assumed that the soil surrounding the root, under the action of the disturbing force $H\sin\omega t$, performs forced oscillations according to the same harmonic law with the amplitude that is determined by elastic properties of the soil. Then the intensity $P(x, t)$ in N m^{-1} of resistance of the soil to movement of the root in point x will be:

$$P(x,t) = 2\pi cx \cdot \tan\gamma \cdot \sin\omega t, \tag{14}$$

Respectively, we will have the following relation for longitudinal external load:

$$Q(x,t) = Q_{зб.}(x,t) - P(x,t).$$

Given the expressions (9), (10), (13) and (14), the Hamilton-Ostrogradski functional (8) will look as follows:

$$\begin{aligned} S = \frac{1}{2}\int_{t_1}^{t_2}\int_{0}^{h}\Bigg\{ & \rho \cdot \pi x^2 \tan^2\gamma\left(\frac{\partial y}{\partial t}\right)^2 - E\pi x^2 \cdot \tan^2\gamma\left(\frac{\partial y}{\partial x}\right)^2 + \\ & + \left[H\sin\omega t \cdot \sigma_1(x - x_1) - 2\pi cx \cdot \tan\gamma \cdot \sin\omega t\right] y(x,t)\Bigg\}\, dx\, dt. \end{aligned} \tag{15}$$

In order to find natural forms and frequencies of longitudinal oscillations of the root in the soil, the Ritz method can be applied (Babakov, 1968). According to the given method we will need to find harmonic longitudinal oscillations of the root as follows:

$$y(x,t) = \varphi(x)\sin(pt + \alpha), \tag{16}$$

where $\varphi(x)$ is the natural form of primary oscillations, i.e. the function that determines continuous population of amplitude longitudinal deviations of cross section of the root from their equilibrium positions, and p is the natural frequency of primary oscillations.

Since natural forms and natural frequencies are related to free oscillations of the system, in the functional (15) we must highlight the part that specifically describes free oscillations of the system. Obviously the functional will look as follows:

$$S_1 = \frac{1}{2}\int_{t_1}^{t_2}\int_{o}^{h}\left[\rho\pi x^2 \tan^2\gamma\left(\frac{\partial y}{\partial t}\right)^2 - E\pi x^2 \tan^2\gamma\left(\frac{\partial y}{\partial x}\right)^2\right]dx\,dt \tag{17}$$

The expression (16) will now be inserted into the functional (17), and we will get:

$$\begin{aligned} S_1 = \frac{1}{2}\int_{t_1}^{t_2}\int_{0}^{h}\Big\{ & \rho\cdot\pi x^2\cdot\tan^2\gamma\cdot\varphi^2(x)\cdot p^2\cdot\cos^2(p\,t+\alpha) - \\ & - E\pi x^2\cdot\tan^2\gamma\left[\varphi'(x)\right]^2\sin^2\Big(p\,t+\alpha\Big)\Big\}\,d\,x\,d\,t. \end{aligned} \tag{18}$$

The expression (18) will be integrated over t within the limits of one period $T = \frac{2\pi}{p}$, and we will have:

$$S_2 = \frac{\pi}{2p}\int_{0}^{h}\left\{\rho\cdot\pi x^2\cdot\tan^2\gamma\cdot\varphi^2(x)p^2 - E\,\pi x^2\cdot\tan^2\gamma\left[\varphi'(x)\right]^2\right\}dx. \tag{19}$$

The basis of the Ritz method is reduction of the variational problem to the problem of search of extremum of function of any independent variables.

According to the Ritz method the value of the functional (19) is analysed on population of linear combinations of functions, i.e. expressions looking as follows:

$$\psi(x) = \sum_{i=1}^{n}\alpha_i\cdot\psi_i(x), \tag{20}$$

where: α_i are the parameters, variations of which enable us to obtain the required class of allowed functions; $\psi_i(x)$ are the basis functions that are specifically chosen and are known functions, that correspond to geometrical boundary conditions of the problem.

Respectively, we insert the expression (20) into the expression (19), and get:

$$\begin{aligned} S_2 = \frac{\pi}{2p}\int_{0}^{h}\Bigg\{ & \rho\cdot\pi x^2\tan^2\gamma\left[\sum_{i=1}^{n}\alpha_i\psi_i(x)\right]^2 p^2 - \\ & - E\pi x^2\tan^2\gamma\left[\left(\sum_{i=1}^{n}\alpha_i\cdot\psi_i(x)\right)'\right]^2\Bigg\}dx. \end{aligned} \tag{21}$$

After respective transformations the functional (21) will look as follows:

$$S_2 = \frac{\pi}{2p}\int_0^h \left[\rho\pi x^2 \tan^2\gamma\, p^2 \sum_{i,k=1}^{n} \psi_i(x)\psi_k(x)\,\alpha_i\alpha_k - \right.$$
$$\left. - E\pi x^2 \tan^2\gamma \sum_{i,k=1}^{n} \psi_i'(x)\psi_k'(x)\,\alpha_i\alpha_k \right] dx. \qquad (22)$$

The following symbols will now be entered:

$$\int_0^h \rho\cdot\pi x^2\cdot\tan^2\gamma\cdot\psi_i(x)\cdot\psi_k(x)\,dx = T_{ik},$$
$$\int_0^h E\pi x^2\cdot\tan^2\gamma\cdot\psi_i'(x)\cdot\psi_k'(x)\,dx = U_{ik}, \qquad (23)$$
$$(i,k = 1,2,...,n).$$

By inserting (23) into (22), we will get a functional as a function from parameters $a_1, a_2, \ldots, a_n$:

$$S_2(\alpha_1, \alpha_2, ..., \alpha_n) = \frac{\pi}{2p} p^2 \sum_{i,k=1}^{n} T_{ik}\alpha_i\alpha_k - \frac{\pi}{2p}\sum_{i,k=1}^{n} U_{ik}\alpha_i\alpha_k. \qquad (24)$$

The extremum analysis of the functional (24) will now performed. For this purpose we will differentiate the expression (24) with respect to parameters α_i, $(i = 1,\ 2, \ldots, n)$ and equate the obtained particular derivatives to zero. As a result of that we will get a set of linear homogeneous equations with respect to the unknowns $a_1, a_2, \ldots, a_n$, from which, in turn, we can find the Ritz frequencies equation for longitudinal oscillations of the root attached in the soil:

$$\begin{vmatrix} U_{11} - p^2T_{11} & U_{12} - p^2T_{12} & \ldots & U_{1n} - p^2T_{1n} \\ U_{21} - p^2T_{21} & U_{22} - p^2T_{22} & \ldots & U_{2n} - p^2T_{2n} \\ \ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots & \ldots & \ldots\ldots\ldots\ldots \\ U_{n1} - p^2T_{n1} & U_{n2} - p^2T_{n2} & \ldots & U_{nn} - p^2T_{nn} \end{vmatrix} = 0 \qquad (25)$$

It is known, that with $n > 4$ the given equation cannot be solved in radicals, that's why it is necessary to apply numerical methods using a PC.

However, in reality, as a rule, only the lower frequencies are determined, most often the first and the second ones, which have the most significant action on the technological process that is being analysed.

Therefore, the first and the second frequencies of natural oscillations of the root will now be determined.

For the purpose of determination of the first and the second frequencies the equation (25) will look as follows:

$$\begin{vmatrix} U_{11}-p^2T_{11} & U_{12}-p^2T_{12} \\ U_{21}-p^2T_{21} & U_{22}-p^2T_{22} \end{vmatrix} = 0 \tag{26}$$

As a result of the solution of the given equation we will obtain expressions for finding the value of the first (primary) frequency:

$$p_1 = \frac{0.662422}{h}\sqrt{\frac{E}{\rho}}, \tag{27}$$

and the second frequency:

$$p_2 = \frac{27.931592}{h}\sqrt{\frac{E}{\rho}}. \tag{28}$$

Now the calculation will be performed of the values of the first and the second frequencies for the beetroot having the following parameters (Pogorely et al., 1983) $h = 250$ mm; $E = 18.4 \cdot 10^6$ N m^{-2}; $\rho = 1{,}300$ kg m^{-3}. As a result of the calculations we get:

$$p_1 = \frac{0.662422}{250 \cdot 10^{-3}}\sqrt{\frac{18.4 \cdot 10^6}{1{,}300}} = 315 \text{ s}^{-1},$$

$$p_2 = \frac{27.931592}{250 \cdot 10^{-3}}\sqrt{\frac{18.4 \cdot 10^6}{130}} = 13{,}292 \text{ s}^{-1}.$$

Next, the analysis of the forced oscillations of the root will be discussed. The exclusively forced oscillations will happen according to the following law:

$$y(x,t) = \varphi(x)\sin\omega t, \tag{29}$$

where φ(x) is the form of the forced oscillations.

In order to determine the form of the forced oscillations of the root the expression (29) will now be entered into the functional (15), and we will get the following functional:

$$\begin{aligned} S_3 = \frac{1}{2}\int_{t_1}^{t_2}\int_0^h \{ & \rho\pi x^2 \tan^2\gamma\omega^2\varphi^2(x)\cos^2\omega t - E\pi x^2\tan^2\gamma[\varphi'(x)]^2\sin^2\omega t + \\ & + [H\sigma(x-x_1) - 2\pi c x\tan\gamma]\ \varphi(x)\sin^2\omega t\}\, dx\, dt. \end{aligned} \tag{30}$$

By integrating the expression (30) over t within the limits of one period $T = \frac{2\pi}{\omega}$, we will get:

$$S_4 = \frac{\pi}{2\omega}\int_0^h \left\{ \rho \pi x^2 \tan^2 \gamma \varphi^2(x)\omega^2 - E\pi x^2 \tan^2 \gamma \left[\varphi'(x)\right]^2 + \right.$$
$$\left. + H\sigma_1(x - x_1)\varphi(x) - 2\pi c x \tan\gamma\varphi(x) \right\} dx. \tag{31}$$

According to the Ritz method let's analysis will now performed of the value of the functional (31) with respect to population of linear combinations of the following type:

$$\varphi(x) = \alpha\psi(x) \tag{32}$$

where: α is the parameter, variations of which let us obtain the class of the allowed functions; $\psi(x)$ is the basis function.

The expression (32) will now be inserted into the functional (31), and we will obtain:

$$S_4 = \frac{\pi}{2\omega}\int_0^h \left\{ \rho \pi x^2 \tan^2 \gamma \alpha^2 \psi^2(x)\omega^2 - E\pi x^2 \tan^2 \gamma \alpha^2 \left[\varphi'(x)\right]^2 + \right.$$
$$\left. + H\sigma_1(x - x_1)\alpha\psi(x) - 2\pi c x \tan\gamma\alpha\psi(x) \right\} dx. \tag{33}$$

The following symbols will now be inserted:

$$\int_0^h \rho \pi x^2 \cdot \tan^2 \gamma \cdot \psi^2(x)\, dx = T, \tag{34}$$

$$\int_0^h E \pi x^2 \cdot \tan^2 \gamma \cdot \left[\psi'(x)\right]^2 dx = U, \tag{35}$$

$$\int_0^h \left[H\sigma_1(x - x_1)\cdot\psi(x) - 2\pi c x \tan\gamma\psi(x)\right] dx = L. \tag{36}$$

The expressions (34), (35), (36) will now be inserted into (33), and we will have:

$$S_4(\alpha) = \frac{\pi}{2\omega}\left(\omega^2 T\alpha^2 - U\alpha^2 + L\alpha\right) \tag{37}$$

So, in the population of functions, (32) the functional (33) is transformed into the function of the independent variable α, looking as (37).

The necessary condition of the stationary functional (37) (i.e. existence of the extremum) is that its first variation equals zero:

$$\frac{\partial S_4}{\partial \alpha}\cdot\delta\alpha = 0 \tag{38}$$

from which we receive the following equation:

$$2\omega^2 T\alpha - 2U\alpha + L = 0 \tag{39}$$

from which we find the required value of the parameter α . It will be:

$$\alpha = \frac{L}{2\left(U - \omega^2 T\right)} \tag{40}$$

The form of the forced longitudinal oscillations of the rod with the constant cross section with one end firmly attached, emerging under the action of the longitudinal harmonic force of frequency ω, applied at the point $x = x_1$ will now be assumed as the basis function $\psi(t)$.

According to Babakov (1968) the form of the forced oscillations of the given rod looks as follows:

$$\psi\left(x\right) = D_1 \sin ax \quad \text{with} \quad x \le x_1 \tag{41}$$

$$\psi\left(x\right) = D_2 \cos a\left(h - x\right) \quad \text{with} \quad x > x_1, \tag{42}$$

where,

$$D_1 = -\frac{1}{a\,EF} \cdot \frac{\cos a\left(h - x_1\right)}{\cos ah} \tag{43}$$

$$D_2 = -\frac{1}{aEF} \cdot \frac{\sin ax_1}{\cos ah} \tag{44}$$

$$a = \omega\sqrt{\frac{\mu}{EF}} \tag{45}$$

μ is the mass per length of the rod; F is the area of the longitudinal section of the rod; E is the Young's module for material of the rod; h is the length of the rod; ω is the frequency of the forced oscillations of the rod.

Having calculated the parameters of T, U and L according to the expressions (34), (35) and (36), we obtain the required value of the parameter α according to the expression (40), in case of which the functional (33) will have a stationary value.

Taking into consideration the expressions (32), (41) and (42), we get expression for the form of the forced oscillations of the root attached in the soil. They look as follows:

$$\begin{aligned} &\varphi\left(x\right) = \alpha \cdot D_1 \sin a\,x\,, \quad \text{with} \\ &x \le x_1,\ \varphi\left(x\right) = \alpha \cdot D_2 \cos a\left(h - x\right), \quad \text{with } x > x_1\,. \end{aligned} \tag{46}$$

Having inserted the expressions (46) into (29), we get the final law of the forced oscillations of the root attached in the soil. If we take into consideration the action of the disturbing force $H \sin \omega t$, the given law will look as follows:

$$\begin{aligned} y(x,t) &= D_1 \alpha \sin ax \cdot \sin \omega t, \quad \text{with } x \le x_1, \\ y(x,t) &= D_2\, \alpha \cos a\,(h-x) \cdot \sin \omega t, \quad \text{with } x > x_1. \end{aligned} \tag{47}$$

Based on the results of the theoretical research of the forced oscillations of the beetroot attached in the soil we will perform specific calculations of the amplitude of the given oscillations.

The length of the root h, its cone angle γ, Young's module E for the body of the root, density ρ of the root, coefficient of elastic deformation of the soil c will be assumed to be equal, according to Pogorely et al. (1983): $h = 250 \cdot 10^{-3}$ m; $\gamma = 14°$; $E = 18.4 \cdot 106$ N m^{-2}; $\rho = 1{,}300$ kg m^{-3}; $c = 1 \cdot 10^{-5}$ N m^{-2}.

The amplitude H of the disturbing force will be chosen within the limits 100...600 N. We will assume the frequency ω of the disturbing force, according to (Vasilenko et al., 1970), to equal $\omega = 20$ Hz.

The calculation is performed using the Mathcad program in order to determine the relations between the amplitude of the forced longitudinal oscillations of the body of the root and changes of the disturbing force within the range 100...600 N for different cross sections of the root.

The result of the given calculation is the graph shown in Fig. 2.

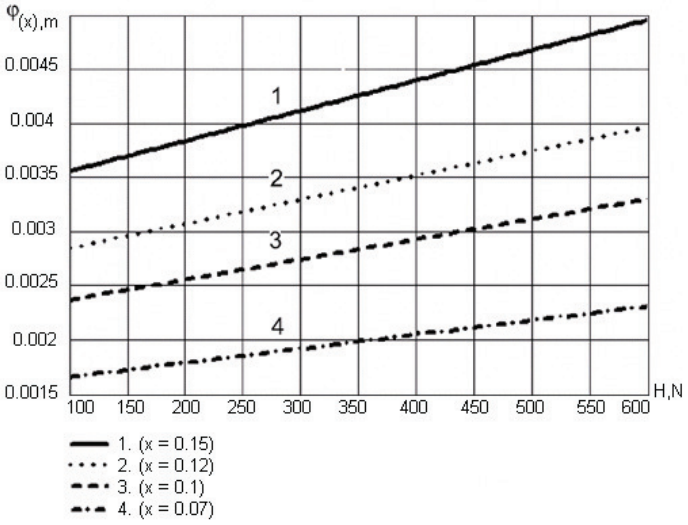


Figure 2. Relation between the amplitude of forced longitudinal oscillations of the body of the root and the value of the disturbing force.

As it is seen from the given graph, increase of the value of the disturbing force leads to the increase of the amplitude of the longitudinal forced oscillations of the body of the root according to the linear law.

It should also be noted, that with increase of the distance of the area of cross section of the root from the origin of coordinates O the amplitude is also increased. For example, with x = 0.07 m the amplitude is within the limits of 1.7…2.3 mm with x = 0.1 m – within the limits of 2.3…3.5 mm, with x = 0.12 m – within the limits of 2.8…3.9 mm, with x = 0.15 m (the point of gripping) – within the limits 3.2…4.8 mm.

Further, analysis will be presented of the calculation performed on a PC of the amplitude of longitudinal oscillations of the body of the root attached in the soil from the coefficient c of the elastic deformation of the soil surrounding the root, and the distance of the cross section of the root from the conditional point of its attachment for the frequency of the disturbing force $\nu = 10\,\text{Hz}$ and $\nu = 20\,\text{Hz}$.

On the basis of the calculations we get the following graphs (Fig 3).

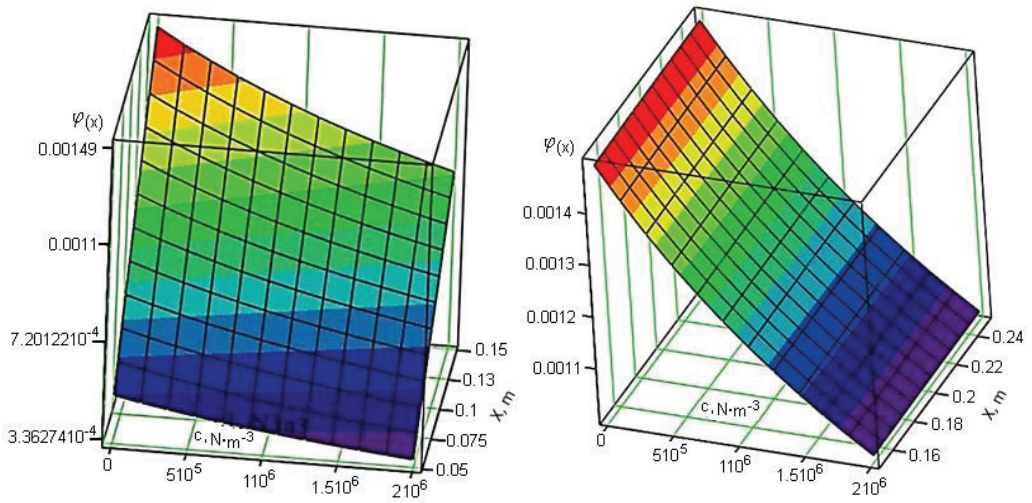


Figure 3. Relation between the amplitude of the forced longitudinal oscillations of the root as an elastic body attached in the soil, and the coefficient c of the elastic deformation of the surrounding soil, and between the distance x of the cross section of the root and the conditional point of attachment: a) $x \le x_1,$) b) $x \ge x_1$), (x_1 – point of gripping, $\nu = 20$ Hz).

As it is seen from the graphs stated above, in case of increase of the coefficient c of the elastic deformation of the surrounding soil, the amplitude of the forced oscillations of the root is reduced, and in case of increase of the distance x of the cross section of the root from the point of conditional attachment with $x \le x_1$ it is increased, and with $x \ge x_1$ it almost doesn't change.

Fig. 4 shows the given relation for a number of specific cross sections of the root, in particular: for x = 0.07 m; 0.1 m; 0.12 m; 0.15 m (point of gripping).

On the given graph we can quite clearly see the tendency of increase of the amplitude of the forced longitudinal oscillations in case of increase of the distance of the cross section from the conditional point of attachment and the tendency of its reduction due to increase of the coefficient c of the elastic deformation of the surrounding soil.

For example, with x = 0.07 m and change of the coefficient c within the limits c = 0…20·105 N m-3, the amplitude is changed within the limits of 0.7…0.47 mm; with x = 0.1 m – within the limits of 0.99…0.67 mm; with x = 0.12 m – within the limits of 1.19…0.81 mm; with x = 0.15 m (point of gripping) – within the limits of 1.49…1.01 mm.

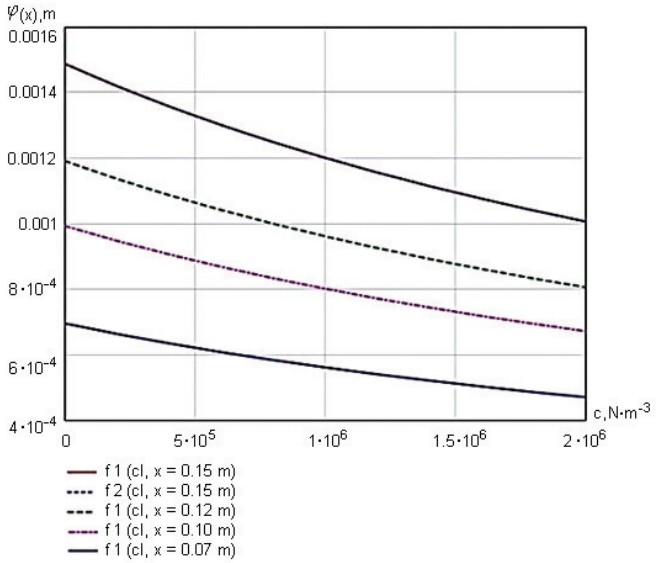


Figure 4. Relation between the amplitude of the forced longitudinal oscillations of the root as an elastic body and the distance x of the cross section of the conditional point of attachment $x \le x_1$, $\nu = 20$ Hz.

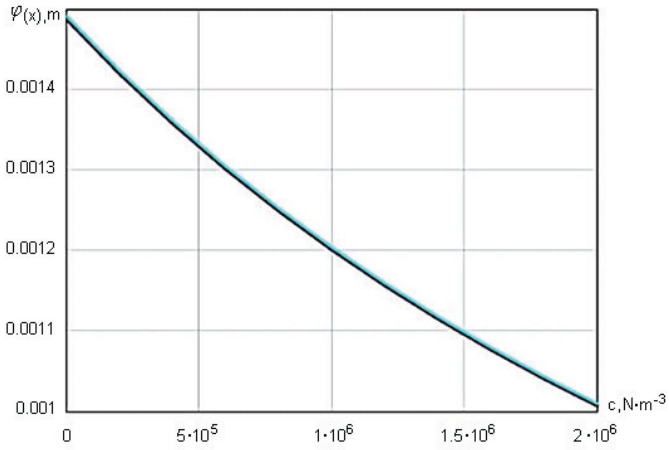


Figure 5. Relation between the amplitude of the forced longitudinal oscillations of the root as an elastic body and the distance x of the cross section from the conditional point of attachment $(x \ge x_1)$, $\nu = 20$ Hz.

However, as the graph in Fig. 5 shows, for cross section of the root above the point of gripping ($x \geq 0.15$ m) the amplitude of forced oscillations of the body of the root with increasing distance of the cross section from the conditional point of attachment almost doesn't change and remains the same as in case of x = 0.15 m. However, the tendency of decrease of the amplitude from increase of the coefficient c is the same as for the sections below the point of gripping (x ≤ 0.15).

In case of the frequency of the disturbing force $\nu = 10$ Hz values of the amplitude are slightly lower. For example, with x = 0.07 m the value of the amplitude remains within the limits of 0.66…0.45 mm; with x = 0.1 – within the limits of 0.94…0.65 mm; with x = 0.12 m – within the limits of 1.13…0.78 mm; with x = 0.15 m (point of gripping) – within the limits of 1.41…0.97 mm.

Also, we have obtained the estimated relation between the amplitude of the forced longitudinal oscillations of the body of the root and the amplitude of the disturbing force for the frequency of the disturbing force $\nu = 20$ Hz (Figs 6 and 7).

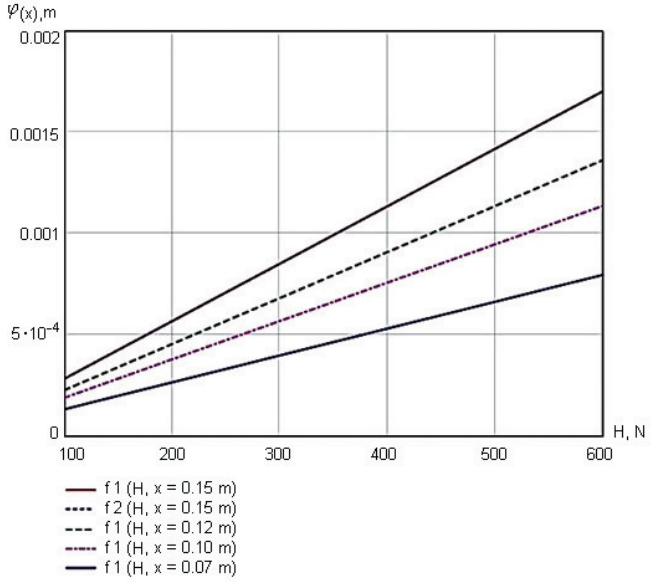


Figure 6. Relation between the amplitude of the forced longitudinal oscillations of the body of the root and the amplitude of the disturbing force ($x \leq x_1$, $\nu = 20$ Hz).

As it is seen from the presented graphs, increase of the amplitude of the disturbing force leads to increase of the amplitude of longitudinal forced oscillations of the body of the root according to the linear law.

It should also be noted, that below the point of gripping ($x \leq 0.15$ m), with increase of the distance of the cross section of the root from the conditional point of attachment O the amplitude also increases (Fig. 6). For example, with x = 0.07 m the amplitude remains within the limits of 0.13...0.8 mm; with x = 0.1 m – within the limits of 0.19...1.14 mm; with x = 1.12 m – within the limits of 0.23...1.36 mm; with x = 0.15 m (point of gripping) – within the limits of 0.28...1.7 mm. However, above the point of gripping ($x \geq 0.15$ m), in case of increase of distance of the cross section

from the conditional point of attachment O the amplitude almost doesn't change, as it is shown on the graph in Fig. 7.

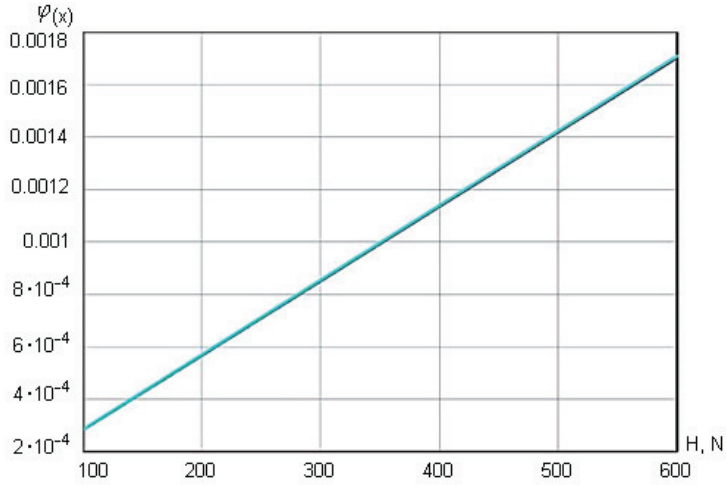


Figure 7. Relation between the amplitude of the forced longitudinal oscillations of the body of the root and the amplitude of the disturbing force ($x \geq x_1$, $\nu = 20$ Hz).

In case of the frequency of the disturbing force $\nu = 10$ Hz the obtained values of amplitudes were a little bit lower, however for $\nu = 10$ Hz they were the same. For example, with $x = 0.07$ m the amplitude remains within the limits of 0.12...0.76 mm; with $x = 0.1$ m – within the limits of 0.18...1.08 mm; with $x = 0.12$ m – within the limits of 0.21...1.3 mm; with $x = 0.15$ m (point of gripping) – within the limits of 0.27...1.62 mm.

Respectively, the obtained values of the frequencies of natural longitudinal oscillations and amplitudes of the forced longitudinal oscillations of the body of the root foster the process of intense knocking of the soil that adhered to the roots off their surface, and in case of such values of the amplitudes tearing of the bodies of the roots is unlikely.

CONCLUSIONS

1. The new theory has been developed with regard to longitudinal oscillations of the root of sugar beet as a body attached in the soil, as an elastic body in an elastic environment, that emerges under the action of the vertical disturbing force that changes according to the harmonic law. The Hamilton-Ostrogradski variational principle of stationary action is used for longitudinal oscillations of the root taking into account the physical and mechanical properties of the root of sugar beet as an elastic body and the surrounding soil.

Using the Ritz direct variational method the Ritz frequencies equation has been obtained, from which different frequencies of free longitudinal oscillations of the root as an elastic body are determined. This, for example, allowed to obtain the analytical expression for calculation of the first natural frequency depending on the physical and

mechanical properties of the root and elasticity of the soil surrounding it, which plays the main role in destruction of the tights of the root with the soil. According to the calculations performed, when the coefficient c of the elastic deformation of the soil is changed, the first frequency of natural oscillations of the body of the root is monotonously increased within the limits of 76.4…93.4 Hz, which sufficiently precisely corresponds to the experimental data stated in (Pogorely et al., 1983; Pogorely & Tatyanko, 2004). At the same time the second frequency is changed within the limits of 528…532 Hz, i.e. it has little dependency on the coefficient c of the elastic deformation of the soil.

2. The Hamilton-Ostrogradski functional for forced longitudinal oscillations of the root as an elastic body was constructed, on the basis of which the theory of forced oscillations of the beetroot in the soil was created. The results of theoretical research of the forced oscillations of beetroot attached in the soil were the basis for finding of the algorithm for calculation on a PC of the specified oscillations, in particular, finding of the law of the forced longitudinal oscillations and amplitude under the condition of prevention of damage (tearing) of the beetroot depending on the coefficient c of the elastic deformation of the soil and the amplitude of the disturbing force.

3. It was analytically established that the amplitude of the forced oscillations of the body of the root decreases in case of increase of the coefficient c of elastic deformation of the soil, and increases in case of increase of distance of the cross section of the beetroot from the conditional point of its attachment in the soil. For example, with $x = 0.07$ m and the change of the coefficient c within the limits of $c = 0…20 \cdot 10^5$ N m^{-3}, the amplitude is measured within the limits of 0.7…0.47 mm; with $x = 0.1$ m – within the limits of 0.99…0.67 mm; with $x = 0.12$ m – within the limits of 1.19…0.81 mm; with $x = 0.15$ m (point of gripping) – within the limits of 1.49…1.01 mm.

However, for the cross sections of the root above the point of gripping ($x \geq 0.15$ m) the amplitude of the forced oscillations of the body of the root almost doesn't change in case of increase of the distance of the cross section from the conditional point of attachment and remains the same as in case of $x = 0.15$ m. However, the tendency of decrease of the amplitude from increase of the coefficient c is the same as for sections below the point of gripping ($x \leq 0.15$ m).

4. The paper also presents the calculations performed of the amplitude of forced longitudinal oscillations in case of change of the amplitude of the disturbing force within the limits of 100…600 N. As the calculations demonstrated, the increase of the amplitude of the disturbing force leads to the increase of the longitudinal forced oscillations of the body of the beetroot according to the linear law, and increase of the distance of the area of cross section of the root from the conditional point of its attachment in the soil also leads to increase of the amplitude.

For example, with $x = 0.07$ m, the amplitude remains within the limits of 0.13…0.8 mm, with $x = 0.1$ m – within the limits of 0.19…1.14 mm, with $x = 0.12$ m – within the limits of 0.23…1.36 mm, with $x = 0.15$ m (point of gripping) – within the limits of 0.28…1.7 mm. However, above the point of gripping in case of increase of the distance of the cross section from the conditional point of attachment the amplitude almost does not change.

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