Analysis of mechanical behaviour of *Jatropha curcas* L. bulk seeds under compression loading with aid of reciprocal slope transformation method and tangent curve method

D. Herak

Czech University of Life Sciences Prague, Faculty of Engineering, Department of Mechanical Engineering, Kamycka 129, 16521, Prague, Czech Republic; e-mail: herak@tf.czu.cz

Abstract. This study is focused on the utilization of the reciprocal slope transformation method (RST) and tangent curve method (TCM) for description of mechanical behaviour of *Jatropha* bulk seeds under compression loading. The experimental data derived from a compression test was done using compression device and pressing vessel with diameter 60 mm at compression speed of 1 mm s$^{-1}$ and compressive force between 0 kN and 100 kN. Measured dependency between compressive force and deformation was fitted by RST method and TCM method and they were statistically analysed by ANOVA. Comparison of both methods in individual pressing regions is an integral part of this manuscript. It was determined that reciprocal slope transformation method describes precisely the beginning of deformation characteristics and it can cover the reorganizing process in the bulk seeds. From the conducted study it also follows that tangent curve method is suitable for description of mechanical behaviour at bulk deformations in which the reorganization process in the bulk seeds is finished yet.

Key words: stress, strain, oilseed, mathematical model, deformation characteristic, oil point.

INTRODUCTION

Mathematical description of deformation characteristic is important to understand the mechanical behaviour of bulk oilseeds under compression loading which could be used for technological improvement. There have been already published two mathematical models describing the mechanical behaviour of *Jatropha* bulk seeds under compression loading as one pressed unit (Herak et al., 2014a; Petru et al., 2014). First model is based on the utilization of tangent curve (Herak et al., 2013b) and second model is derived from reciprocal slope transformation (Blahovec, 2011), these two models have been already deeply described in the literature and also they have been mutually compared (Herak et al. 2014b). In published studies their applications were analysed throughout deformation characteristics however it is very well known that deformation characteristic of *Jatropha curcas* bulk seeds has non-linear course and it can be divided into few regions in which the compressed bulk seeds shows different mechanical behaviour (Kabutey et al., 2013a). From practical point of view it means that the mathematical model can fit properly whole deformation characteristic although the beginning of deformation curve or other deformation regions can be described statistically insignificant. It has been already published that deformation characteristics
of Jatropha curcas bulk seeds has three general regions of mechanical behaviour: under lower oil point – the oil is not gained yet, between lower oil point and upper oil point – the leakage of oil is running, and above upper oil point – theoretically the oil leakage is stopped (Herak et al., 2013a; Kabutey et al., 2013a). The aim of this study is to compare utilization of reciprocal slope transformation (RST) and tangent curve method (TCM) for mathematical description of mechanical behaviour of Jatropha curcas L. bulk seeds under compression loading at regions under lower oil point and above lower oil point.

MATERIALS AND METHODS

Samples
Samples of bulk Jatropha curcas L. seeds, variety IPB2, obtained from North Sumatra, Indonesia were used for the experiment. The general physical properties of the oilseed crop are given in Table 1. The moisture content of the samples was determined using standard moisture measurement equipment (Farm Pro, model G, Czech Republic). The mass of sample was determined using an electronic balance (Kern 440–35, Kern & Sohn GmbH, Balingen, Germany). The porosity was calculated from the relationship between the bulk and true densities (Blahovec, 2008). The bulk density was determined from the mass of the sample divided by initial pressing volume. The true seed density was determined gravimetrically (Blahovec, 2008). The results obtained were expressed as mean of three replicates.

Table 1. Physical properties of bulk seeds of Jatropha; data in the table are means ± SD

<table>
<thead>
<tr>
<th>$M_c$ (%)</th>
<th>$m$ (g)</th>
<th>$V$ (mm$^3$)</th>
<th>$P_f$ (%)</th>
<th>$\rho_b$ (kg m$^{-3}$)</th>
<th>$\rho_t$ (kg m$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.5 ± 0.2</td>
<td>87.95 ± 1.19</td>
<td>226224 ± 6340</td>
<td>59.98 ± 1.26</td>
<td>388 ± 12</td>
<td>980 ± 12</td>
</tr>
</tbody>
</table>

$M_c$ – moisture content of bulk seeds in dry basis, $m$ – mass of bulk seeds, $V$ – initial volume of bulk seeds, $P_f$ – porosity of bulk seeds, $\rho_b$ – bulk density, $\rho_t$ – true density.

Compression test
To determine the relationship between compressive force and deformation characteristic curves, a compression device (Labortech, model 50, Czech Republic) was used to record the course of deformation function. A single pressing vessel diameter, 60 mm with plunger (Fig. 1.) was used.

Initial pressing height 80 mm of Jatropha bulk seeds were tested with a compression speed of 1 mm s$^{-1}$ under temperature of 20 °C. The compressive force was between 0 and 100 kN. The experiment was repeated three times. Recorded deformation characteristic was divided into five regions according to the study already published by Herak et al. (2013a) and Kabutey et al. (2013a) such as shown in Fig. 2. Individual regions were defined by bulk deformations: region I. from 0 mm to 10 mm, region II. from 10 mm to 40 mm, region III. from 40 mm to 50 mm, region IV. from 50 mm to 55 mm and region V. from 55 mm to 60 mm.
Reciprocal slope transformation (RST)

Dependency between compressive force \( F_{\text{RST}} \) (N) and corresponding deformation \( x \) (mm) was described by Eq. 1 using reciprocal slope transformation method (RST) (Herak et al., 2014), where \( a \) (N\(^{-1}\) mm\(^{-2}\)), \( b \) (N\(^{-1}\) mm\(^{-1}\)), \( c \) (N\(^{-1}\)) and \( d \) (N\(^{-1}\) mm) are coefficients of the RST method. The coefficients were determined by the least squares method applicable in MathCAD software.

\[
F_{\text{RST}}(x) = \frac{x}{ax^3 + bx^2 + cx + d} \quad (1)
\]

Tangent curve method (TCM)

Dependency between compressive force \( F_{\text{TCM}} \) (N) and corresponding deformation \( x \) (mm) was described by tangent curve method (TCM) (Herak et al., 2013b) which is given by Eq. 2.

\[
F_{\text{TCM}}(x) = A \cdot \left[ \tan(B \cdot x) \right]^2, \quad (2)
\]

where: \( A \) (N) is force coefficient of mechanical behaviour and \( B \) (mm\(^{-1}\)) is coefficient of mechanical deformation behaviour. These coefficients were determined by Marquardt Levenberg approximation process (Marquardt, 1963) using MathCAD software.

RESULTS AND DISCUSSION

The measured characteristic was fitted by Eq. 1 and Eq. 2 such as presented in Fig. 3 with determined coefficient of variation \( CV = 10\% \) and the coefficients of this function are presented in Table 2 and Table 3. ANOVA analysis of the measured data using MathCAD 14 software was statistically significant at significance level 0.05, that is, the values of \( F_{\text{crit}} \) (critical value comparing a pair of models) were higher than the \( F_{\text{rat}} \) values (value of the F – test) and values of \( P \) (significance level at which it can be rejected)
the hypothesis of equality of models) (Table 4) were higher than 0.05. The validity of these equations is limited to the region where the deformation of seeds varies from zero to maximum. It is evident that both method (RST) and (TCM) can be used for appropriate description of mechanical behaviour of *Jatropha* bulk seeds which was confirmed by already published studies (Kabutey et al., 2013a; Divisova et al., 2014; Herak et al., 2014a; Petru et al., 2014; Sigalingging et al., 2014). From the graphical presentations (Fig. 3) and also from statistical analyses (Table 4) it follows that used methods Eq. 1 and Eq. 2 show different possibility of their utilization for description mechanical behaviour in individual regions.

**Figure 3.** Measured amounts of mechanical characteristic of *Jatropha* bulk seeds with displayed amounts of standard deviation and their fitted functions by Eq. 1 and Eq. 2. *F* is measured force, *F RST* is force determined by RST, *F TCM* is force determined by TCM, I., II., III., IV., V. are individual regions, F. CH. is full deformation characteristics.
Table 2. Determined coefficients of RST method

<table>
<thead>
<tr>
<th>A</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N(^{-1}) mm(^{-2}))</td>
<td>(N(^{-1}) mm(^{-1}))</td>
<td>(N(^{-1}))</td>
<td>(N(^{-1}) mm)</td>
</tr>
<tr>
<td>2.14·10(^{-7})</td>
<td>-2.85·10(^{-5})</td>
<td>0.93·10(^{-3})</td>
<td>16.34·10(^{-5})</td>
</tr>
</tbody>
</table>

a, b, c, d – coefficients of reciprocal slope transformation

Table 3. Determined coefficients of TCM method

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N)</td>
<td>(mm(^{-1}))</td>
</tr>
<tr>
<td>2320</td>
<td>0.026</td>
</tr>
</tbody>
</table>

A – force coefficient of mechanical behavior, B – deformation coefficient of mechanical behavior

It is clear (Fig. 3) that for small bulk seeds deformations up to 40 mm the RST method is more appropriate than TCM method. From mechanical behaviour in regions I. (Fig. 3) it implies that RST method describes precisely origin of the bulk seeds reorganization which is in accordance to the study of Petru et al. (2014), the TCM method ignores this origin of the change of bulk seeds mechanical behaviour under small deformations which is confirmed by statistical analysis in Table 4. ANOVA shows statistical significances between measured data and both method used for description of mechanical behaviour in region II. From Table 4 and Fig. 3 follows that RST method more appropriately describes the whole course of deformation characteristics in region II. which is obvious from the amount of \(P_{value}\) (Table 4).

Table 4. ANOVA statistical analysis

<table>
<thead>
<tr>
<th>Region</th>
<th>(F_{rat}) (-)</th>
<th>(F_{crit}) (-)</th>
<th>(P_{value}) (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F. CH.</td>
<td>RST 0.062</td>
<td>3.919</td>
<td>0.803</td>
</tr>
<tr>
<td></td>
<td>TCM 0.005</td>
<td>3.918</td>
<td>0.942</td>
</tr>
<tr>
<td>I.</td>
<td>RST 0.001</td>
<td>4.747</td>
<td>0.971</td>
</tr>
<tr>
<td></td>
<td>TCM 24.244</td>
<td>4.747</td>
<td>0.001</td>
</tr>
<tr>
<td>II.</td>
<td>RST 0.006</td>
<td>4.170</td>
<td>0.938</td>
</tr>
<tr>
<td></td>
<td>TCM 3.748</td>
<td>4.170</td>
<td>0.062</td>
</tr>
<tr>
<td>III.</td>
<td>RST 0.281</td>
<td>4.351</td>
<td>0.602</td>
</tr>
<tr>
<td></td>
<td>TCM 0.355</td>
<td>4.351</td>
<td>0.558</td>
</tr>
<tr>
<td>IV.</td>
<td>RST 4.024</td>
<td>4.026</td>
<td>0.038</td>
</tr>
<tr>
<td></td>
<td>TCM 4.015</td>
<td>4.026</td>
<td>0.036</td>
</tr>
<tr>
<td>V.</td>
<td>RST 67.324</td>
<td>5.317</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>TCM 3.376</td>
<td>5.317</td>
<td>0.103</td>
</tr>
</tbody>
</table>

\(F_{rat}\) – value of the F test, \(F_{crit}\) – critical value that compares a pair of models, \(P_{value}\) – the significance level at which it can be rejected the hypothesis of equality of models

For bulk seeds deformations between 40 mm and 50 mm the both methods can be used without limitations such as follows from graphical presentation of region III. (Fig. 3) and from statistical analysis (Table 4). From the course of deformation characteristics (Fig. 3) implies that description of mechanical behaviour in region IV. shows differences between models given by Eq. 1 and Eq. 2 and measured data. The amounts of differences are approaching to the limit of the applicability of both models.
which is also given by very low amount of $P_{value}$ such as small difference between $F_{rat}$ and $F_{crit}$ however from the statistical point of view both methods can be used for description of mechanical behaviour in region IV. From the determined coefficient of variation of deformation curve it is clear that *Jatropha* bulk seeds show variances between mechanical behaviour of individual seeds which is given by their biological nature. Taking into account of this variability which is graphically presented by standard deviations in Fig. 3 the data determined by Eq. 1 and Eq. 2 are still in the range bounded by standard deviations. The lowness limit amount of bulk deformation (55 mm) in region IV. was determined in previously published studies as lower oil point (Willems et al., 2008; Herak et al., 2013a). In the region V. it is clear that RST method doesn’t appropriate describe mechanical behaviour above bulk deformations of 55 mm which is also evident from the statistical analysis (Table 4.) and also from Fig. 3 where the RST data are out of standard deviations. From the conducted study it follows that RST method given by Eq. 1 describes more precisely the beginning of the pressing process and it provides a detailed view of the process of reorganizing of individual seeds in the compressed bulk seeds. Deviations from deformation curve at larger deformations can be eliminated by using a multiple-member polynomial in Eq. 1. The RST method in the present form given by Eq. 1 is suitable for description of the mechanical behaviour of *Jatropha* bulk seeds under compression loading at deformation equal to or less than lowness oil point. The TCM method describes appropriately mechanical behaviour of *Jatropha* bulk seeds under compression loading at deformations in which the reorganization process of compressed bulk seeds was finished and the bulk seeds show mechanical behaviour similar to the one compressed unit. Practically it means the compression above bulk deformation of 40 mm. From this study it is also evident that using derivation of Eq. 1 and Eq. 2 the dependency between modulus of elasticity and bulk deformation can be determined (Herak et al., 2011; Sigalingging et al., 2014). Used models show that the developed mathematical equations Eq. 1 and Eq. 2 take into account the experimental boundary conditions of the linear compression of *Jatropha* bulk seeds for the mechanical behaviour description. The boundary conditions means that the origin of the deformation curve starts from zero force and zero deformation which is followed by an increasing function within the whole range of pressing process and when compressive force is approaching infinity then deformation reaches a maximum limit (Herak et al., 2013b; Kabutey et al., 2013a; Raji & Favier, 2004; Sayyar et al. 2009). It is important to mention here that both methods are suitable for mechanical behaviour description of bulk oilseeds of *Jatropha curcas L.* (Herak et al., 2014a; Herak et al., 2014b). RST method using least square method for data fitting and its coefficients can be determined by standard computer software (Blahovec, 2011; Blahovec & Yanniotis, 2009), however TCM method using Marquardt Levenberg approximation process is determined by the MathCAD software (Herak et al., 2011; Sigalingging et al., 2014). On the other hand by the integration of Eq. 1 and Eq. 2 the deformation energy of compressed bulk seeds can be determined and from mutual comparison published by Herak et al. (2014b) it is clear that determined energies are significant and booth models can be also applied for deformation energy determination. The results of this study are in accordance with already published theories related to the tangent curve method (Petru et al., 2012; Herak et al., 2013b; Sigalingging et al., 2014) and to the reciprocal slope transformation (Blahovec and Yanniotis, 2009; Blahovec 2011; Herak et al., 2014a). The advantage of using both methods is that it is not conditional to resolve individual
particles and their properties and relationships between particles since both methods use the bulk seeds as a unit which is affected by constrains between the pressing vessel and bulk oilseeds and the pressing process. These theories can be used for determining the mechanical behavior of different materials under compression loading such as oil palm kernels, rapeseeds, sunflower seeds, wood chips, paper chips and other (Herak, et al., 2011; Kabutey et al., 2012; Kabutey et al., 2013b; Divisova et al., 2014, Sigalingging et al., 2014). The mechanical behaviour of bulk oilseeds can be described also by different solution methods generally based on the Darcy’s Law (Fasino & Ajibola, 1990) and fluid flow through porous media (Singh & Kulshreshtha, 1996) or by methods based on the Terzaghi’s model (Shirato et al., 1986; Willems et al., 2008) or the energetic balance model (Zheng et al., 2005). In terms of the tangent curve method (Herak et al., 2013b) and reciprocal slope transformation (Blahovec 2011; Herak et al., 2014a), the bulk oilseeds are considered as a unit which is most relevant from construction engineering and economical point of view. Generally, the results of the present study were in agreement with the results published by different authors (Mrema & Mc Nulty, 1985; Omobuwajo et al., 1998; Braga et al., 1999; Guner et al., 2003). Using of RST method as well as TCM method has its justification for description of mechanical behaviour of Jatropha curcas bulk seeds under compression loading and the advantage both methods is using the bulk seeds as a unit which is affected by constrains between the pressing vessel and bulk oilseeds and the pressing process.

CONCLUSIONS

The reciprocal slope transformation method and tangential curve method were used for mathematical description of Jatropha curcas L. bulk oilseeds. From the statistical analysis, it is evident that both models can be significantly used for description of mechanical behaviour of Jatropha bulk seeds under compression loading as well as for deformation energy determination. It was determined that reciprocal slope transformation method describes precisely the beginning of pressing process and it can cover the reorganizing process in the bulk seeds. From the conducted study it follows that tangent curve method is suitable for description of mechanical behaviour at bulk deformations in which the reorganization process in the bulk seeds is finished yet. A suitable determination of coefficients of these two mathematical models can be used for a more accurate description of deformation characteristics in individual pressing regions. Both methods consider bulk oilseeds as a unit which is important from technological and economical point of view.

REFERENCES


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