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Determination of the mass diffusion coefficient of wood by thin-layer drying kinetics

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Abstract. The aim of this paper is to present the method of using thin layer drying under controlled conditions, to obtain variable moisture diffusion coefficient expression of wood drying. The obtained relationship parameters can then be used for a larger sample drying process simulation. This paper includes the theoretical study of changes of moisture content determination in wood in response to high temperature of the drying air (105 °C). A 1-D diffusion model with a variable concentration-dependent diffusion coefficient is considered. This problem is solved, using the differential scheme. Paper described theory and experimental results of wood drying by the high temperature at 105 °C. For studies have selected five types of wood: oak (*Quercus robur*), beech (*Fagus silvatica*), spruce (*Picea abies*), scots pine (*Pinus silvestris*), and larch (*Larix decidua*). Experimental measurements and modelling results are given.

Key words: wood pieces, oven, natural convection, mass diffusion coefficient.

INTRODUCTION

The aim of this paper is to present the methodology of determination of the mass diffusion coefficient of wood by thin-layer drying kinetics and show some results of this measurement. Using a wood measurement of a small wood sample in drying process, it is possible to obtain information on the diffusion coefficient depending on the moisture concentration in the sample. Wood heat treatment is a compulsory technological process for gluing materials, plywood, building materials and so on. This process is energy-intensive, complex and long-lasting, and therefore it is necessary to scientifically justify it. The usual drying methods are based on low-temperature convective drying and the drying process lasts long time (Simpson, 1983). To reduce the drying time without decreasing the quality of wood, the drying temperature of product is above the boiling point of water. This paper describes the experiments and experimental results of wood drying at 105 °C.

This research is focused on the most important wood used as a timber or in the industry in European conditions. The tested samples are made from spruce (*Picea abies*), scots pine (*Pinus silvestris*), larch (*Larix decidua*), oak (*Quercus robur*) and beech (*Fagus silvatica*).

There are some research studies that are limited to 'catching' the drying curves (Hua et al., 2016; Anisimov et al., 2017). One of the basic drying parameters is the diffusion coefficient. Knowing its value and using Fick's second law is possible to predict moisture concentration change in timeand get the moisture distribution in the sample during drying. The several fundamental multiphase models has been compiler and shown how these models can be applied to the wood drying process (Ciegis & Starikovicius, 2002). The mass transfer process is studied in four widely used Central African tropical woods using a climatic chamber permanently maintained at 59% air relative humidity (Simo et al., 2016). This study shows that diffusion coefficient correlate with the density of wood by an exponential function.

Mass diffusion coefficients for pine, oak, spruce, beech and larch are calculated at initial samples moisture 14% wet basis (Aboltins et al., 2017a) drying at a temperature105 °C. These results show, that average diffusion coefficients in first 10 drying hours are higher more that 20% than average diffusion coefficients in all drying time - 24 hours. For example, for spruce this difference reaches 21%, for larch 25% and oak only 16%.These results show that water diffusion coefficient depends on the water concentration in wood sample.

The main objective of this article was to using thin-layer drying kinetics of wood for determination of coefficient of diffusion. This paper includes the theoretical study of changes of moisture content determination in wood in response to high temperature of the drying air. The aim of this research was to investigate theoretical background of wood drying by high temperatures and determination of changed diffusion coefficients.

MATERIALS AND METHODS

The tested samples are made from five types of wood: oak (*Quercus robur*), beech (*Fagus silvatica*), spruce (Picea abies), scots pine (*Pinus silvestris*), and larch (*Larix decidua*).

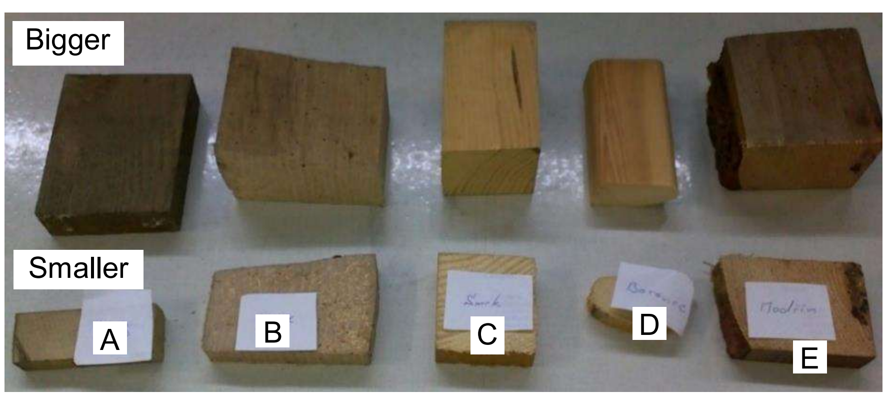

Figure 1. Bigger and smaller samples of wood used for drying measurement. Bigger samples on upper row and smaller samples on bottom row: A – oak (*Quercus robur*); B – beech (*Fagus silvatica*); C – spruce (*Picea abies*); D – scots pine (*Pinus silvestris*); E – larch (*Larix decidua*).

To be able to study different way of drying up the water from the wood, two samples of each type of wood were examined (larger and smaller length) (Fig. 1). Small samples are cut from the large sample perpendicular to the fibres direction. The wood density depends not only on moisture content of the wood but also on growth conditions. The exact dimensions and properties of tested samples are measured and presented in Tables 1 and 2. The length of the sample is measured in the direction of the fibres.

Table 1. The characteristics of bigger wood samples used for drying measurements

Type of wood	Weight (g)	Density (kg m^{-3})	Sample profile (mm x mm)	Length (mm)
Oak	229.35	725	93 x 33	103
Beech	367.18	720	100 x 60	85
Spruce	190.96	450	67 x 68	93
Pine	100.00	535	55 x 34	100
Larch	295.21	590	88 x 65	88

Table 2. The characteristics of smaller wood samples used for drying measurements

Type of wood	Weight (g)	Density (kg m^{-3})	Sample profile (mm x mm)	Length (mm)
Oak	46.33	725	93 x 33	21
Beech	110.99	720	100 x 60	25
Spruce	48.90	450	67 x 68	24
Pine	18.39	535	55 x 34	18
Larch	79.17	590	88 x 65	23

The determination of wood moisture was carried out by gravimetric method, that is direct method and the results are very accurate. There was used for drying of samples of wood the electric oven Memmert UNB with automatic control of temperature and natural flow of air inside the chamber. Samples were weighed on the digital laboratory balance KERN-440-35N with maximum load weight 400 g and with resolution 0.01 g. The experiments lasted 30 hours, measurements were made every 1.25 hours. No charges in samples weight after 24 hours were observed. This indicates that the water in the samples has been removed. Wood moisture is determined on a wet basis.

Using the experimental data with natural convective drying method in the laboratory conditions under high temperature 105 °C (near the boiling point of water) there were calculated the theoretical drying coefficients, useful for description and modelling of the drying process, calculated theoretical results of moisture removal and compared with experimental results obtained from the measurements. The results of drying of different samples dimensions and small mass of wood pieces are compared. The obtained results of this research are parameters, which can be used for the future research work and for improvement of the whole drying process. This destructive method of measurement can be also used for laboratory control of another method of measurement, e.g. non-destructive sensor tests, based on other physical principles (capacitive or resistive) sensors (Papez & Kic, 2013).

Mathematical model with changing coefficient of diffusion

In order to determinate the effective moisture diffusion in solid wood is used mass maintenance law usually presented in the following form:

$$\frac{\partial \tilde{c}}{\partial t} = div(D grad \tilde{c}) \tag{1}$$

D – coefficient of diffusion ($m^2 s^{-1}$), $\tilde{c}(x,y,z,t)$ – concentration of moisture in wood in wet basis (g $100^{-1}$$g^{-1}$), x,y,z – space coordinates (m), t – time (s).

Since the diffusion of vapours in wood fibre direction is several times greater than in radial and tangential ones (Aboltins et al., 1999) and surface of fibre direction of samples is greater that surfaces of radial and tangential directions, we choose 1-dimentional model with $D_x$in fibre direction (diffusion in a plane sheet $\tilde{c}(x, y, z, t) \approx c(x, t)$.

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(D_x \frac{\partial c}{\partial x}\right) \tag{2}$$

There is case, where diffusion occurs through all surfaces of samples and is possible to assume that the diffusion coefficient D_x depending from concentration c i.e. $D_x = D(c)$. For modelling assumed that in the moment $t = 0$ concentration of moisture in sample of wood is constant c_s. The water vapour concentration on surfaces is constant $c(x, t) = 0$ during drying time. The diffusion process in this case can considered as symmetrical situation and get mathematical problem:

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(D(c) \frac{\partial c}{\partial x}\right) \quad -l < x < l, \qquad t > 0 \tag{3}$$

$$c|_{t=0} = c_s \tag{4}$$

$$c|_{x=-l} = \ c|_{x=l} = 0 \tag{5}$$

where $2l$ – islength of sample in x direction.

Choose the simplest and widely used for small concentrations linear (Fox et al., 1968; Rowland, 1984) relationship $D(c) = a + b \cdot c(x, t)$ and mathematical problem (3)–(5) transforms as (6)–(8):

$$\frac{\partial c}{\partial t} = a \frac{\partial^2 c}{\partial x^2} + b \frac{\partial}{\partial x}\left(c \frac{\partial c}{\partial x}\right) \quad -l < x < l, \qquad t > 0 \tag{6}$$

$$c|_{t=0} = c_s \tag{7}$$

$$c|_{x=-l} = \ c|_{x=l} = 0 \tag{8}$$

Analytical solution to the problem (6)–(8) does not exist. Moreover, not know the coefficients a and b of expression of $D(c)$. These expression's coefficients a and b we define from experimental data. For solving (6)–(8) we can use difference schemes (Samarskii 1988; Aboltins & Morozovs, 2003).

If take condition (5) as $c|_{x=-l} = c|_{x=l} = c_0$ and use substitution $u(x, t) = c(x, t) - c_0$ is obtained Cauchy problem (6')–(8')

$$\frac{\partial u}{\partial t} = (a - b \cdot c_0) \frac{\partial^2 u}{\partial x^2} + b \frac{\partial}{\partial x}\left(u \frac{\partial u}{\partial x}\right) \qquad -l < x < l, \qquad t > 0 \tag{6'}$$

$$u|_{t=0} = c_s - \ c_0 \tag{7'}$$

$$u|_{x=-l} = u|_{x=l} = 0 \tag{8'}$$

As shown the systems (6)–(8) and (6')–(8') differ only with constant coefficients.

RESULTS AND DISCUSSION

For solving (6) – (8) we discretize the region of space-time where we want to obtain a solution. For discretization and numerical calculation have been taken domain $\Omega = \big((x,t)\colon 0 \le x \le 2l,\ 0 \le t \le T\big)$. We have put a bound on time T because in practise we only solve a problem up until a finite time. Discretizing means defining a lattice of points in this space-time region by

$$x_i = ih, \qquad t_j = j\tau, \quad i = 0,1,\dots,N, \qquad j = 0,1,\dots,K,$$

where the fixed numbers h and τ are the spatial and temporal step sizes, respectively. Here, $h=2l/N$ and $\tau=1/K$. The integer N is the number of subintervals in $0 \le x \le 2l$ and K is the number of time steps to be taken. Using forward difference approximation for the first and second derivates (Samarskii 1988; Aboltins & Morozovs, 2003) Cauchy problem can solve as difference problem (9)–(11):

$$\frac{c_i^{j+1} - c_i^j}{\tau} = a\frac{c_{i-1}^j - 2c_i^j + c_{i+1}^j}{h^2} + b\frac{c_{i+1}^j \frac{\partial c}{\partial x} - c_{i-1}^j \frac{\partial c}{\partial x}}{2h}, \tag{9}$$

$$i = 1,2,\dots,N-1; \qquad j = 0,1,\dots,K$$

$$c_i^0 = c_s, \qquad i = 0,1,2,\dots,N \tag{10}$$

$$c_0^j = c_N^j, \qquad j = 1,2,\dots,K \tag{11}$$

Solve c_i^{j+1} from (9),

$$\begin{aligned} c_i^{j+1} &= c_i^j + \frac{a\tau}{h^2}\left(c_{i-1}^j - 2c_i^j + c_{i+1}^j\right) + \\ &+ \frac{b\tau}{2h^2}\left(\left(c_{i+1}^j\right)^2 - c_i^j\left(c_{i-1}^j + c_{i+1}^j\right) + \left(c_{i-1}^j\right)^2\right), \end{aligned} \tag{12}$$

$$i = 1,2,\dots,N-1; \qquad j = 0,1,2,\dots,K$$

The difference Eq. (12) gives the approximate solution at the node (x_i, t_{j+1}) in terms of approximations at three earlier nodes.

To solve the problem (10) – (12) need to determine the values a and b for each tree type. These values can be determined from experimental data of small samples of each type of wood. This determination methodology is similar to the drying rate determination (Aboltins, 2013). Assume that $D_x = const$. If M_t denotes the amount of diffusing moisture which has come out from the material at time t, and M_∞ the corresponding quantity after infinite time, then (Crank, 1956):

$$\frac{M_t}{M_\infty} = 1 - \sum_{n=0}^{\infty} \frac{8}{(2n+1)^2\pi^2} \exp\left(-\frac{D_x(2n+1)^2\pi^2 t}{4l^2}\right) \tag{13}$$

The first we must estimate D_x. Looking at the series (13), it converges very fast. The first member ($n = 0$) of series (13) is selected for estimation of approximated D_x, taking into account inaccuracy of this assumption at the beginning of drying process (Aboltins et al., 2017b):

$$\frac{8}{\pi^2} \cdot \exp\left(-\frac{D_x \cdot \pi^2 t}{4l^2}\right) = 1 - \frac{M_t}{M_\infty} \tag{14}$$

Since only smaller samples (Fig. 1) are used in the experiment, it is assumed that thickness of the samples has small effect on the diffusion coefficient and it is not taken into account. Right-hand side of the Eq. (14) known (experimental data at time $= t_k$, $k = 1,2,...,k_o$) and is possible to express coefficient of diffusion $D_x = D_x^k$ for each $t = t_k$ and $M_t = M_{t_k}$:

$$D_x = -\frac{4l^2 ln\left(\frac{\pi^2(M_\infty - M_t)}{8 \cdot M_\infty}\right)}{\pi^2 \cdot t} \tag{15}$$

Computed coefficient values corresponding to the literature considered (Shubin, 1990; Simo et al., 2016). The calculation results for four types of wood shown Figs 2–5:

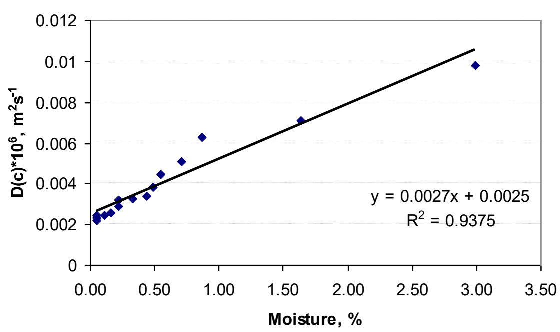

Figure 2. Water diffusion coefficient depending from moisture concentration in sample of pine.

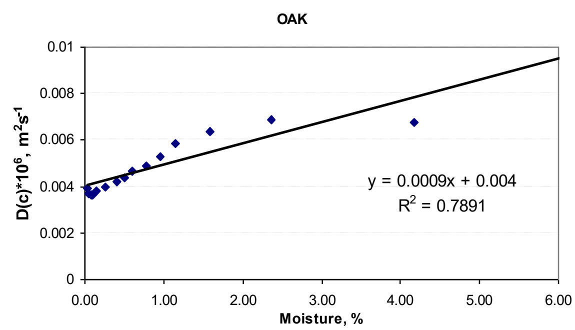

Figure 3. Water diffusion coefficient depending from moisture concentration in oak sample.

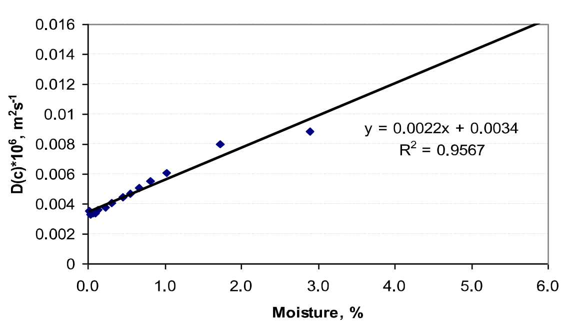

Figure 4. Water diffusion coefficient depending from moisture concentration in larch sample.

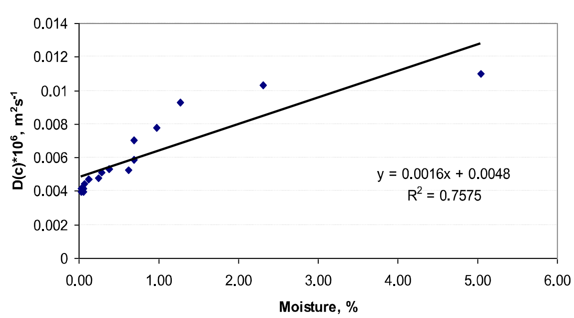

Figure 5. Water diffusion coefficient depending from moisture concentration in spruce sample.

Using the experimental data processing (Figs 2–5) it is possible to determine the expressions $D(c) = a + b \cdot c(x,t)$ constants a and b for each tree species examined. Using problem's (6)–(8) numerical solution (10)–(12) and calculated diffusion coefficients for oak $D(c) = (0.004 + 0.0009 \cdot c(x,t)) \cdot 10^{-6}$ and spruce $D(c) = (0.0048 + 0.0016 \cdot c(x,t)) \cdot 10^{-6}$samples (Fig. 3, Fig. 5) the moisture concentration distribution inside longer samples is possible calculated. Results for 0.1 m length samples are shown at Fig. 6.

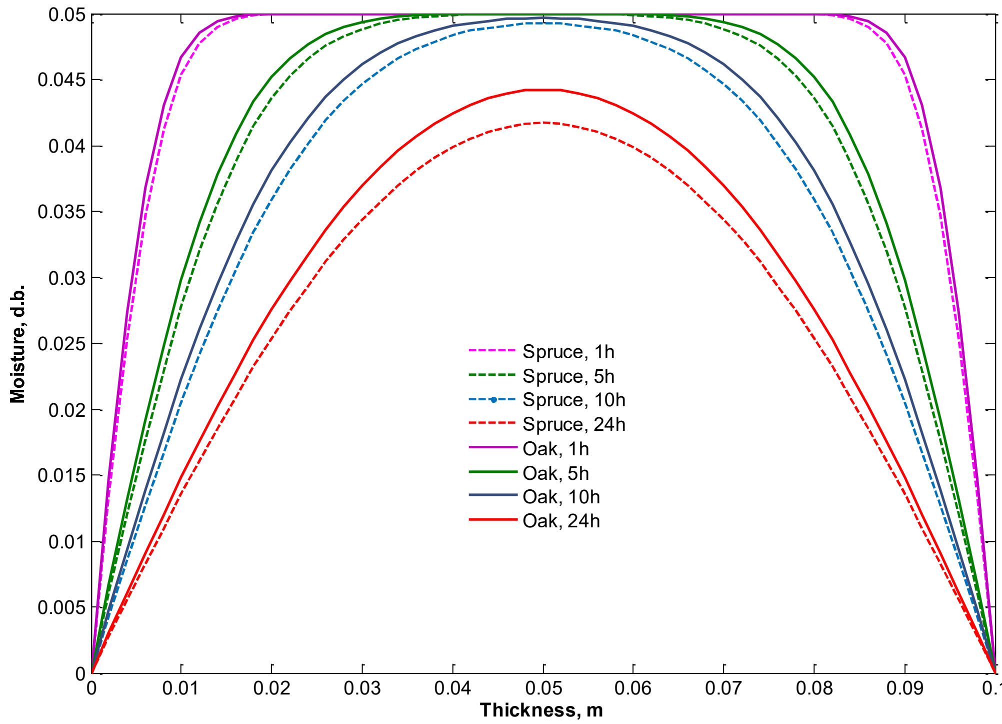

Figure 6. Comparison of moisture content distribution in oak and pine samples at different drying time.

It should be noted that in this case it is considered that the water removal takes place only in the longitudinal direction. Since the drying occurred at 105 °C, it is then assumed, that moisture concentration on surface is zero Eq. (11). Of course, in real situation at lower temperatures, we must take into account equilibrium moisture content and capillarity at higher wood moisture. It will also produce amore complicated mathematical model with more unknown parameters.

The effect of moisture concentration on the diffusion coefficient of spruce wood is almost 2 times higher than of oak wood, which could be explained by different wood densities. As density of oak is higher as spruce wood moisture diffusion goes quicker in spruce. It means the spruce wood dry quicker as oak wood (Fig. 6).

CONCLUSIONS

The proposed methodology, originally used for determination of drying coefficient of wood and can be used to determinate diffusion coefficient depending from concentration.

Using offered difference scheme is possible calculate moisture distribution in solid body at different drying time.

Using the proposed methodology, more accurate results will be achieved if the cross-sectional area / perpendicular to the direction of the capillary / is greater than the lateral surface area, or if the side surfaces are isolated.

It is necessary to realize different measurements in variable wood moisture range in order to specify changes of moisture diffusivity.

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