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Research of the movement of agricultural aggregates using the methods of the movement stability theory

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Abstract. The theory of the movement stability is of crucial practical importance for mobile agricultural machines and machine aggregates, since it determines how qualitative and stable their performance is in a particular technological process. It is especially urgent To ensure stable movement for operation at high speeds of contemporary agricultural aggregates. The aim of this investigation is detailed examination of criteria for the stability assessment of a mechanical system used in agriculture, enabling their wide application in order to study the performance of the system in the case when it is affected by random forces that were not taken into account in the original model. The considered calculation methods and examples of their application make it possible to evaluate the performance of complex dynamic systems without numerical solution of complicated differential equations of the movement in the presence of external disturbances. The considered example of the stability determination of the movement of a trailed cultivator showed that this research method can be successfully used for practical purposes. Besides, a differential equation of disturbed movement has been composed for an actually symmetrical trailed agricultural machine with a particular mass, which moves at a constant forward speed under the impact of summary resistance force running along the symmetry axis of the cultivator and is applied at its centre of gravity. Reduced to normal Cauchy form, this equation was solved on the PC, which made it possible to determine immediately the conditions for stable movement of the trailed cultivator.

Key words: agricultural aggregates, movement stability, theoretical research.

INTRODUCTION

The movement stability is of great practical importance for the agricultural machinery, especially for agricultural aggregates (Schwabik, 1992; Bulgakov et al., 2017; 2018;). It is widely applied in scientific research and in the calculations and design of automatic control systems, navigation instruments, airplanes, spacecraft, various kinds of engines (Matignon, 1996; Zaslavsky & Edelman, 2004).

The concept of stability in a broad sense is interpreted as the ability of an object to maintain its state, not submitting to earlier unforeseen external disturbances. This concept occupies one of the most important places in physics and technology. Depending on the nature of a specific process to be considered, there are also various implementations of this concept. This particularly concerns investigations of the movement stability of a number of mechanical systems for agricultural purposes for which observance of the stability conditions allows one to ensure high-quality execution of technological processes.

In the works by (Zhukovsky, 1948; Merkin et al., 1997), a number of general questions about the movement stability are considered. The foundations of the stability theory are outlined in the work by A.M. Lyapunov ‘A General Problem of the Movement Stability’, published in 1892 (Lyapunov, 1980; Momani & Hadid, 2004). Lyapunov presented a precise definition of the movement stability; he obtained a complete solution for the problem of stable movement; he proposed two methods for the investigation of the movement stability characterised by simplicity and efficiency.

From a physical point of view the equilibrium state is called stable if at sufficiently small initial deviations and velocities during the movement the system does not go beyond the limits of an arbitrarily small environment of the equilibrium state, while having arbitrarily small velocities. An elementary example is the physical pendulum. In the lower vertical position, it has stable equilibrium (after a series of vibrations it returns to its original rest position). In the upper vertical position the physical pendulum occupies an unstable equilibrium: with an arbitrarily small deviation it will move, moving towards a stable lower position.

In our time Lyapunov's methods are deepened; new application areas are emerging, in which general methods are being developed for studying the movement stability of individual broad classes of systems: automatic regulation systems, controlled systems, etc. (Matignon, 1996; Zaslavsky & Edelman, 2004).

The wide application of the theory of automatic control to modern agricultural machines and aggregates determines the creation of research methods ensuring the movement stability of the systems, which is one of the main tasks of this science. Yet, regardless of this, such investigations have not been developed to a sufficient degree in the field of soil tillage mechanics in which the mechanical systems of agricultural machines and machine aggregates should ensure high-quality execution of technological operations.

Consequently, a need arises to solve a very important scientific problem how to expand possibilities for an accurate analytical study of the movement stability of complex multi-mass mechanical systems, such as agricultural machines and aggregates since they are under constant impact of external disturbing influences. Therefore, in-depth consideration of the methods of studying the stability of movement, reduction of the basic assumptions of the classical theory of the movement stability to their specific application for the research of agricultural machines will help to improve further their dynamic, kinematic and design parameters.

Purpose of the research - to determine the criteria for the assessment of the movement stability of agricultural machines and aggregates which will be most efficient for the study of the plane-parallel movement of a trailed cultivator and the oscillations of a self-propelled tool frame in the longitudinal-vertical plane.

MATERIALS AND METHODS

The investigations have been carried out using methods of the theory of the movement stability, the theory of agricultural machines, as well as theoretical mechanics and higher mathematics (Halanay, 1966). There are used methods of the classical theory of the movement stability, based on the methods of constructing, solving and studying the systems of differential equations for the movement of agricultural machines and aggregates. Besides, there are considered differential equations of perturbed movement, and from them, by integrating in a closed form, the movement of the considered agricultural machine or aggregate is evaluated as stable or unstable. There are also used methods for estimating the movement stability without solving the systems of differential equations of the movement when finding the corresponding stability criteria. There are methods applied to linearise the differential equations of perturbed movement, as well a method of constructing special Lyapunov functions, which does not require solving these equations; in this case only the roots of the characteristic equations are investigated.

In order to achieve successful theoretical study of the movement stability of a concrete agricultural machine, it is necessary to consider and specify some general provisions for the stability of mechanical systems (Hale & Verduyn Lunel, 1993).

Sufficient conditions for the equilibrium stability of a system are reflected by the Lagrange – Dirichlet theorem (Malkin, 1996): ‘If in the equilibrium state the potential energy of a holonomic stationary system, being in the field of conservative forces, has an isolated minimum, then this equilibrium state is stable’.

For a conservative system, there is a law of mechanical energy conservation in force:

$$T_0 + \Pi_0 = T + \Pi \tag{1}$$

where T_0, Π_0, T, Π – the kinetic and potential energy in the state of equilibrium and at disturbance. Since always $T \geq 0$, then from expression (1) we have

$$T = T_0 + \Pi_0 - \Pi \geq 0 \tag{2}$$

From where

$$\Pi \leq T_0 + \Pi_0 \tag{3}$$

Inequalities (2) and (3) show that the movement of the system after its deviation from the equilibrium position occurs in the vicinity of the equilibrium position. Increase in the potential energy is limited by inequality (3) so much that it will be one of the values of the potential energy in the vicinity of the equilibrium position. Based on expression (2), we can assume that according to the indicated initial conditions the speeds of all points of the system are limited by the module: when T_0 and Π_0 decreasing to zero, T and Π also approach zero.

The Lagrange-Dirichlet theorem provides only sufficient conditions for the stability of the equilibrium state. The solution of the problem of the equilibrium instability of a conservative system is based on two well-known A.M. Lyapunov’s theorems (Lyapunov, 1980) on the equilibrium instability. The essence of the Lyapunov theorem on equilibrium instability is that instability takes place if:

1) the potential energy does not have a minimum that can be established by the terms of the second order in the layout of the potential energy in the Maclaurin series;

2) the potential energy has a maximum, and this can be established by the terms of the lowest order of smallness included in the Maclaurin series.

As it is known from the course of analytical mechanics, the expression of potential energy for a holonomic stationary system can be obtained in the quadratic form as a function of generalised coordinates:

$$\Pi = \frac{1}{2}\sum_{k=1}^{N}\sum_{j=1}^{N} C_{kj} q_k q_j \tag{4}$$

where C_{kj} – generalised stiffness coefficients (coefficients of the Maclaurin series); $q_1, \dots, q_N$ – generalised coordinates of a mechanical system.

In expression (4) it is taken into account that the generalised coordinates and the potential energy in the equilibrium position are zero $\left(q_j = 0;\ \Pi(0) = 0\right)$. In addition, the generalised forces in the equilibrium position are also equal to zero:

$$\left(\frac{\partial \Pi}{\partial q_1}\right)_0 = \left(\frac{\partial \Pi}{\partial q_2}\right)_0 = 0 \tag{5}$$

Since in the equilibrium position the potential energy is zero $(\Pi(0) = 0)$, then it has a minimum in this position if $\Pi(\bar{q})$ is explicitly a positive function. The sign of a quadratic form is determined by Sylvester's theorem (Malkin, 1996).

For a positive-definite quadratic form it is necessary and sufficient that all the main diagonal minors of the matrix of a quadratic form be positive.

Let us write a matrix of coefficients of expression (4):

$$\begin{vmatrix} C_{11} & C_{12} & C_{13} & \dots & C_{1N} \\ C_{21} & C_{22} & C_{23} & \dots & C_{2N} \\ C_{31} & C_{32} & C_{33} & \dots & C_{3N} \\ \dots & \dots & \dots & \dots & \dots \\ C_{N1} & C_{N2} & C_{N3} & \dots & C_{NN} \end{vmatrix} \tag{6}$$

Let us create the main diagonal minors of the matrix (6):

$$\Delta_1 = C_{11},\quad \Delta_2 = \begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix},\quad \Delta_3 = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}, \dots ,\ \Delta_N = \begin{vmatrix} C_{11} & \dots & C_{1N} \\ \dots & \dots & \dots \\ C_{N1} & \dots & C_{NN} \end{vmatrix}. \tag{7}$$

According to Sylvester's criterion the quadratic form is positive-definite, and hence there will be a minimum of potential energy in the equilibrium position if the main diagonal minors of the coefficient matrix are positive:

$$\Delta_1 > 0,\ \Delta_2 > 0,\ \dots,\ \Delta_N > 0; \tag{8}$$

The movement stability of a mechanical system, for example, a car, an airplane, a projectile, etc., depends on the acting forces and the initial conditions of the movement (coordinates and velocities of the points of the system at the starting moment the movement). Knowing the forces and initial conditions, one can theoretically calculate how the mechanical system will move. A movement that agrees with the calculation is called undisturbed (Samoilenko & Perestyuk, 1995).

Due to certain inaccuracy in the measurement of the initial conditions their actual values, as a rule, differ from the calculated ones. Besides, the mechanical system during its movement may occur under random influences of various forces, which also equivalently change the initial conditions (Schwabik, 1984). Deviation of the initial conditions arising because of a different reason, is called the initial disturbance, and the movement that the mechanical system performs in the presence of disturbances is called a disturbed movement. As a result of the above-mentioned, the following definition can be given: ‘If at sufficiently small initial disturbances any of the characteristics of the movement during the whole time differs little from the value that it should have during the undisturbed movement, then the movement of the system with respect to this characteristic is called stable’. The conditions under which the movement of a mechanical system is stable are called stability criteria. There are such kinds of stability: the stability of the equilibrium position and the stability of the movement.

Next let us consider the problem of the movement stability and give a definition for the stability of a mechanical system (Federson & **S**chwabik, 2006). Suppose that the motion of a mechanical system is described in the Cauchy form by a system of differential equations in the following way:

$$\frac{dy_k}{dt} = Y_k(t,\ y_1,\ y_2,\ ...,\ y_n),\ k = 1,\ 2,\ ...,\ n, \tag{9}$$

where y_k – some parameters that are connected with the movement, for example, coordinates, velocity projections, with the initial conditions at $t = 0$, equal to:

$$y_k(t_0) = y_{k0},\quad k = 1,\ 2,\ ...,\ n. \tag{10}$$

Let a certain solution of system (9) corresponds to some fixed initial conditions (10):

$$y_k = f_k(t),\quad k = 1,\ 2,\ ...,\ n, \tag{11}$$

which describes a predetermined movement, but we may not know this movement because of the impossibility of integration.

Solution (11) that satisfies the initial conditions (10) and describes the predetermined movement is called an undisturbed movement of the mechanical system.

Further let us assign to the initial conditions y_{k0} some small increments $\delta_k, k = 1, 2, ..., n$ behind the module, which are called the initial disturbances. Let the new partial solution of system (9) correspond to the new initial value $y_{k_1} = y_{k_0} + \delta_k$:

$$y_k = \varphi_k(t),\ k = 1,2,...,n. \tag{12}$$

Solution (12) obtained taking into account the initial disturbances δ_k, and the respective movement of the system is called a disturbed movement.

Proceeding from solutions (11) and (12), we define their increments:

$$\delta_{yk} = \varphi_k(t) - f_k(t) = u_k(t),\quad k = 1,2,...,n, \tag{13}$$

which are called variations of the movement parameters.

Let us consider the movement in coordinates $u_1, u_2, ..., u_n$. In the stability theory space $u_1, u_2, ..., u_n$ is called the phase space, the coordinates - the phase coordinates, and their totality, which determines a certain state of the system, which is investigated - the phase of the system, the coordinates u_k are the phase coordinates, and their totality that determines a certain state of the investigated system is the phase of the system.

Any undisturbed movement is represented in the coordinate system $u_1, u_2, ..., u_n$ by a fixed point M_0 (0,...,0) which coincides with the origin of the coordinates (all) $u_k \equiv 0$. Point M_0 is called the equilibrium point of the system. The totality of values $u_1(t), ..., u_n(t)$ at an arbitrary point of time t determines the respective phase state or phase of the system. The geometric interpretation of the change in the phase coordinates determines the phase path L_k of the depicted point M_k in n-dimensional space $u_k$with the origin at point M_0 that corresponds to the origin of coordinates during the undisturbed movement. Proceeding from the above mentioned considerations, we denote the movement stability according to Lyapunov (Lyapunov, 1980).

If to an arbitrarily predetermined positive number ε, however small it may be, a second positive number $\delta = \delta(\varepsilon)$ can be put in correspondence, such that at any initial disturbances:

$$\delta_1 = u_1(t_0),\ \delta_2 = u_2(t_0),\ \ldots, \delta_n = u_n(t_0), \tag{14}$$

which satisfy at inequalities $t = t_0$:

$$|u_1(t_0)| \le \delta,\ |u_2(t_0)| \le \delta,\ \ldots, |u_n(t_0)| \le \delta, \tag{15}$$

for all $t = t_0$ the following inequalities are fulfilled:

$$|u_1(t_0)| < \varepsilon,\ |u_2(t_0)| < \varepsilon,\ \ldots, |u_n(t_0)| < \varepsilon, \tag{16}$$

this undisturbed movement is called stable.

In a flat phase subspace (u_1, u_2) this definition can be given a geometric interpretation (Fig. 1). The phase path L_1 of point M_1 belongs to steady movement.

A separate group of stable movements is formed from asymptotically stable movements which can be defined in this way. If the undisturbed movement of the system is stable and, in addition, any disturbed movement at sufficiently small initial disturbances tends to an undisturbed movement, i.e. if then such an undisturbed movement is called an asymptotically stable movement (path L_3 of point M_3 in Fig. 1).

$$\lim_{t\to\infty} \sum_{k=1}^{n} u_k^2(t) = 0, \tag{17}$$

In expression (17), the sum of squares of the phase coordinates u_k is taken as a measure of deviations of the disturbed movement from the undisturbed one. If the movement parameters of the system do not satisfy this definition, then such a movement is unstable (the phase path L_2 of point M_2 on (Fig. 1).

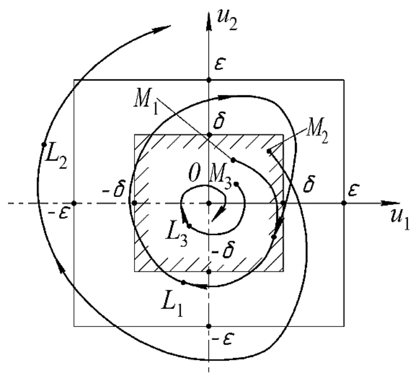

Figure 1. A scheme to the geometric interpretation of the movement stability of a mechanical system.

From the geometrical point of view conditions (17) are understood in the following way: at asymptotic stability the depicted point M of the phase path, without going beyond the boundaries of the radius sphere ε, must approach unlimited to the origin of coordinates 0 (line L_3 of point M_3 in Fig. 1). This means that the physical system, the movement of which is investigated, is trying to return to its original balanced state.

The peculiarities of determining the movement stability according to Lyapunov:

– disturbances are considered small;

– only initial conditions are subject to disturbances, i.e. at a certain point in time an instantaneous change in the movement parameters of the system takes place, after which its disturbed movement occurs under the action of the previous forces;

– the movement stability is studied in an infinite period of time.

In order to investigate the disturbed movement in accordance with its definition in the system of phase coordinates $u_1, u_2, \dots, u_n$, it is appropriate to reduce differential equations (9) to new variables $\delta_{y_k}(t) = u_k(t)$, where $k = 1, \dots, n$. Substituting the parameters of the disturbed movement $\varphi_k = f_k + u_k$ into equation (9), we obtain a new system of equations:

$$\begin{aligned} \dot{u}_k &= Y_k(t, \varphi_1, \dots, \varphi_n) - Y_k(t, f_1, \dots, f_n) = \\ &= Y_k(t, f_1 + u_1, \dots, f_n + u_n) - Y_k(t, f_1, \dots, f_n) = \\ &= U_k(t, u_1, \dots, u_n), \quad k = 1, \dots, n, \end{aligned} \tag{18}$$

In the theory of the movement stability equations (18) are called the differential equations of the disturbed movement.

To each disturbed movement of the investigated object there corresponds a certain partial solution of system (18). It is known that zero values of phase coordinates $u_k(t)$ correspond to any undisturbed movement, i.e. a trivial solution $u_1 = u_2 = \dots = u_n = 0$ of system (18), which it must have. For this purpose it is necessary that functions $U_k(t, u_1, \dots, u_n)$ change into zero at $u_1 = u_2 = \dots = u_n = 0$.

Consequently, investigation of the stability of any undisturbed movement can be reduced to the stability research of a trivial solution of system (18). The physical sense of the system of equations (18) is that it determines the velocity vector of the depicted point M along the phase path L:

$$\bar{u}_M = \{u_1, u_2, \dots, u_n\} = \{U_1, U_2, \dots, U_n\}, \tag{19}$$

Equalities $U_k = U_k(t)$ may be considered as parametric equations of the motion of a point.

A system (18) in which the right-hand parts of equations depend on time $U_k = U_k(t)$ is called a no stationary or non-autonomous one, like the physical system itself the movement of which is described by this system of equations. The corresponding movement of the physical system is unsteady.

However, in many cases, the right-hand parts of the equations of disturbed movement do not explicitly depend on time:

$$\dot{u}_k = U_k(u_1, \dots, u_n), \quad k = 1, 2, \dots, n. \tag{20}$$

System (20) is called stationary or autonomous, and its movement is steady. It is these systems that are discussed further.

Assuming that the right-hand parts of equations (20) are decomposed into a Taylor (Maclaurin) series by powers $u_k(t)$, we write:

$$\dot{u}_k = p_{k1}u_1 + p_{k2}u_2 + \dots + p_{kn}u_n + \overset{*}{U}_k(t, u_1, \dots, u_n), \quad k = 1, 2, \dots, n, \tag{21}$$

where coefficients $p_{ki} = p_{ki}(t) = \left(\frac{\partial U_k}{\partial u_j}\right)_0$ in a general case are functions of time t (for autonomous systems, steady); $\overset{*}{U}_k$ – a totality of all the terms of decomposition of higher orders of smallness (starting from the second one) in relation to U_k.

Neglecting the higher order terms in equations (20), we obtain a linear homogeneous system for a steady movement.

$$\dot{u}_k = p_{k1}u_1 + p_{k2}u_2 + ... + p_{kn}u_n, \quad k = 1, ..., n. \tag{22}$$

RESULTS AND DISCUSSION

Theoretical investigation of disturbed movement of a symmetric trailed agricultural machine

Let us create a differential equation of a disturbed motion of the symmetric trailed agricultural machine (trailed cultivator) with mass *m* moving at a constant forward speed under the impact of the force of total resistance $\bar{R}$, which runs along the axis of symmetry and is applied to the centre of mass *O*. Force $\bar{R}$ coincides with the direction of the traction force of the tractor applied at point $D(x_1, y_1)$ (Fig. 2). The moment of inertia of the cultivator l_0 relative to the centre of mass.

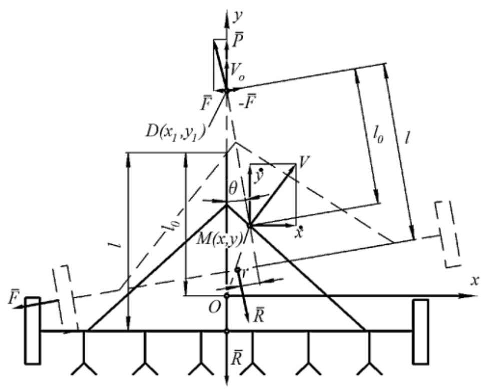

Figure 2. Plane-parallel movement of the trailed symmetric cultivator.

Because of the random lateral forces the total resistance $\bar{R}$ of the cultivator has shifted. As a result of this, pair of forces have arisen under the impact of which the entire aggregate turns counterclockwise. The pair is partly compensated by a reactive pair $(\bar{F}, -\bar{F})$ which arises from the lateral resistance of the wheels and the working parts of the cultivator.

The cultivator is under the impact of the total disturbed moment:

$$M = R \cdot r - F \cdot l, \tag{23}$$

where r – deviation of force $\bar{R}$ from the line of symmetry; L – the arm of the reactive pair $(\bar{F}, -\bar{F})$.

Confining to a small angle θ, which we take for the generalized coordinate, we will assume that:

$$F = R \cdot \tan\theta \approx R \cdot \theta, \tag{24}$$

Therefore, equation (23) will be as follows:

$$M = R\left(r - l \cdot \theta\right). \tag{25}$$

Let us write the equation of the link as distance which always remains preserved between the hitch point $D(x_1, y_1)$ and the centre of mass $M(x_1, y_1)$, L_0 is the distance between the indicated points:

$$(x_1 - x)^2 + (y_1 - y)^2 = l_0^2. \tag{26}$$

Since $x_1 = 0$, $y_1 = V_0 \cdot t + l$, equation (26) will change:

$$x^2 + \left(V_0 \cdot t + l_0 - y\right)^2 = l_0^2. \tag{27}$$

The Cartesian coordinates of the centre of mass, expressed in terms of the generalised coordinate θ, are equal to:

$$x = l_0 \cdot \sin\theta, \quad y = V_0 \cdot t + l_0\left(1 - \cos\theta\right). \tag{28}$$

Taking the time derivative from expression (28), we have:

$$\dot{x} = l_0\, \dot{\theta} \cdot \cos\theta; \; \dot{y}_0 = V_0 + l_0 \cdot \dot{\theta} \cdot \sin\theta. \tag{29}$$

The machine is a system with one degree of freedom, so the Lagrange equation can be written as:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} = Q_\theta, \tag{30}$$

where T – the kinetic energy, Q_θ – a generalised force, $\dot{\theta}$ – generalised speed.

Let us determine the kinetic energy of the machine:

$$T = \frac{1}{2} m \cdot V^2 + \frac{1}{2} I_0 \cdot \dot{\theta}^2 = \frac{1}{2} m\left(\dot{x}^2 + \dot{y}^2\right) + \frac{1}{2} I_0 \cdot \dot{\theta}^2. \tag{31}$$

By substituting expression (29) into (31), we have:

$$T = \frac{1}{2} m\left(l_0^2 \cdot \dot{\theta}^2 + V_0^2 + 2V_0 \cdot l_0 \cdot \dot{\theta} \cdot \sin\theta\right) + \frac{1}{2} I_0 \cdot \dot{\theta}^2. \tag{32}$$

We find the partial derivatives from expression (32). We have:

$$\frac{\partial T}{\partial \dot{\theta}} = \left(m \cdot l_0^2 + I_0\right)\dot{\theta} + m \cdot V_0 \cdot l_0 \cdot \sin\theta, \; \frac{\partial T}{\partial \theta} = m \cdot V_0 \cdot l_0 \cdot \cos\theta \cdot \dot{\theta}. \tag{33}$$

In order to determine the generalised force Q_θ, we write the expression of the elementary work of the applied forces on the possible displacements of the points of the system:

$$\delta A = M \cdot \delta\theta = R\left(r - l \cdot \theta\right)\delta\theta, \tag{34}$$

hence:

$$Q_\theta = R\left(r - l \cdot \theta\right). \tag{35}$$

Substituting all the values that we found into expression (30), we have:

$$\left(m \cdot l_0^2 + I_0\right)\ddot{\theta} = R\left(r - l \cdot \theta\right), \tag{36}$$

or

$$\ddot{\theta} + \lambda^2 \cdot \theta = \lambda^2 \cdot k, \tag{37}$$

Where $\lambda = \sqrt{\dfrac{R \cdot l}{m \cdot l_0^2 + I_0}}$; $k = \dfrac{r}{l}$.

It is equation (37) that is the differential equation for the disturbed movement of the trailed cultivator.

Let us reduce the differential equation (37) to the normal Cauchy form. We have:

$$\dot{x}_1 = x_2; \quad \dot{x}_2 = -\lambda^2 x_1 + \lambda^2 k . \qquad (38)$$

Thus, a differential equation of the disturbed movement of a trailed symmetric cultivator has been compiled, which allows it to be used for further studies of the movement stability, as well as to determine its optimal design and kinematic parameters.

Investigation of the movement stability of the system

Let us further discuss methods for the investigation of the movement stability of the system. If the differential equation of the movement is integrated in a closed form, then the study of the stability movement occurs without complications. Yet such cases are practically very rare. As a rule, Lyapunov's predecessors used the linearisation method of the equations of the movement. Its essence is to replace equations (18) of the system to be investigated by a linear system (21). The solution of the problem was greatly simplified, especially for autonomous systems, the equation of the movement of which is integrated in a closed form, and with constant coefficients $p_{kj} = \alpha_{kj} = \text{const}$ it will look like:

$$\dot{u}_k = a_{k_1} u_1 + a_{k_2} u_2 + ... + a_{k_n} u_n , \quad k = 1, ..., n. \qquad (39)$$

However, such a replacement means substitution of one task with another. Although the research of linearity, or as a first approximation, sometimes solves the problem correctly, in other cases, this method leads to incorrect conclusions. A question arises: What are the conditions for the credibility of the answer obtained on the basis of the study of the movement stability that will be in the first approximation?

For the first time the answer to this question was given by Lyapunov (Lyapunov, 1980). He received a complete solution of the problem for the steady-state and periodic movements, as well as for a wide range of unsteady movements. He also considered some of the main cases when one cannot confine to the first approximation.

Lyapunov divided all methods for studying motion on stability into two categories:

- the first method, which concerns systems the movement of which is described by nonlinear differential equations, is based on the study of linearised equations of disturbed movement or differential equations of the first approximation;

the second (direct) method is connected with the construction of special Lyapunov functions, which have properties on the basis of which it can be concluded that the movement is stable without solving differential equations.

The study of the movement stability in the first approximation. Lyapunov’s theorems. Let us consider a linearised system of the first approximation (39) in an expanded form, replacing $u_k = x_k$:

$$\begin{aligned}
\dot{x}_1 &= \frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n, \\
\dot{x}_2 &= \frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n, \\
&\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\
x_n &= \frac{dx_n}{dt} = a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n.
\end{aligned} \qquad (40)$$

We remind that for the autonomous system, which is discussed here, all the coefficients of equations (40) α_k – are constant numbers. As it is known, a particular solution of linear systems is sought in the form:

$$x_1 = A_1 e^{\lambda t}, \quad x_2 = A_2 e^{\lambda t}, \; \dots , \quad x_n = A_n e^{\lambda t}. \tag{41}$$

We substitute solution (41) into equation (40), and, after grouping the terms, we will have:

$$\begin{aligned} &(a_{11} - \lambda)A_1 + a_{12}A_2 + \ldots + a_{1n}A_n = 0 \; ; \\ &a_{21}A_1 + (a_{22} - \lambda)A_2 + \ldots + a_{2n}A_n = 0 \; ; \\ &\dots \; \dots \; \dots \; \dots \; \dots \; \dots \; \dots \; \dots \; \dots \; \dots \\ &a_{n1}A_1 + a_{n2}A_2 + \ldots + (a_{nn} - \lambda)A_n = 0 \; . \end{aligned} \tag{42}$$

In order the system of algebraic equations (42) had a solution, different from zero, it is necessary that its determinant be zero:

$$\begin{vmatrix} a_{11} - \lambda & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} - \lambda & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} - \lambda \end{vmatrix} = 0 \tag{43}$$

The determiner (43), which is composed for the system (40), is called a characteristic. Expanding this determinant by the elements of the first line, we obtain an equation in relation to λ, which is called a characteristic and contains the unknown λ in the degree n, having roots $(\lambda_1, \lambda_2, \dots, \lambda_n)$.

We formulate the main conditions on the basis of the Lyapunov stability theorems in the first approximation:

1. If the valid parts of all roots of the characteristic equation are negative, then the undisturbed movement is asymptotically stable.
2. If among the roots of the characteristic equation there is at least one root, the valid part of which is positive, then the undisturbed movement is unstable.
3. If the valid parts of some roots of the characteristic equation are zero, and the valid parts of other roots are negative, then the undisturbed movement is stable, but not asymptotically stable.

The presented Lyapunov theorems on the movement stability in the first approximation completely solve the problem of the stability of movement. The assessment of the stability of the movement is also carried out applying the Hurwitz criterion. From the foregoing it is clear that, in order to make a conclusion about the movement stability it is of great importance to know what is the sign of the valid parts of the roots of the characteristic equation, that is, it is important to know the necessary and sufficient conditions under which the roots of the equation have negative valid parts. Such conditions must satisfy the Hurwitz criterion. Such conditions must satisfy the Hurwitz criterion.

Let us open determiner (43) by grouping the terms by powers λ:

$$a_0 \lambda^n + a_1 \lambda^{n-1} + \ldots + a_{n-1}\lambda + a_n = 0. \tag{44}$$

In order to determine the movement stability using the equations of the first approximation, it is necessary to previously know when the valid parts of all the roots of the characteristic equation are negative, without solving the characteristic equation, without calculating its roots. For this it is necessary to construct the Hurwitz matrix from the roots of the characteristic equation $\alpha_0, \alpha_1, \dots, \alpha_n$ (44):

$$\begin{Vmatrix} a_1 & a_3 & a_5 & \dots & 0 \\ a_0 & a_2 & a_4 & \dots & 0 \\ 0 & a_1 & a_3 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & a_n \end{Vmatrix}. \quad (45)$$

We compose from the matrix (45) the main diagonal minors:

$$\Delta_1 = a_1; \ \Delta_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix}; \ \dots ; \ \Delta_n = a_n \Delta_{n-1}. \quad (46)$$

In order all the roots of the characteristic equation (44) had negative valid parts, it is necessary and sufficient that all the main diagonal minors (46) were positive, that is:

$$\Delta_1 > 0; \ \Delta_2 > 0; \ \dots, \ \Delta_{n-1} > 0; \ \Delta_n > 0. \quad (47)$$

Let us discuss further the direct Lyapunov's method. Therefore, it is necessary to compile Lyapunov functions. This method is most suitable to examine the stability of the movement of autonomous systems. The direct or the second Lyapunov's method is characterised by the fact that in its application there is no need to integrate the differential equations of the disturbed movement. This method is connected with a search for some functions V of the disturbance variables $t, \ x_1, x_2, \dots, x_N$, where $x_j = y_j - f_j(t)$ – disturbance, y_j – a partial solution of the disturbed movement, $f_j(t)$ – a partial solution of the undisturbed programmed movement (basis). The method also involves the study of the properties of these functions, which are called Lyapunov functions, and the properties of their derivatives. Let us treat only a steady-state (stationary) movement (autonomous systems), for which $V = V(x_1, x_2, \dots, x_N)$ in the environment $|x_j| < h(j = 1, 2, \dots, N)$, where h is a sufficiently small positive number, considering these functions to be continuously differentiated, unambiguous, and such that they change into zero at the origin of the coordinates $x_{1o} = x_{2o} = \dots = x_{No} = 0$.

In the stability theory the direct method is considered the main one. It is a qualitative method since it does not need any solution of the equations of the movement but studies the properties of the 'test' functions, i.e. Lyapunov functions.

The simplest example of a 'test' function may be an expression of the potential energy of the system with which it is possible to establish stability or instability of equilibrium.

The derivative of the Lyapunov function is determined from the expression:

$$\frac{dV}{dt} = \sum_{j=1}^{N} \frac{\partial V}{\partial x_j} \frac{\partial x_j}{\partial t}. \quad (48)$$

In addition, the Lyapunov functions may have special properties. Function V is called a positively-defined function in the environment $|x_j| < h$ if at any point in this environment, except for the origin of coordinates (where function V is zero), the condition $V > 0$ is fulfilled. If $V < 0$, then function V is called a negatively-defined function. In these two cases, function V is called a sign-definite function. If in this environment $|x_j| < h$ function V acquires the value of only one sign ($V \geq 0$)$\leq$ or $V \leq 0$, but can change into zero not only at the origin of the coordinates, then it is called a sign-fixed (positive or negative) function; if function V acquires both positive and negative values, then it is called an alternating function in this environment.

For example, function $V = x_1^2 - x_2^2$ at $N = 2$ is an alternating function, and function $V = x_1^2 + x_2^2$ is positively-definite, function $V = x_1^2$ is a sign- fixed function since it changes into zero on axis O_{x_2}, but beyond the boundaries of this axis it is positive.

So, if V it is a quadratic form, then definiteness of the sign can be established using the Sylvester criterion. If V is a form of an unpaired degree, then it is clear that it is a sign-alternating function. Consequently, Lyapunov functions are functions of variables $x_1, x_2, \dots, x_N$, each of which in a certain n -measurable region, containing the origin of space coordinates, is a sign-definite, a fixed or an alternating function, and in this region it has continuous first-order partial derivatives of the first order with respect to variables $x_1, x_2, \dots, x_N$, i.e. it has a full differential. The issue about the stability of undisturbed movement is solved on the basis of an investigation of the behaviour of function $V(x_1, x_2, \dots, x_N)$ and its derivatives in time. It should be taken in account that variables $x_1, x_2, \dots, x_N$ are solutions of differential equations of the disturbed movement. The study of the behaviour of function V along the path of the system allows one to make a conclusion about the behaviour of the paths of a mechanical system being investigated, i.e. to solve the problem of the stability or instability of the movement.

Since the issue about the sign-definiteness of a quadratic form is solved quite simply (Sylvester's criterion (8)), when constructing Lyapunov functions, the sign-determined quadratic form is chosen as the basis, adding, if necessary, forms of higher orders. The resulting function will have the same sign-definiteness properties as the original quadratic form.

Theoretical investigation of the movement stability of a model of the self-propelled chassis

Let us investigate by the direct Lyapunov's method the movement stability of a model of the self-propelled chassis with mass m and the inertia moment in relation to the transverse axis that passes through its centre of mass – mr^2, where r is the inertia radius of the chassis body, C_p, C_z – the rigidity coefficients of the frontal and rear springs of the self-propelled chassis (Fig. 3).

We will discuss the longitudinal oscillations of the self-propelled chassis. In the process of oscillations its position is determined by two generalised coordinates: the vertical displacement of the centre of mass (point C) and the turning angle Θ of the frame. The kinetic energy of the self-propelled chassis is:

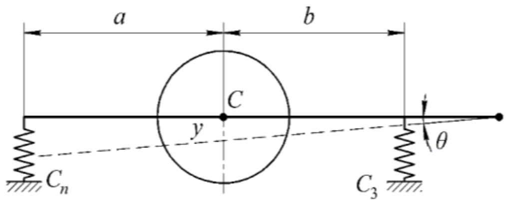

Figure 3. An equivalent scheme of the oscillation pattern of the self-propelled chassis.

$$T = \frac{1}{2} m \cdot \dot{y}^2 + \frac{1}{2} I_c \cdot \dot{\theta}^2 = \frac{1}{2} m \left(\dot{y}^2 + r^2 \cdot \dot{\theta}^2 \right) \tag{49}$$

The potential deformation energy of the chassis wheels:

$$\Pi = C_p \left(y + a \cdot \theta \right)^2 + C_z \left(y - b \cdot \theta \right)^2 . \tag{50}$$

Let us compile the movement equations of the self-propelled chassis using for this the initial equations in the Lagrange form of the II-nd kind:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}}\right)-\frac{\partial T}{\partial y}=-\frac{\partial \Pi}{\partial y} \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right)-\frac{\partial T}{\partial \theta}=-\frac{\partial \Pi}{\partial \theta}. \tag{51}$$

Substituting into equations (51) a derivative from T and Π, we obtain differential equations of the oscillatory movement of the self-propelled chassis:

$$\left.\begin{aligned} &m\ddot{y}+2C_p\left(y+a\cdot\theta\right)+2C_z\left(y-b\cdot\theta\right)=0,\\ &mr^2\ddot{\theta}+2C_p\left(y+a\cdot\theta\right)a+2C_z\left(y-b\cdot\theta\right)b=0.\end{aligned}\right\} \tag{52}$$

We transform differential equations (52) in the Cauchy normal form:

$$\begin{aligned} &\dot{x}_1=x_2,\\ &\dot{x}_2=-\frac{1}{m}\left[2C_p\left(x_1+a\cdot x_3\right)+2C_z\left(x_1-b\cdot x_3\right)\right],\\ &\dot{x}_3=x_4,\\ &\dot{x}_4=-\frac{1}{mr^2}\left(2C_p\left(x_1+a\cdot x_3\right)a+2C_z\left(x_1-b\cdot x_3\right)b\right)\end{aligned} \tag{53}$$

These differential equations (53) are called the disturbed movement equations of the self-propelled chassis.

Let us select a Lyapunov function in the form of full mechanical energy:

$$V=T+\Pi=\frac{1}{2}m\left(\dot{y}^2+r^2\cdot\dot{\theta}^2\right)+C_p\left(y+a\cdot\theta\right)^2+C_z\left(y-b\cdot\theta\right)^2 \tag{54}$$

We write the Lyapunov function in the new variables. We have:

$$V=\frac{1}{2}m\left(x_2^2+r^2\cdot x_4^2\right)+C_p\left(x_1+a\cdot x_3\right)^2+C_z\left(x_1-b\cdot x_3\right)^2 \tag{55}$$

Let us take a full derivative of the Lyapunov function (55) with respect to time. We will have:

$$\frac{dV}{dt}=\frac{\partial V}{\partial x_1}\dot{x}_1+\frac{\partial V}{\partial x_2}\dot{x}_2+\frac{\partial V}{\partial x_3}\dot{x}_3+\frac{\partial V}{\partial x_4}\dot{x}_4. \tag{56}$$

By virtue of the equations of disturbed movement we have $\frac{dV}{dt}=0$. In this case the movement of the self-propelled chassis will be stable.

CONCLUSIONS

1. Since the stability of movement is one of the most important categories in the theoretical research of functioning of various mechanical systems, including the agricultural machines and equipment, finding methods for their efficient use is an important scientific task. Application of the basic assumptions of the classical theory of the movement stability in the analytical research of agricultural machines and aggregates is connected with considerable difficulties when integrating nonlinear differential equations in a closed form. However, it is possible to apply efficiently the criteria of the movement stability in order to evaluate how an agricultural machine will continue to move if it is accidentally subject to external forces that had not been taken into account in the model. The latter is equivalent to a change in the initial conditions on which the pattern of the movement of the agricultural machine or aggregate directly depends.

2. Presented scientific problem has been solved concerning the development of methods and their efficient application in the analytical research of agricultural aggregates, which provides an opportunity to consider the behaviour of a machine without using complex differential equations of the movement in the case of perturbations.

3. Application of various methods of the theory of the movement stability is considered in the research of the movement of a trailed cultivator and a self-propelled agricultural tool frame.

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