The theory of vibrational wave movement in drying grain mixture

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Abstract. This paper outlines a theory that involves the vibrational wave transportation of bulk grain during the course of passing that grain under an infrared radiation source, in a working thermal radiation drying chamber, and using a vibrational wave transporter belt that has been developed by the authors of this paper. The main outstanding feature of the proposed design is the presence of mechanical off-centre vibration drives which generate the vibration in the working rollers at a preset amplitude and frequency, thereby generating a mechanical wave on the surface of the flexible transporter belt which ensures the movement of bulk grain along the processing zone which itself is being subjected to infrared radiation. A calculation method was developed for the oscillation system that is used in conjunction with the vibrational transportation of the grain mass, in order to be able to determine the forces that may be present in the vibrational system and to prepare the differential calculations for the movement of the vibrational drive’s actuators, utilising for this purpose Type II Lagrange equations. The solving of the aforementioned integral equations on a PC yielded a number of graphical dependencies in terms of kinetic and dynamic parameters for the vibrational system described above; the analysis of those dependencies provided a rational structural, along with kinetic and dynamic indicators. According to the results that were taken from theoretical and experimental studies on the functioning of the developed infrared grain dryer combined with a vibrational exciter, stable movement for its working roller takes place if the angular velocity of a drive shaft is changed within the range of between 50–80 rads⁻¹, whereas the amplitude of the indicated oscillations falls within the range of 3.0–4.0 mm. It has been discovered that a rational speed when transporting soy seeds during infrared drying falls between the range of between 0.15–0.60 cm s⁻¹, whereas the amplitude of the indicated oscillations falls within the range of 3.0–4.0 mm. An increase of this parameter within the stated limits increases the time that it takes to achieve the stage in which a constant drying soy speed is reached by more than 2.5 times (from 205 seconds to 520 seconds), stabilising the figure at a level of 520 seconds, which makes it possible to recommend a range of transport speeds of between 0.15–0.40 cm s⁻¹ under infrared radiation for the seeds in order to achieve the required moisture content with a single pass of the produce on the wave transporter.
With that in mind, the power consumption levels for the vibrational exciter do not exceed 50W, while the angular velocity of the drive shaft’s rotation falls within the range of between 100–120 rad s⁻¹. The results of the experimental study that has been conducted indicated that a rational transportation speed for the soy seeds on the wave transporter under infrared radiation is between 0.15–0.40 cm s⁻¹

**Key words:** drying, grain, kinetic and dynamic parameters, model, quality, transportation.

**INTRODUCTION**

The drying of cereal and oil crop seeds holds an important place in the technology that covers their processing and storage. The use of high-performance grain dryers significantly shortens the time needed to prepare the seeds for long-term storage, while also reducing losses, and creating the necessary conditions for further processing.

On the other hand, the preparation of seeds for long-term storage and subsequent sowing requires significant consumption of heat and mechanical energy. There are currently a good many different designs of dryer in use.

The convective dryer, which is widely used worldwide, has significant shortcomings, such as the fact that the evaporation of moisture from the material that is being processed takes place rather slowly. Drying by means of thermal radiation (under an infrared radiator) eliminates that shortcoming thanks to its ensuring the coincidence of the heat beam’s gradient and the material’s moisture content which, together with the high energy absorption levels, makes sure this process has one of the best outcomes of all of the methods to be used when intensifying the **bulk’s heat transfer**.

The only limitation in drying by means of thermal radiation is the transparency of the layer of seeds for the infrared radiation, i.e. the radiation must penetrate the entire layer, for which purpose the layer’s thickness must not exceed a depth of 15mm.

The best method for ensuring the necessary layer thickness of the material that is being dried and its movement along the working area is the use of vibrating technology in the course of the drying process.

But, in our opinion, the best outlook is displayed by thermal radiation dryers with infrared technology, as used in the process of drying grain and seed material. And yet the bulk grain needs to be moving along the drying chamber’s working area while it undergoes the process of infrared irradiation of the bulk.

The drying of agricultural materials, in particular grain, requires constant transportation. Therefore the use of belt, conveyor, screw, or other types of transporter is not entirely suitable. Upon ordinary transportation, bulk grain does not become mixed; in addition, such transporters need more metal and energy to make and operate.

Therefore, nowadays the method used in vibrational transportation is also widely used in the transportation of various bulk materials while drying them. There are a number of vibrational transporter designs in use for this purpose, all of which ensure the effective vibrational transportation of bulk grain. For example, in machines which employ a process of constant vibration, a vibrating belt conveyor is used as the transportation element of the design, providing for guaranteed movement of the bulk material. Despite this popularity, all of the aforementioned designs of vibration equipment have significant shortcomings.
Therefore, we have developed a thermal radiation dryer which employs a somewhat novel design, one which uses a vibrational wave transporter belt that moves the bulk grain through the drying chamber’s working area in which infrared irradiation and therefore the drying of the bulk grain all takes place. It must be noted that the belt conveyor itself remains stationary (not performing a translational movement); the bulk material is shifted only by a mechanical wave that is generated along its elastic belt by means of vibrational exciters.

Figure 1 provides an overall view of the developed thermal radiation dryer and its novel design which uses a vibrational wave transporter belt (a), and a diagram of its technological design (b).

Figure 1. Infrared dryer with a vibrational wave transporter belt: a—overall view; b—technological diagram: 1—frame; 2—elastic elements; 3—rollers; 4—elastic belt; 5—tensioning cylinder; 6—roller axes; 7—vibration dampers; 8—off-centre drive shafts; 9—counterweights; 10—bulk grain; 11—infrared radiator; 12—feeder; 13—receiving hopper.

The main benefits in using this dryer are its simple design, reliable functioning, and low energy consumption for its operation.

The infrared dryer with its vibrational wave transporter belt has the following design features: the frame (1) houses two vibrational exciters in the form of rollers (3) which are installed onto shafts (6) that are connected to the frame (1) via elastic elements (2). These two rollers (3), installed at the same level as each other, are surrounded by an elastic belt (4). The elastic belt has a horizontal section at the top, between two rollers (3), and a tensioning cylinder (5) at the bottom; the cylinder pulls the belt down under gravity due to its own weight, thereby generating preliminary tensioning in the horizontal top part of the belt.

The insides of the rollers (3) are penetrated by the off-centre drive shafts (8), which have attached counterweights (9) on the outside of one of the side elements of the rollers. The off-centre drive shafts are installed on the frame with the aid of vibration dampers (7). This way, the off-centre drive shafts are connected to the frame via the vibration dampers at the bottom and to the frame via axes (6) and elastic elements (2) at the top. The counterweights (9) offset the inertial forces emerging while the rollers work. Upon the off-centre drive shafts (8) within the rotating rollers, each shaft (3) produces vibrational movement that, if certain parameters have been selected regarding frequency and amplitude, makes it possible for each roller to generate a dynamic wave on the surface of the elastic belt (4) (in the form either of a standing or running wave). This makes it possible to transport the seed bulk (10), but also to mix its layers, and to dry the seed bulk in the working area of the infrared radiator (11). Moreover, when varying the
vibration parameters of the two vibrational exciters in the form of the rollers, the speed of movement of the seed bulk that is coming from the feeder (12) in the working area and the intensity of its mixing can both be changed. Thereafter, the dried seed bulk (10) is moved via vibrational waves into the receiving hopper (13) at the transporter’s other end.

In this way, when employing vibrational wave transportation for seed bulk, the bulk is also loosened due to the forces being applied to it by means of their alternating directions, leading to a decrease in the bulk’s internal friction and viscosity, as well as to its uniform thermal treatment.

The infrared dryer that has been developed for bulk grain, using a vibrational transporter, is a combination of a belt transporter (without the translational movement of the transporter belt) and a vibration machine with a combined kinetic manner in which it is able to generate oscillations. It establishes the conditions for constant movement and the simultaneous infrared treatment of the continually-fed seed bulk, ensuring its loosened state and guaranteed its drying. This is accompanied by a reduction of the oscillating mass of the vibration drive.

By means of this method - using the equipment that has been developed and which is now being proposed by us - with its two vibrational exciters, it becomes possible to reduce the oscillating mass of the entire drive and provides the levelling of any unwanted oscillations by means of the incorporated flexible elements. This type of design for a drive unit, together with a wave transporter with a deforming transportation element, makes it possible to provide a significantly improved system for balancing the dryer’s vibrational drive.

Therefore the implementation of the proposed design for a vibrational dryer with its kinetic method of providing vibrational excitation provides a significantly increased intensive process for removing free and physically-bound moisture from the fed seed bulk on account of having creating a pseudo-levitating state of the material that is being processed, and also makes it possible to ensure lower stress levels in the equipment and lower costs, as well as ensuring the conditions for the effective balancing of the oscillation system.

An analysis of various methods being used in the process of drying cereal and oil crops is provided in previous papers (Boyce, 1965; Safarov, 1991; Bruce & Giner, 1993; Malin, 2005; Moroz et al., 2011; Abolins & Upitis, 2012; Hemis et al., 2012; Rudobashta et al., 2016; Gilmore et al., 2019; Giner, 2019; Kliuchnikov, 2019; Rugovskii et al., 2019), where the basis for classifying these methods is the one that involves the transferral of heat to the material that is being dried. The convective dryer, which is widely used worldwide, has significant shortcomings, such as the fact that the evaporation of moisture from the material that is being processed happens rather slowly. Drying by means of thermal radiation (under an infrared radiator) eliminates that shortcoming thanks to its ensuring the coincidence of the heat beam’s gradient and the material’s moisture content which, together with the high energy absorption levels, makes sure this process has one of the best outcomes of all of the methods to be used when intensifying the bulk’s heat transfer.

The method used in drying plant materials utilising short-wave infrared radiation is presented in previous papers (Filonenko & Grishin, 1971; Bezbakh & Bakhmutyan, 2006). The only limitation in drying by means of thermal radiation is the transparency of the layer of seeds for the infrared radiation, i.e. the radiation must penetrate the entire layer, for which purposes the layer’s thickness must not exceed a depth of 15mm.
The best method for ensuring the necessary layer thickness of the material that is being dried and its movement along the working area is the use of vibrating technology in the course of the drying process.

The problem of creating a high-performance drying unit is discussed in previous papers (Goncharevich & Frolov, 1981; Grochowski et al., 2004; El Hor et al., 2005; Palamarchuk et al., 2018), which present the mathematical model for the drying processes, along with the results of experimental studies, the design diagrams for wave-based and vibrational transportation equipment, and proof that one of the most effective methods of intensifying the processes that have been studied is the use of a vibrational field.

The goal of the paper is to increase the productivity and quality in terms of drying the grain and seeds of various crops by means of developing and scientifically reasoning out the rational parameters of a new type of thermal radiation dryer.

**MATERIALS AND METHODS**

For a theoretical description of the vibrational wave-based movement of bulk grain in the dryer that has been developed by us, it is first necessary to discuss the functioning of the kinetic vibrational exciter which serves to generate the oscillations in an elastic belt, and which effects the process of wave-based movement in the bulk grain that is fed onto it from one end.

First of all, based on the technological design diagram for the thermal radiation dryer with its vibrational transporter belt (Fig. 1), and on the description of its functioning that has been provided above, it is necessary to develop a calculation diagram for the kinetic vibrational exciter that is to be used with the elastic transporter belt’s oscillations in its initial position (in a balanced state) and, thereafter, in any of its working positions, i.e. when offsetting the work roller’s centre of mass from its balanced state. The relevant calculation diagrams are presented in Figs 2 and 3.

The diagram in Fig. 2 indicates these parameters for the oscillating system: $m_1$ – the drive shaft’s mass; $m_2$ – is the roller’s mass; $m_3$ – is the counterweight’s mass; $C_1$ – is the elastic support’s rigidity levels; $C$ – is the vibration damper’s rigidity levels; $e$ – the offset from centre, which is defined by distance $l_{12} = O_1O_2$; $l_{13} = O_1O_3$ – the distance from a drive shaft (point $O_1$) to a counterweight’s axis (point $O_3$).

For further study of the movement of the oscillating system being discussed, we shall establish an equivalent diagram which indicates the offset for a roller’s centre of mass from the balanced state when carrying out
any oscillating movements. For this purpose, we shall select and display on the equivalent diagram a fixed Cartesian coordinate system \(O_{2xy}\), the starting point of which is \(O\); which is located at the roller’s centre (in its initial position), while the axis \(O_{2x}\) is directed horizontally towards the right, and the axis \(O_{2y}\) is directed vertically towards the upwards position.

The equivalent diagram in Fig. 3 displays the following indicators:
- \(x_1\) — linear horizontal offset for a drive shaft’s centre of mass from the balance position;
- \(y_1\) — linear vertical offset for a drive shaft’s centre of mass from the balance position;
- \(\phi_1\) — angle of rotation for the vibrational exciter’s drive shaft from the balance position;
- \(x_i\), \(y_i\) — coordinates for point \(O_i\) when attaching the left-hand end of the belt to the left-hand elastic support in a random position; \(x_r\), \(y_r\) — coordinates for point \(O_r\) when attaching the right-hand end of the belt to the right-hand elastic support in a random position.

Furthermore, we shall create the differential equations for the movement of the oscillating system being discussed, utilising Type II Lagrange equations for this very purpose.

Within this context it has to be noted that the aforementioned oscillating system is characterised by three degrees of freedom: the linear shifts \(x_1\), \(y_1\) of a drive shaft’s centre of mass (point \(O_1\)) in relation to the axes of the coordinates \(O_{2x}\) and \(O_{2y}\), respectively, and the rotational angle \(\phi_1\) of the vibrational exciter’s drive shaft (measured from the balance position).

**RESULTS AND DISCUSSION**

It must also be noted that the oscillating system incorporates three masses:
- \(m_1\) — a drive shaft’s mass;
- \(m_2\) — a roller’s mass;
- \(m_3\) — a counterweight’s mass.

To be able to establish the differential equations of movement by means of the aforementioned method, we shall first determine the kinetic energy of the oscillating system.

It is evident that the summary kinetic energy \(T\) of the oscillating system equals:

\[
T = T_1 + T_2 + T_3,
\]

(1)

where \(T_1\) — is the kinetic energy of the translational movement of a drive shaft; \(T_2\) — is the kinetic energy of the parallel plane movement of a roller; \(T_3\) — is the kinetic energy of the parallel plane movement of a counterweight.

Within that context, the kinetic energy \(T_1\) equals:
where \( m_1 \) – is a drive shaft’s mass; \( V_1 \) – is a drive shaft’s translational movement’s speed.

The kinetic energy \( T_2 \) of a roller’s parallel plane movement is determined as the kinetic energy of the module \( O_1O_2 \), namely:

\[
T_2 = \frac{m_2 V_2^2}{2} + \frac{I_2 \omega_1^2}{2},
\]

where \( m_2 \) – is the mass of a roller (module \( O_1O_2 \)); \( V_2 \) – is the speed of the translational movement of a roller (module \( O_1O_2 \)); \( \omega_1 \) – is the angular velocity of a drive shaft; \( I_2 \) – is the moment of inertia in a roller (module \( O_1O_2 \)) in relation to the point \( O_1 \).

The kinetic energy \( T_3 \) is determined as the kinetic energy of the module \( O_1O_3 \) in its parallel plane of movement:

\[
T_3 = \frac{m_3 V_3^2}{2} + \frac{I_3 \omega_1^2}{2},
\]

where \( m_3 \) – is the mass of a counterweight (module \( O_1O_3 \)); \( V_3 \) – is the translational movement’s speed of a counterweight (module \( O_1O_3 \)); \( \omega_1 \) – is the angular velocity of a drive shaft; \( I_3 \) – is the moment of inertia of a counterweight (module \( O_1O_3 \)) in relation to the point \( O_1 \).

Furthermore, we shall determine the kinetic characteristics (linear and angular velocities) of the points and modules of the moving masses \( m_1, m_2 \), and \( m_3 \).

It is evident that, within the coordinate system of \( O_{2xy} \), the square of speed \( V_1 \) is:

\[
V_1^2 = \dot{x}_1^2 + \dot{y}_1^2
\]

To determine the speeds \( V_2 \) and \( V_3 \), we shall establish calculation diagrams for the modules \( O_1O_2 \) and \( O_1O_3 \), making parallel plane movements. These calculation diagrams are provided in Fig. 4.

Figure 4. Calculation diagrams to determine the speeds of the oscillating system’s modules: a) module \( O_1O_2 \); b) module \( O_1O_3 \).

As the provided calculation diagrams indicate, the following expressions are yielded for the speed \( V_2 \). The vector form of this is:

\[
\vec{V}_2 = \vec{V}_1 + \vec{V}_{21}
\]

Considering that the value of the speed \( V_{21} \) equals:

\[
V_{21} = I_{21} \cdot \dot{\phi}_1
\]
and after a number of translations, we get to the following:

\[ V_1^2 = V_i^2 + 2l_{12} \cdot \phi \left( 0.5 \cdot l_{12} \cdot \phi_i - \dot{x}_i \cos \phi_i - \dot{y}_i \sin \phi_i \right). \]  

(8)

Similarly, from the calculation diagram in Fig. 4, the expression for the speed \( V_3 \) is ultimately:

\[ V_3^2 = V_i^2 + 2l_{13} \cdot \phi_i \left( 0.5 \cdot l_{13} \cdot \phi_i + \dot{x}_i \cos \phi_i + \dot{y}_i \sin \phi_i \right) \]  

(9)

Furthermore, we shall determine the moments of inertias \( I_1 \) and \( I_2 \) of the modules \( O_1O_2 \) and \( O_1O_3 \), in relation to a drive shaft’s axis (point \( O_1 \)), which are, respectively:

\[ I_2 = m_s \cdot l_{12}^2 \]  

(10)

\[ I_s = m_s \cdot l_{13}^2 \]  

(11)

Inserting the expressions (5), (8) – (11) into the expressions (2), (3), (4), we get the following expressions for the relevant kinetic energies:

\[ T_1 = \frac{m_s (\dot{x} + \dot{x}^2)}{2} \]  

(12)

\[ T_2 = \frac{m_s}{2} \left[ \dot{x}^2 + \dot{y}^2 + 2l_{12} \cdot \phi \left( 0.5 \cdot l_{12} \cdot \phi_i - \dot{x}_i \cos \phi_i - \dot{y}_i \sin \phi_i \right) \right] \]  

(13)

and

\[ T_3 = \frac{m_s}{2} \left[ \dot{x}^2 + \dot{y}^2 + 2l_{13} \cdot \phi_i \left( 0.5 \cdot l_{13} \cdot \phi_i + \dot{x}_i \cos \phi_i + \dot{y}_i \sin \phi_i \right) \right] \]  

(14)

Inserting the expressions (12), (13) and (14) into the expression (1), we get the expression for determining the summary kinetic energy of the oscillating system:

\[ T = 0.5 \cdot \left( m_1 + m_2 + m_3 \right) \cdot \left( \dot{x}^2 + \dot{y}^2 \right) + m_2 \cdot l_{12} \cdot \phi \left( 0.5 \cdot l_{12} \cdot \phi_i - \dot{x}_i \cos \phi_i - \dot{y}_i \sin \phi_i \right) + m_3 \cdot l_{13} \cdot \phi_i \left( 0.5 \cdot l_{13} \cdot \phi_i + \dot{x}_i \cos \phi_i + \dot{y}_i \sin \phi_i \right) + 0.5 \left( m_2 \cdot l_{12}^2 + m_3 \cdot l_{13}^2 \right) \cdot \phi_i^2. \]  

(15)

To determine the generalised forces that are included on the right-hand side of the Type II Lagrange equation, it is necessary to establish an equivalent diagram of forces for the oscillating system that is being studied (Fig. 5).

This equivalent diagram indicates the following forces and inertias:

\( \vec{G}_1 \) – the force of a drive shaft’s weight;

\( \vec{G}_2 \) – the force of a working roller’s weight;

\( \vec{G}_3 \) – the force of a counterweight’s weight;

\( \vec{S}_1 \) – the tensioning force of the elastic belt’s left-hand section;

\( \vec{S}_2 \) – the tensioning force of the elastic belt’s right-hand section;

\( \vec{M}_T \) – the torque moment in a drive shaft.

Figure 5. An equivalent diagram of forces in the oscillating system that is being studied.
Additionally, the diagram indicates:
- \( \varphi_1 \) – the turning angle of a drive shaft;
- \( \beta_1 \) – the angle between the elastic belt’s left-hand section and the vertical;
- \( \beta_2 \) – the angle between the elastic belt’s right-hand section and the vertical.

Furthermore, we shall enter the expression that will be used to determine all forces and moments indicated into the equivalent diagram of Fig. 5:

\[ G_i = m_i \cdot g \cdot \]  \hspace{0.5cm} (16)

where \( m_1 \) – a drive shaft’s mass; \( g \) – gravitational acceleration;

\[ G_i = m_i \cdot g \cdot \]  \hspace{0.5cm} (17)

where \( m_2 \) – is a roller’s mass;

\[ G_i = m_i \cdot g \cdot \]  \hspace{0.5cm} (18)

where \( m_3 \) – is a counterweight’s mass.

The belt’s tensioning forces \( S_1 \) and \( S_2 \) are equivalent to the elastic forces occurring upon the deformation of the elastic elements; therefore we shall determine them as follows:

\[ S_1 = C_1 \cdot \Delta l_1 \]  \hspace{0.5cm} (19)

\[ S_2 = C_1 \cdot \Delta l_2 \]  \hspace{0.5cm} (20)

where \( C_1 \) – is an elastic support’s levels of ‘hardness’; \( \Delta l_1 \) and \( \Delta l_2 \) – is the linear deformations of the left-hand and right-hand elastic support respectively.

The deformations \( \Delta l_1 \) and \( \Delta l_2 \) in the elastic elements are determined on the basis of a geometric analysis in the oscillating system.

As a result:

\[ S_1 = C_1 \left( \sqrt{(x_1 - x_i - l_{12} \cdot \sin \varphi_1)^2 + (y_1 - y_i - l_{12} \cdot \cos \varphi_1)^2} - R \right) - \]  \hspace{0.5cm} (21)

\[ -\sqrt{x_1^2 + y_1^2 - R^2} \],

and

\[ S_2 = C_1 \left( \sqrt{(x_1 - x_i + l_{12} \cdot \sin \varphi_1)^2 + (y_1 - y_i - l_{12} \cdot \cos \varphi_1)^2} - R \right) - \]  \hspace{0.5cm} (22)

\[ -\sqrt{x_1^2 + y_1^2 - R^2} \],

where \( R \) – is a roller’s radius.

Next we shall determine the generalised forces in the oscillating system, corresponding to each of the independent generalised coordinates.

For the generalised coordinate \( \varphi_1 \), the generalised force \( Q_{\varphi} \) is equal to the arithmetic sum of the moments of all forces in relation to the point \( O_1 \), namely:

\[ Q_{\varphi} = M_{\varphi} - (G_2 \cdot l_{12} - G_3 \cdot l_{13}) \cos \varphi_1 + S_1 (R + l_{12}) \cdot \sin (\varphi_1 + \beta_1) - \]  \hspace{0.5cm} (23)

\[ -S_2 (R + l_{12}) \cdot \sin (\varphi_1 - \beta_2) . \]

For the generalised coordinate \( x_1 \), the generalised force \( Q_{x1} \) is equal to the arithmetic sum of the projections of all forces indicated in Fig. 5 on the axis \( O_{2x} \), namely:

\[ Q_{x1} = S_1 \cdot \sin \beta_1 - S_2 \cdot \sin \beta_2 - C_1 \cdot x_1 \]  \hspace{0.5cm} (24)

where \( C_1 \) – is a vibration damper’s hardness in the direction of the axis \( O_{2x} \), and \( x_1 \) – is a vibration damper’s linear deformation in the direction of the axis \( O_{2x} \).
For the generalised coordinate \( y_1 \), the generalised force \( Q_y \) is equal to the arithmetic sum of the projections of all forces indicated in Fig. 5 on the axis \( O_2 y \), namely:

\[
Q_y = S_1 \cdot \cos \beta_1 + S_2 \cdot \cos \beta_2 - C_y \cdot (y_1 - \delta y) - G_1 - G_2 - G_3
\]

(25)

where \( C_y \) is a vibration damper's levels of ‘hardness’ in the direction of the axis \( O_2 y \); \( y_1 \) is a vibration damper’s linear deformation in the direction of the axis \( O_2 y \); and \( \delta y \) is a vibration damper's static deformation.

In this way, the generalised forces are determined for each generalised coordinate of this oscillating system.

In addition, we shall determine the necessary partial differentiations of the kinetic energy \( T \) in this oscillating system which corresponds to the conditions that are contained in the left-hand sections of the Type II Lagrange equations. We get the following expression for the partial differentiation of the generalised coordinate \( \phi_1 \):

\[
\frac{\partial T}{\partial \phi_1} = m_1 \cdot l_{12} \cdot (0.5 \cdot l_{12} \cdot \ddot{\phi}_1 - \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi) + 0.5 \cdot m_2 \cdot l_{12} \cdot \dot{\phi}_1 + m_1 \cdot l_{13} \cdot \dot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi) + 0.5 \cdot m_3 \cdot l_{13} \cdot \dddot{\phi}_1 + \left\{m_2 \cdot l_{13}^2 + m_3 \cdot l_{13}^2 \right\} \dot{\phi}_1
\]

(26)

also:

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \phi_1} \right) = m_1 \cdot l_{12} \cdot (0.5 \cdot l_{12} \cdot \dddot{\phi}_1 - \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi \cdot \dot{\phi}_1 - \dddot{y} \cdot \sin \varphi \cdot \dot{\phi}_1 - \dddot{y} \cdot \cos \varphi \cdot \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi \\
+ m_3 \cdot l_{13} \cdot \dddot{\phi}_1 + \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi \cdot \dot{\phi}_1 + \dddot{y} \cdot \cos \varphi \cdot \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi \\
+ \dddot{x} \cdot \dddot{y} \cdot \cos \varphi \cdot \phi_1 + 0.5 \cdot m_2 \cdot l_{13} \cdot \dddot{\phi}_1 + \left\{m_2 \cdot l_{13}^2 + m_3 \cdot l_{13}^2 \right\} \dddot{\phi}_1 = 0.5 \cdot m_2 \cdot l_{13}^2 + m_1 \cdot l_{13} \cdot \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi \cdot \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi \cdot \dot{\phi}_1 + 0.5 \cdot m_2 \cdot \dddot{x} \cdot \cos \varphi \cdot \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi \cdot \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi \\
+ \dddot{x} \cdot \dddot{y} \cdot \cos \varphi \cdot \phi_1 + 0.5 \cdot m_2 \cdot l_{13} \cdot \dddot{\phi}_1 + \left\{m_2 \cdot l_{13}^2 + m_3 \cdot l_{13}^2 \right\} \dddot{\phi}_1 = 2 \left(m_2 \cdot l_{13}^2 + m_3 \cdot l_{13}^2 \right) \dddot{\phi}_1 + m_1 \cdot l_{13} \cdot \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi \cdot \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi \cdot \dot{\phi}_1 + 0.5 \cdot m_2 \cdot \dddot{x} \cdot \cos \varphi \cdot \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi \cdot \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi \\
- \dddot{x} \cdot \dddot{y} \cdot \cos \varphi \cdot \phi_1 + \dddot{x} \cdot \dddot{y} \cdot \cos \varphi \cdot \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi \cdot \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi \cdot \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi \\
+ \dddot{x} \cdot \dddot{y} \cdot \cos \varphi \cdot \phi_1 + \dddot{x} \cdot \dddot{y} \cdot \cos \varphi \cdot \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi \cdot \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi \cdot \dddot{x} \cdot \cos \varphi + \dddot{y} \cdot \sin \varphi

\]

(27)

The partial differentiation of the kinetic energy of the generalised coordinate \( \phi_1 \) is:

\[
\frac{\partial T}{\partial \phi_1} = m_1 \cdot l_{12} \cdot \dddot{\phi}_1 \left( \dddot{x} \cdot \sin \varphi - \dddot{y} \cdot \cos \varphi \right) + m_1 \cdot l_{13} \cdot \dddot{\phi}_1 \left( -\dddot{x} \cdot \sin \varphi + \dddot{y} \cdot \cos \varphi \right)
\]

(28)

For the generalised coordinate \( x \), we get the following expressions of partial differentiations in the oscillating system’s kinetic energy \( T \):
\[
\frac{\partial T}{\partial \dot{x}_i} = (m_1 + m_2 + m_3) \cdot \dot{x}_1 - m_2 \cdot l_{12} \cdot \dot{\phi}_1 \cdot \cos \phi_1 + m_3 \cdot l_{13} \cdot \dot{\phi}_1 \cdot \cos \phi_1 \quad (29)
\]
also:
\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{x}_1} \right) = (m_1 + m_2 + m_3) \cdot \ddot{x}_1 - m_2 \cdot l_{12} \cdot \ddot{\phi}_1 \cdot \cos \phi_1 +
\]
\[
+ m_2 \cdot l_{12} \cdot \sin \phi_1 \cdot \dot{\phi}_1 \cdot \dot{\phi}_1 + m_1 \cdot l_{13} \cdot \dot{\phi}_1 \cdot \cos \phi_1 - m_2 \cdot l_{12} \cdot \dot{\phi}_1 \cdot \sin \phi_1 ;
\]
and finally:
\[
\frac{\partial T}{\partial \dot{\phi}_i} = 0 \quad (31)
\]

For the generalised coordinate \( \gamma_1 \), we get the following expressions of partial differentiations in the kinetic energy \( T \):
\[
\frac{\partial T}{\partial \dot{\gamma}_1} = (m_1 + m_2 + m_3) \cdot \dot{\gamma}_1 - m_2 \cdot l_{12} \cdot \dot{\phi}_1 \cdot \sin \phi_1 + m_3 \cdot l_{13} \cdot \dot{\phi}_1 \cdot \sin \phi_1 \quad (32)
\]
also:
\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\gamma}_1} \right) = (m_1 + m_2 + m_3) \cdot \ddot{\gamma}_1 - m_2 \cdot l_{12} \cdot \ddot{\phi}_1 \cdot \sin \phi_1 -
\]
\[
-m_2 \cdot l_{12} \cdot \cos \phi_1 \cdot \dot{\phi}_1 + m_1 \cdot l_{13} \cdot \dot{\phi}_1 \cdot \sin \phi_1 + m_2 \cdot l_{12} \cdot \dot{\phi}_1 \cdot \cos \phi_1 ;
\]
and finally:
\[
\frac{\partial T}{\partial \dot{\phi}_1} = 0 ;
\]

When inserting the resulting expressions (23), (24), (25), (27), (28), (30), (31), (33) and (34) into the initial Type II Lagrange equation for each generalised coordinate and performing the required translations, we get the following system of differential equations for the oscillating system’s movement:
\[
\begin{align*}
2 \left( m_2 \cdot l_{12}^2 + m_3 \cdot l_{13}^2 \right) \ddot{\phi} &= -m_2 \cdot l_{13} \cdot (\ddot{x}_1 - \cos \phi_1 \cdot \dot{\phi}_1 + \sin \phi_1 \cdot \dot{\phi}_1 -
\]
\[
- \ddot{\gamma}_1 \cdot \sin \phi_1 - \dot{\gamma}_1 \cdot \cos \phi_1 \cdot \dot{\phi}_1 ) - m_1 \cdot l_{13} \cdot (\ddot{x}_1 \cdot \cos \phi_1 - \dot{\phi}_1 \cdot \sin \phi_1 \cdot \dot{\phi}_1 ) +
\]
\[
+ \ddot{\gamma}_1 \cdot \sin \phi_1 + \dot{\gamma}_1 \cdot \cos \phi_1 \cdot \dot{\phi}_1 + m_1 \cdot l_{13} \cdot \dot{\phi}_1 \cdot (\ddot{x}_1 + \cos \phi_1 + \ddot{\phi}_1 \cdot \sin \phi_1 ) +
\]
\[
+ m_2 \cdot l_{12} \cdot \dot{\phi}_1 \cdot (\ddot{x}_1 \cdot \sin \phi_1 + \dot{\phi}_1 \cdot \cos \phi_1 ) + M - (m_2 \cdot l_{12} -
\]
\[
-m_3 \cdot l_{13} \cdot g \cdot \cos \phi_1 + S_1 \cdot (\ddot{R} + l_{12}) \cdot \sin \left( \phi_1 + \beta_1 \right) - S_2 \cdot (\ddot{R} + l_{12}) \cdot \sin \left( \phi_1 - \beta_1 \right),
\end{align*}
\]
\[
\begin{align*}
(m_1 + m_2 + m_3) \ddot{x}_1 &= m_2 \cdot l_{12} \cdot \ddot{\phi}_1 \cdot \cos \phi_1 - m_2 \cdot l_{13} \cdot \sin \phi_1 \cdot \dot{\phi}_1 -
\]
\[
- m_3 \cdot l_{13} \cdot \dot{\phi}_1 \cdot \cos \phi_1 + m_1 \cdot l_{13} \cdot \dot{\phi}_1 \cdot \sin \phi_1 + S_1 \cdot \sin \beta_1 + S_2 \cdot r \cdot \cos \beta_1 - C \cdot x_1 ,
\end{align*}
\]
\[
\begin{align*}
(m_1 + m_2 + m_3) \ddot{\gamma}_1 &= m_2 \cdot l_{12} \cdot \ddot{\phi}_1 \cdot \sin \phi_1 + m_2 \cdot l_{12} \cdot \cos \phi_1 \cdot \dot{\phi}_1 -
\]
\[
- m_3 \cdot l_{13} \cdot \dot{\phi}_1 \cdot \sin \phi_1 - m_1 \cdot l_{13} \cdot \cos \phi_1 \cdot \dot{\phi}_1 + S_1 \cdot \cos \beta_1 + S_2 \cdot \cos \beta_1 -
\]
\[
- C \cdot (\gamma_1 - \delta_\alpha ) - (m_1 + m_2 + m_3) g .
\end{align*}
\]
This system of differential equations (35) is one that involves non-linear differential equations of Type II, and can be solved using digital methods on a PC such as, for example, with the MathCAD software.

The initial conditions for the system of differential equations are as follows:
If $t = 0$:

$$x_{10} = l_{12}, \; y_0 = 0, \; \dot{x}_{10} = 0, \; \dot{y}_{10} = 0, \; \text{and} \; \varphi_{10} = 0 \quad (36)$$

For digital calculations in the MathCAD software, the following values were set up for the constant parameters that were included in the system for differential equations (35). Based on the calculations that have been recorded during the design and subsequent manufacture of the experimental specimen of the infrared dryer, but also based on the results of testing and redesigning the dryer, the following values have been taken for constant parameters:

- a drive shaft’s mass $m_1 = 5.4 \text{ kg}$;
- a roller’s mass $m_2 = 6.2 \text{ kg}$;
- a counterweight’s mass $m_3 = 1.2 \text{ kg}$;
- the distance from a drive shaft’s axis to a roller’s axis (the offset rate) $l_{12} = 0.003 \text{ m}$;
- the distance from a drive shaft’s axis to a counterweight’s axis is determined by this condition. From $m_3 \cdot l_{13} = m_1 \cdot l_{12}$, the following results:

$$l_{13} = \frac{m_1 \cdot l_{12}}{m_3} = \frac{5.4 \cdot 0.003}{1.2} = 0.0135 \text{ m}.$$  

On the basis of our designing, manufacturing, and testing the new type of vibrational dryer with its vibrational wave transporter belt, which confirmed its functioning, we also take the following values as constants for the following parameters:
- a vibration damper’s ‘hardness’ in the direction of the axis $O_{3x}$: $C_x = 9,800 \text{ N m}^{-1}$
- a vibration damper’s ‘hardness’ in the direction of the axis $O_{3y}$: $C_y = 31,000 \text{ N m}^{-1}$;
- a drive shaft’s rotational moment $M_r = 4.2 \text{ kNm}$.

The digital calculations for the system of differential equations as carried out on a PC (35) provided dependencies for the movement (trajectory) of the rollers along the axis of the coordinates $O_{3x}$ and $O_{3y}$, as depicted in Fig. 6.

Figure 6. The trajectories of a roller’s centre of mass along the coordinates $O_{3x}$, $O_{3y}$, depending on the angular velocity $\omega_1$.  

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We also used digital modelling on a PC to get the dependence of the excitation force $F_d$ on the angular velocity $\omega_1$ (Fig. 7) and also the dependence of the power consumption $N$ on the angular velocity $\omega_1$ (Fig. 8).

![Figure 7. The dependence of the excitation force $F_d$ on the angular velocity $\omega_1$.](image)

As the graphs in Fig. 6 indicate, a stable oscillating movement in a roller takes place when changing a drive shaft’s angular speed $\omega_1$ within the range of 100–120 rad s$^{-1}$. With that occurring, a roller’s oscillation amplitude changes within the limits of 1.0–4.0·10$^{-3}$ m in the direction of the axis $O_2x$ and within the limits of 1.0–2.0·10$^{-3}$ m in the direction of the axis $O_2y$, which ensures the generation of a necessary wave on the elastic belt’s surface, i.e., the necessary vibrational wave-type movement of the bulk grain along the elastic belt’s surface, simultaneously with the bulk grain being mixed under the infrared radiator that is drying the bulk grain.

The graphical dependencies pictured in Fig. 7 indicate that the value of the excitation force $F_d$ starts increasing significantly (going above 1·10$^3$ N) if a drive shaft’s angular velocity $\omega_1$ is increased to 220 rad s$^{-1}$. At a drive shaft’s angular velocity $\omega_1$ reaching 120 rad s$^{-1}$, the value of the aforementioned force does not exceed 350 N (0.35 kN). Therefore, when considering the reliability of the vibrational exciter, it is not recommended that the angular velocity $\omega_1$ be increased above 120 rad s$^{-1}$, because the power consumption level may increase to infinity.

Moreover, the graphs in Fig. 8 indicate that, upon increasing a drive shaft’s angular velocity $\omega_1$ to 120 rad s$^{-1}$, the power consumption level of the vibrational exciter reached 50 W, which is more acceptable in a situation which requires minimum energy spend in return for the high quality performance of the technological process.

Therefore, all of the graphs that have been discussed above indicate that the rational values for the angular velocity $\omega_1$ of the vibrational exciter’s drive shaft must fall within the range of 100–120 rad s$^{-1}$.

We also carried out experimental studies to investigate the process involved in the infrared drying of soy seeds on a vibrational transporter that had been developed by us.

An experimental installation was established on the basis of that concept, and a diagram of it is provided in Fig. 9. The operating principle of the experimental installation is similar to that of the industrial model of an infrared dryer with a novel design, incorporating into it a vibrational belt transporter.
In the course of the experimental studies, portions of soy seeds with fixed moisture parameters were taken out at moisture levels of between 0.1% to 7.0%. The speed at which the bulk grain was being transported was also set at levels of between 0.15 to 0.6 cm·s⁻¹. The time spent by each portion of the seeds under the infrared lamps was recorded. The data received from this enabled us to determine the drying speed (the speed at which moisture is removed). For each portion of the seeds and to ensure individually-noted transportation speed readings, triple tests were carried out. The measurement and calculation results were processed with specially developed software on a PC.

Based on the results of the experimental studies, the speed at which the soy seed can be dried in the aforementioned dryer was all assessed. The experimental data was used to prepare a table which indicated the data of the infrared drying speed (removing moisture from the dried bulk grain over time) at various speeds of transporting the bulk grain on the vibrational belt and at various levels of moisture content in the bulk grain (Table 1).

The details given in the table indicates that the speed of infrared drying (moisture removal) for bulk seed depends upon both its initial moisture content and the speed of its vibrational transport. Within that context, the drying time for the bulk grain also changes. At a constant speed of vibrational transport for bulk grain, and with an increase of its initial relative moisture content, the time actually lengthens for the stage at which a constant drying speed can be achieved for drying soy seeds. As can be seen from the table above, for a vibrational transportation speed of 0.15 cm·s⁻¹, when the relative moisture content changes from 0.70% to 6.80%, the time needed to attain a constant speed of 12.1–13.2·10⁻³ %s⁻¹ in terms of drying the bulk grain changes from 85 s to 520 s.

**Figure 9.** A diagram of an experimental installation in order to study the process of the infrared drying of soy seeds: 1) elastic belt; 2) rollers; 3) roller drive; 4) tensioning roller; 5) feeder hopper; 6) collector for dried seeds; 7) infrared radiator; 8) soy seeds.

<table>
<thead>
<tr>
<th>Specific moisture content of the seeds, ( \Delta W ), %</th>
<th>Speed of vibrational transportation of the seeds, ( V ), cm·s⁻¹</th>
<th>Drying time, ( \tau ), s</th>
<th>Speed at which moisture is removed, ( dW/(dt) ), %·s⁻¹·10⁻³</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.70</td>
<td>0.15</td>
<td>85</td>
<td>8.24</td>
</tr>
<tr>
<td>2.70</td>
<td>0.15</td>
<td>205</td>
<td>13.17</td>
</tr>
<tr>
<td>4.60</td>
<td>0.15</td>
<td>380</td>
<td>12.11</td>
</tr>
<tr>
<td>6.80</td>
<td>0.15</td>
<td>520</td>
<td>13.10</td>
</tr>
<tr>
<td>0.10</td>
<td>0.40</td>
<td>45</td>
<td>2.22</td>
</tr>
<tr>
<td>0.30</td>
<td>0.40</td>
<td>84</td>
<td>7.97</td>
</tr>
<tr>
<td>1.70</td>
<td>0.40</td>
<td>160</td>
<td>10.63</td>
</tr>
<tr>
<td>2.90</td>
<td>0.40</td>
<td>235</td>
<td>12.34</td>
</tr>
<tr>
<td>0.15</td>
<td>0.60</td>
<td>50</td>
<td>3.00</td>
</tr>
<tr>
<td>0.58</td>
<td>0.60</td>
<td>95</td>
<td>6.10</td>
</tr>
<tr>
<td>1.20</td>
<td>0.60</td>
<td>140</td>
<td>8.57</td>
</tr>
<tr>
<td>2.40</td>
<td>0.60</td>
<td>320</td>
<td>7.50</td>
</tr>
</tbody>
</table>
For a vibrational transportation speed of 0.40 cm s\(^{-1}\), when changing the relative moisture level from 0.10\% to 2.9\% and when the drying speed stabilises at the level of \(12.3 \cdot 10^{-3}\) \(\% \cdot s^{-1}\), the drying time for the soy seeds changes from 45 s to 235 s.

And finally, at a vibrational transportation speed of 0.60 cm s\(^{-1}\) and with the drying time as set out in the table, the drying speed for soy seeds does not exceed \(8.6 \cdot 10^{-3}\) \(\% \cdot s^{-1}\), i.e. a drying speed level of \(12.1 \cdot 10^{-3}\) \(\% \cdot s^{-1}\) is not attained in this case.

Therefore, based on the data gained from the experimental studies that have been carried out here, a rational speed for the vibrational transportation of soy seeds in infrared drying falls within the range of between 0.15 to 0.40 cm s\(^{-1}\).

CONCLUSIONS

1. A thermal radiation dryer of a novel design has been developed using a vibrational wave transport element which generates a mechanical wave on the surface of the flexible transporter belt, which itself ensures the movement of bulk grain along the processing zone that is currently being treated with infrared radiation.

2. On the basis of the calculation diagrams that have been developed, a mathematical model was prepared of the vibrational process that was the subject of the study, namely involving the use of Type II Lagrange equations which resulted in a system of differential equations for the movement of the vibrational exciter with its novel design, as proposed by the authors.

3. The solving of the aforementioned integral equations on a PC yielded a number of graphical dependencies for the kinetic and dynamic parameters of the vibrational system that has been described above.

4. It has been demonstrated that the stable movement of a working roller takes place if the angular velocity \(\omega_1\) is changed within the range of 100–120 rad s\(^{-1}\), while the rational amplitude of the vibrational exciter’s oscillations does not exceed 4 mm.

5. The fact has been identified that a rational speed for transporting soy seeds during infrared drying falls between the range of 0.15–0.60 cm s\(^{-1}\). This parameter within the stated limits increases the time taken until the stage of constant drying speed can be achieved for soy by more than 2.5 times, from 205 s to 520 s, stabilising at a level of 520 s.

6. At a drive shaft’s angular velocity \(\omega_1\) being 120 rad s\(^{-1}\), the value of the excitation force \(F_d\) does not exceed 0.35 kN but, when it does actually exceed it, the excitation force starts increasing significantly.

7. When increasing a drive shaft’s angular velocity \(\omega_1\) to 120 rad s\(^{-1}\), the power consumption of the vibrational exciter reached 50W, which again confirms the finding that the rational range when it comes to changing a drive shaft’s angular velocity falls between 100–120 rad s\(^{-1}\).

8. As shown by the results of the experimental studies that have been conducted, a rational speed for the vibrational transportation of soy seeds in infrared drying falls between 0.15 to 0.40 cm s\(^{-1}\).

REFERENCES


