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Utilization of the elementary mathematical model for description of mechanical behaviour of composites reinforced by Ensete Ventricosium fibres

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Abstract. This article is focused on the utilization of elementary mathematical model for description of mechanical behaviour of composites materials reinforced by fibres of Ensete Ventricosium under tension loading. Elementary mathematical model was derived for unidirectional fibres oriented in the direction of tension loading and it was experimentally verified. As a matrix it was used a two-component resin Gluepox Rapid and as a reinforcement they were used fibres of Ensete Ventricosum. Experimental samples with different volume fibres ratio contained 40, 60, 80, 100, 120 and 140 fibres were tested on tensile equipment MP Test – 5.050. In this study the elementary mathematical model was utilized for description of dependency between modulus of elasticity, rupture stress and volume fibres ratio. From this research follows that data determined from derived elementary mathematical model are significant (on the level of significance 0.05) with experimentally determined data. This derived elementary mathematical model can be used as background for further research related to the modelling of mechanical behaviour of composites reinforced by fibres.

Key words: modulus of elasticity, stress at rupture, natural material.

INTRODUCTION

Natural fibres such as flax, sisal, coconut fibres as well as banana fibres were used in a historical age. Products from these natural fibres were of wide range of use from clothes to a roof of a house. Nowadays these fibres have been evaluated as ecological materials because of their biological dissolubility and renewability (Müller, 2015). Except for this lignocellulose fibres are CO_2 emission neutral. Except for plants which are grown for the production of fibres the fibre of other plants is of secondary or no commercial use. This is a case of a banana plant Ensete Ventricosium which is grown like food of inhabitants. Fibres with very good mechanical properties can be gained from remaining leaves and a bark of the banana plant (Mizera et al., 2015; Müller, 2016).

A modern technology uses the composites various ways including bearing constructions which a resistance to static, dynamic and fatigue loadings. Numerical modelling can help to interpret results gained from common portable equipment's which have been nowadays used for metals. Modifications of these methods can be used for composites (Atuanya et al., 2011).

A current research is focused on the numerical modelling of the problem through hybrid rack to an irregular quadrilateral rack (Magomedov & Kholodov, 1988; Chelnokov, 2006; Beklemysheva et al., 2015). This method is based on characteristic properties of elastically deformable bodies as a set of equations and models for spreading of a reflection and a quarry among areas including their mutual transformation on various boundaries and contact types. The method was verified at various problematic examples, in a comparison with experiments in various scientific branches including a seismology (Petrov et al., 2011; Petrov & Kvasov, 2012), materials science (Beklemysheva et al., 2014; Petrov et al., 2014) and a biomechanics (Agapov et al., 2006; Beklemysheva et al., 2015). One-way carbon fibres of the polymeric composite are modelled as orthotropic medium by means of one direction, namely according to the fibres (Petrov et al., 2014).

The numerical modelling was performed in publications (Daynes et al., 2008; Daynes et al., 2010¸ Petrov et al., 2014). Most of analyses according to theoretical and numerical methods were performed only in limited cases. A primary mistake at a derivation of original loading is stated in the published theoretical study which led to a wrong form of equations (Mostafa et al., 2016). However, most of previous studies used one-way composites in their analyses. Only the paper of Potluri & Thammandra (2007) was focused on the numerical modeling of 2-D fabrics of composites exposed to one-axis and two-axis loading of fibres in a molding material (Kejval & Müller 2013).

By now no publication was dealing with the mathematical description of composites reinforced by Ensete fibres. Ensete fibres have a good mechanical behaviour (Mizera et al., 2016). From already published studies is evident that Ensete ventricosum fiber has very similar rupture stress as other fibers produced from other materials such as vakka, coconut, abaca or sisal (Munawar et al., 2006; Rao & Rao 2007).

This paper is focused on the elementary mathematical model for the description of the mechanical behaviour of composite materials reinforced with Ensete Ventricosium fibres. In order to the elementary mathematical model, mechanical test of composites were performed.

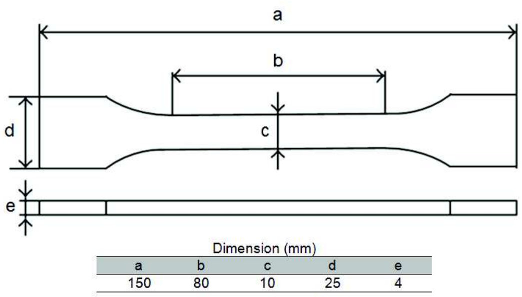
MATERIALS AND METHODS

Materials

As reinforcement of composite material the fibres from the plant *Ensete ventricosum*, obtained from Hawassa region, Ethiopia were used. The moisture content $M_c = 9.12 \pm 1.81\%$ *(w. b.)* of the fibres was determined using standard oven method, ASAE method (ASAE S410.1 DEC97, ASAE, 1998). Samples of *100 g* mass from a batch of Ensete fibres were randomly selected for the moisture content determination. The mass of each sample m_s *(g)* was determined using an electronic balance (Kern 440–35, Kern & Sohn GmbH, Balingen, Germany). The average diameter of fibres $D_f = 205 \pm 17\ \mu m$ was determined by using optical microscope (Zeiss Jenavert, Carl Zeiss, Jena, Germany). The average tension rupture stress of Ensete fibres after 20 tests was 385.12 ± 26.69 MPa. The epoxy resin GlueEpox Rapid was used as a matrix. It is a two-component resin prepared from a bisphenol (A) and epichlorhydrin (B).

Composite preparation

The composite material was created by pre-mixture containing of matrix and added fibres. The matrix was prepared by mixing of the parts A and B in ratio 100:45 (w/w) at room temperature. The final fluid matrix was poured in forms, which were prepared from a material Lukapren N, according to the standards. Into the liquid matrix were inserted the fibres in one direction (Fig. 2). The mixture was then left to cure for 24 h. Finally, the samples were removed from the forms and stored for posterior tests. The dimensions of samples (obtained from forms) are shown in Fig. 1. Composite samples were prepared with a different number of fibres in a sample 40, 60, 80, 100, 120 and 140, which corresponds to the percentage volume of 3.8, 5.8, 7.8, 9.8, 11.8 and 13.8%, respectively. For comparison of measured values of composite materials the pure matrix without fibres were created.



Dimension (mm)

a	b	c	d	e
150	80	10	25	4

Figure 1. Test sample – Tensile strength (CSN EN ISO 3167, 2004).

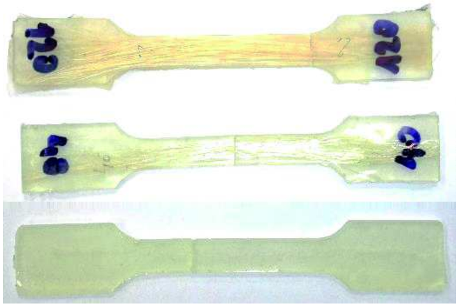


Figure 2. Samples of composite materials.

Laboratory tests

To determine the relationship between tension force and deformation, a device (Labortech, MPTest 5.050, Czech Republic) was used to record the course of deformation function. The tensile test was performed according to (ČSN EN ISO 527-2, 2012). A deformation speed at the tensile test was 10 mm min^{-1}. Determined amounts of tension force and deformation were transformed into stress and strain using Eqs. 2 & 3 respectively.

$$\sigma = \frac{F}{S} \tag{1}$$

where: σ – tensile stress in sample, MPa; F – tensile force, N; S – appropriate cross section area of sample after tensile test, mm^2,

$$\varepsilon = \frac{x}{L_0} \tag{2}$$

where: ε – strain, - ; ε – elongation of sample, mm; L_0 – gauge length, mm.

From measured values was determined the modulus of elasticity by Eq. 3.

$$E = \frac{\sigma}{\varepsilon} \tag{3}$$

where: E – modulus of elasticity, MPa; σ – tensile stress in sample, MPa; ε – strain, -.

Model determination

Simple model of composites assembled from two parts, matrix and fibres, (Fig. 3) was used for model derivation.

Basic assumption of this model is that strain of composite is equal to the strain of matrix as to the strain of fibres and that internal forces acting between the matrix and the fibres were neglected. Therefore it follows that force acting on composites can be described by Eq. 4.

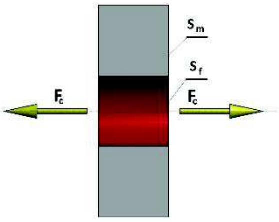


Figure 3. The basic model of composite material.

$$F_c = F_m + F_f \tag{4}$$

Where F_m (N) is force acting on matrix and F_f (N) is force acting on fibres. Previous formula (Eq. 4) can be expressed using Hooke's law as Eq. 5.

$$E_c \times (S_m + S_f) \times \varepsilon = E_m \times S_m \times \varepsilon + E_f \times S_f \times \varepsilon \tag{5}$$

Where E_c (MPa) is modulus of elasticity of composite, E_c (MPa) is modulus of elasticity of matrix, E_f (MPa) is modulus of elasticity of fibres, S_m (mm^2) is cross section area of matrix, S_f (mm^2) is cross section area of fibres and ε (-) is strain of composite. From Eq. 5 it can be expressed formula for modulus of elasticity of composites (Eq. 6) and this equation can be simplified with aid of Eq. 7 into general formula (Eq. 8).

$$E_c = E_m \times \frac{S_m}{(S_m + S_f)} + E_f \times \frac{S_f}{(S_m + S_f)} \tag{6}$$

$$\alpha = \frac{S_f}{S_m} \tag{7}$$

Where α (-) is fibre volume ratio.

$$E_c = E_m \times \frac{1}{1 + \alpha} + E_f \times \frac{\alpha}{1 + \alpha} \tag{8}$$

Strain of composites can be described by Eq. 9, where σ_m (MPa) is tension stress in matrix and σ_f (MPa) is tension stress in fibres.

$$\varepsilon = \frac{\sigma_m}{E_m} = \frac{\sigma_f}{E_f} = \frac{F_m}{S_m \times E_m} = \frac{F_f}{S_f \times E_f} \tag{9}$$

Eq. 10 can be derived using Eq. 9 into Eq. 4.

$$F_c = F_m + F_m \times \frac{S_f \times E_f}{S_m \times E_m} \tag{10}$$

Eq. 10 was divided by cross section area of composites (S$S_m + S_f$) (mm^2) and thus it was transformed in stress form (Eq. 11)

$$\sigma_c = \left(\sigma_m + \sigma_m \times \frac{S_f \times E_f}{S_m \times E_m}\right) \times \frac{S_m}{S_m + S_f} \tag{11}$$

With aid of Eq. 12, where β (-) is modulus of elasticity ratio, the general formula for description tension stress of composites can be derived (Eq. 13).

$$\beta = \frac{E_f}{E_m} \tag{12}$$

$$\sigma_c = \sigma_m \times \frac{1+\alpha\times\beta}{1+\alpha} \tag{13}$$

RESULTS AND DISCUSSION

Measured and determined values of composite materials are shown in Table 1.

Table 1. Measured and determined values of samples

Number of fibres (-)	Fibre volume ratio (-)	Composite cross section area (mm^2)	Rupture force (N)	Rupture strain (-)	Rupture stress (MPa)	Modulus of elasticity (MPa)
40	0.038 ± 0.004	33.23 ± 3.02	1,482 ± 107	0.031 ± 0.002	44.6 ± 6.2	1,439 ± 103
60	0.058 ± 0.006	23.89 ± 3.94	1,075 ± 85	0.028 ± 0.004	45.0 ± 7.3	1,607 ± 115
80	0.078 ± 0.008	36.75 ± 3.93	1,687 ± 56	0.029 ± 0.005	45.9 ± 7.0	1,583 ± 106
100	0.098 ± 0.010	24.36 ± 3.44	1,196 ± 90	0.026 ± 0.004	49.1 ± 6.9	1,888 ± 114
120	0.118 ± 0.012	33.99 ± 2.59	1,893 ± 80	0.030 ± 0.004	55.7 ± 6.1	1,857 ± 137
140	0.138 ± 0.014	38.27 ± 2.20	2,216 ± 95	0.029 ± 0.003	57.9 ± 6.6	1,997 ± 145

Measured values of modulus of elasticity and obtained values from model are shown in Fig. 4. High modulus of elasticity 1997 ± 145 MPa was achieved in the composite material at the maximum number of fibres 140 pcs. Individual samples of the composite material were compared with the pure epoxy resin without fibres. From the Fig. 4 is evident that the modulus of elasticity increased continuously with increase the number of fibres. For statistical comparison of measured and obtained values the statistical analysis ANOVA was used.

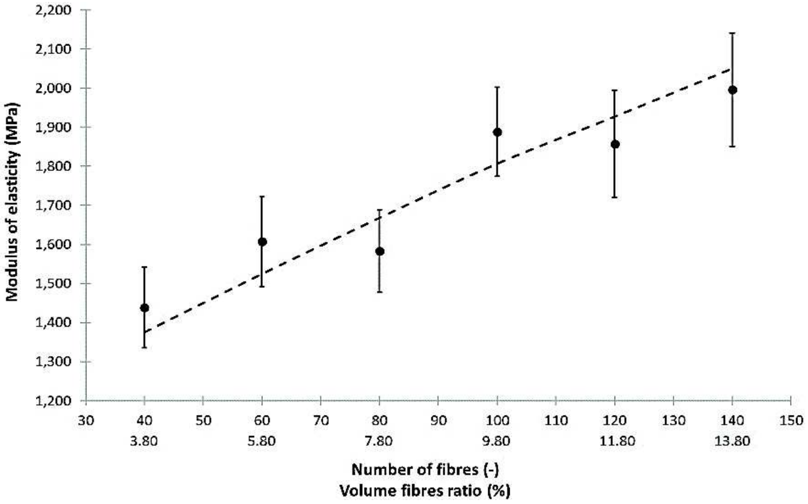


Figure 4. Effect of number of fibres in composite material on modulus of elasticity.

Statistical analysis ANOVA (Table 2) shows that the measured values of modulus of elasticity and the results from the general model (Eq. 8) were statistically significant at significance level 0.05, that is, the values of F_{crit} (critical value comparing a pair of models) were higher than the F_{rat} values (value of the F – test) for all the measured composites and values of P_{value} (significance level at which it can be rejected the hypothesis of equality of models) (Table 2) were higher than 0.05 which is also confirmed by very high coefficients of determination R^2.

Table 2. Statistical analysis of general models

	F_{rat} (-)	P_v (-)	F_{crit} (-)	R^2 (-)
Modulus of elasticity	0.005	0.945	5.32	0.91
Rupture stress	0.489	0.504	5.32	0.86

F_{rat} – value of the F test, *F_{crit}* – critical value that compares a pair of models, *P_{value}* – hypothesis of the study outcomes significant level, *R^2* – coefficient of determination.

Measured and determined values of tension stress at rupture are shown in Fig. 5. The obtained results from general model of tension stress of composite material (Eq. 13) are shown in Fig. 5. The highest stress was obtained at 57.9 ± 6.6 MPa. From the Fig. 5 it is clear that increased number of fibres increased the stress at rupture. Statistical analysis of results is shown in Table 2. From the statistical analysis (Table 2) is evident that the measured values of rupture and the results from the general model (Eq. 13) were statistically significant at significance level 0.05.

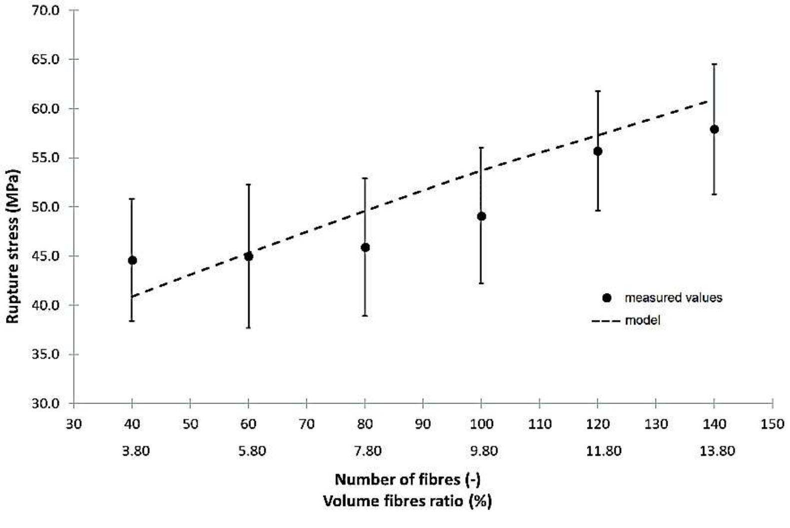


Figure 5. Effect of number of fibres in composite material on stress at rupture.

Modelling of behaviour of composite materials researched also authors Mostafa et al., 2016. They examined the tensile tests of composite materials reinforced by glass fibres. The effect of addition of Ensete fibres into two-component resin on mechanical behaviour of composite material described authors Mizera et al., 2015. Ensete fibres have good mechanical properties and that is why they can have great potential for use in composite materials. By adding various natural fillers in composite materials, can modify their mechanical properties (Müller et al., 2015). Authors Mostafa et al., 2016 also published a mathematical model for synthetic fibres with preloading.

Modelling of mechanical behaviour of composites brings new knowledge which can be used in construction by using of natural materials.

This material can be used in the automotive industry.

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CONCLUSIONS

The aims of this study were to create basic mathematical models describing the dependency between number of Ensete fibres in composite material and modulus of elasticity and stress at rupture. It was found as follows:

- an increasing number of Ensete fibres in composite increases the modulus of elasticity and stress at rupture,
- the general models describing effect the number of Ensete fibres in composite material on modulus of elasticity and on stress at rupture were determined and also statistically confirmed,
- in this study determined models can help to design further models which will describe the mechanical behaviour of composite materials reinforced by natural fibres.

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REFERENCES

Agapov, P.I., Belotserkovskii, O.M. & Petrov, I.B. 2006. Numerical modeling of the effects of mechanical effects on the human brain in cases of head injury (Chislennoe modelirovanie posledstvii mekhanicheskogo vozdeistviia na mozg cheloveka při cherepno-mozgovoi travme=. *Zhurnal vychislitelnoi matematiki i matematicheskoi fiziki* **46**, 1711–1720.

Atuanya, C.U., Ibhadode, A.O.A. & Igboanugo, A.C. 2011. Potential of using recycled low-density polyethylene in wood composites board. *African Journal of Environmental Science and Technology* **5**, 389–396.

Beklemysheva, K.A., Petrov, I.B. & Favorskaya, A.V. 2014. Numerical simulation of processes in solid deformable media in the presence of dynamic contacts using the grid-characteristic method. *Mathematical Models and Computer Simulations* **4**, 294–304.

Beklemysheva, K.A., Danilov, A.A., Petrov, I.B., Vassilevsky, Y.V. & Vasyukov, A.V. 2015. Virtual blunt injury of human thorax: Age-dependent response of vascular system. *Russian Journal Of Numerical Analysis And Mathematical Modelling* **5**, 259–268.

Daynes, S., Potter, K.D. & Weaver, P.M. 2008. Bistable prestressed buckled laminates. *Composites Science and Technology* **68**, 3431–3437.

Daynes, S., Diaconu, C.G., Potter, K.D. & Weaver, P.M. 2010. Bistable prestressed symmetric laminates. *Journal of Composite Materials* **44**, 1119–1137.

Chelnokov, F.B. 2006. Iavnoe predstavlenie setochno-kharakteristicheskikh skhem dlia uravenii uprugosti v dvumernom i trekhmernom prostranstvakh. *Matematicheskoe modelirovanie* **18**, 96–108.

Kejval, J. & Müller, M. 2013. Mechanical properties of multi-component polymeric composite with particles of AL2O3/SIC. *Scientia Agriculturae Bohemica* **44**, 237–242.

Magomedov, K.M. & Kholodov, A.S. 1988. Setochno-kharakteristicheskie chislenye metody. *M.: Nauka*. 1988.

Mizera, Č., Herák, D., Müller, M. & Hrabě, P. 2015. Mechanical behaviour of polymeric composite with fibres of false banana (Ensete ventricosum). *Agronomy Research* **13**, 680–689.

Mizera, Č., Herak, P., Hrabe, P., Muller, P. & Kabutey, A. 2016. Mechanical Behavior of Ensete ventricosum Fiber Under Tension Loading. *Journal of Natural Fibers* **14**, 287–296.

Mostafa, H.N., Ismarrubie, N.Z., Sapuan, M.S. & Sultan, H.T.M. 2016. Fibre prestressed polymer-matrix composites: a review. *Journal of Composite Materials* **51**, 39–66.

Müller, M. 2015. Hybrid composite materials on basis of reactoplastic matrix reinforced with textile fibres from process of tyres recyclation. *Agronomy Research* **13**, 700–708.

Müller, M., Cidlina, J., Dědičová, K. & Krofová, A. 2015. Mechanical properties of polymeric composite based on aluminium microparticles. *Manufacturing Technology* **15**, 624–628.

Müller, M. 2016. Mechanical properties of composite material reinforced with textile waste from the process of tyres recycling. *Research in Agricultural Engineering* **62**, 99–105.

Munawar, S.S., Umenura, S. & Kawai, S. 2006. Characterization of the morphological, physical and mechanical properties of seven nonwood plant fiber bundles. *Journal of Wood Science* **53**, 108–113.

Petrov, I.B., Kvasov, I.E. & Pankratov, S.A. 2011. Numerical simulation of seismic responses in multilayer geologic media by the grid-characteristic method. *Mathematical Models and Computer Simulations* **3**, 196–204.

Petrov, I.B. & Kvasov, I.E. 2012. High-performance computer simulation of wave processes in geological media in seismic exploration. *Computational Mathematics and Mathematical Physics* **52**, 302–313.

Petrov, I.B., Vasyukov, A.V., Beklemysheva, K.A., Ermakov, A.S., Dzuba, A.S. & Golovan, V.I., 2014. Chislennoye modelirovalie dynamicheskih processov pri nizkoskorostnom udare po kompozitnoy stringernoi paneli. *Matematicheskoye modelirovanie* **4**, 96–110.

Petrov, I.B., Favorskaya, A.V., Vasyukov, A.V., Ermakov, A.S., Beklemysheva, K.A., Kazakov, A.O. & Novikov, A.V. 2014. Numerical simulation of wave propagation in anisotropic media. *Doklady Mathematics* **4**, 778–780.

Potluri, P. & Thammandra, V.S. 2007. Influence of uniaxial and biaxial tension on mesoscale geometry and strain fields in a woven composite. *Composite Structures* **7**, 405–418.

Rao, K.M.M. & Rao, K.M. 2007. Extraction and tensile properties of natural fibres: Vakka, date and bamboo. *Composite Structures* **77**, 288–295.